



SNS COLLEGE OF TECHNOLOGY

Coimbatore-27
An Autonomous Institution



Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT 2 – TIME RESPONSE ANALYSIS

TOPIC 2- IMPULSE AND STEP RESPONSE ANALYSIS OF FIRST ORDER SYSTEMS



OUTLINE



- REVIEW ABOUT PREVIOUS CLASS
- RELATION BETWEEN STANDARD TEST SIGNALS
- LAPLACE TRANSFORM OF TEST SIGNALS
- TIME RESPONSE OF CONTROL SYSTEMS
- INTRODUCTION- FIRST ORDER SYSTEM
- IMPULSE RESPONSE OF 1ST ORDER SYSTEM
- ACTIVITY
- STEP RESPONSE OF 1ST ORDER SYSTEM
- RELATION BETWEEN STEP AND IMPULSE RESPONSE
- ANALYSIS OF SIMPLE RC CIRCUIT
- EXAMPLE 1
- SUMMARY



RELATION BETWEEN STANDARD TEST SIGNALS



• Impulse

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$\frac{d}{dt}$

• Step

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$\frac{d}{dt}$

• Ramp

$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$\frac{d}{dt}$

• Parabolic

$$p(t) = \begin{cases} \frac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$



LAPLACE TRANSFORM OF TEST SIGNALS



- Impulse

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$L\{\delta(t)\} = \delta(s) = A$$

- Step

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{u(t)\} = U(s) = \frac{A}{S}$$



LAPLACE TRANSFORM OF TEST SIGNALS



- Ramp

$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{r(t)\} = R(s) = \frac{A}{s^2}$$

- Parabolic

$$p(t) = \begin{cases} \frac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{p(t)\} = P(s) = \frac{A}{s^3}$$



TIME RESPONSE OF CONTROL SYSTEMS



- **Transient response depends** → system poles only & not on the type of input → To analyze the transient response using a step input.
- **The steady-state response depends** → system dynamics & the input quantity → To examine using different test signals by final value theorem.



INTRODUCTION- FIRST ORDER SYSTEM



- The first order system has only one pole.

$$\frac{C(s)}{R(s)} = \frac{K}{Ts + 1}$$

- Where ***K*** is the D.C gain and ***T*** is the time constant of the system.
- Time constant is a measure of how quickly a 1st order system responds to a unit step input.
- D.C Gain of the system is ratio between the input signal and the steady state value of output.



INTRODUCTION- FIRST ORDER SYSTEM



- The first order system given below.

$$G(s) = \frac{10}{3s + 1}$$

- D.C gain is **10** and time constant is **3** seconds.

- For the following system $G(s) = \frac{3}{s + 5} = \frac{3/5}{1/5s + 1}$

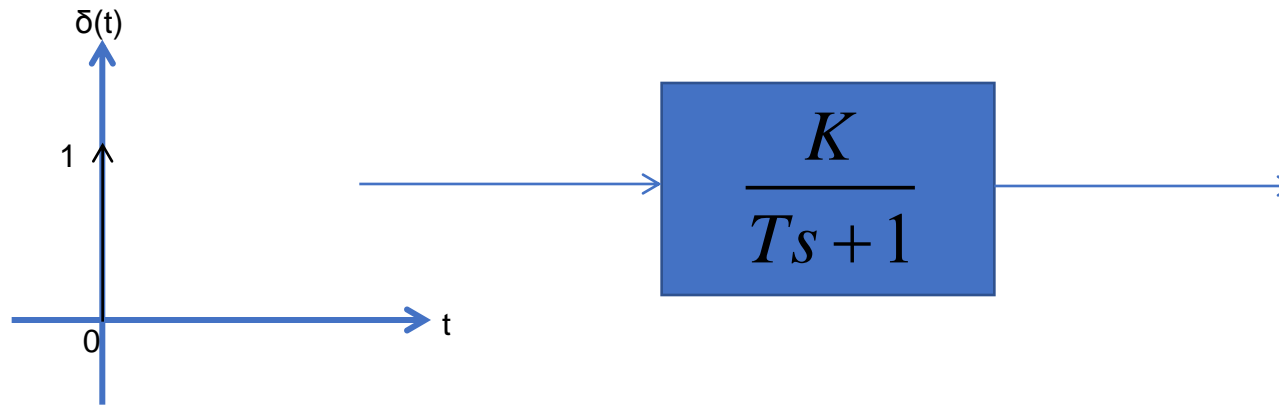
- D.C Gain of the system is **3/5**
- time constant is **1/5** seconds.



IMPULSE RESPONSE OF 1ST ORDER SYSTEM



- Consider the following 1st order system



$$R(s) = \delta(s) = 1$$

$$C(s) = \frac{K}{Ts + 1}$$



IMPULSE RESPONSE OF 1ST ORDER SYSTEM



$$C(s) = \frac{K}{Ts + 1}$$

- Re-arrange following equation as

$$C(s) = \frac{K/T}{s + 1/T}$$

- In order to compute the response of the system in time domain we need to compute inverse Laplace transform of the above equation.

$$L^{-1}\left(\frac{C}{s + a}\right) = Ce^{-at} \quad c(t) = \frac{K}{T} e^{-t/T}$$

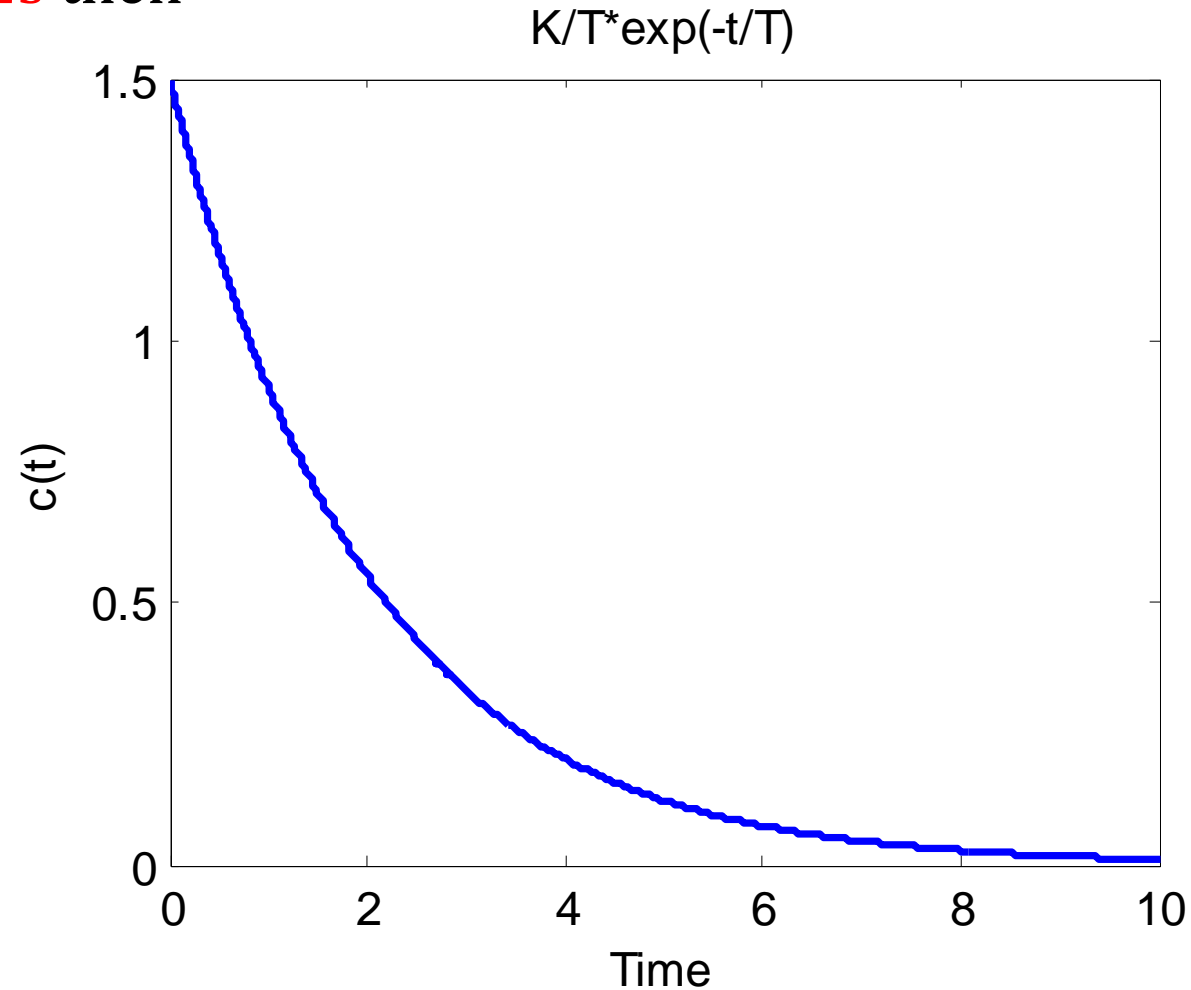


IMPULSE RESPONSE OF 1ST ORDER SYSTEM



- If $K=3$ and $T=2s$ then

$$c(t) = \frac{K}{T} e^{-t/T}$$





ACTIVITY



1. Tsunamis are not caused by

- (a) Hurricanes
- (b) Earthquakes
- (c) Undersea landslides
- (d) Volcanic eruptions

4. Where was the electricity supply first introduced in India

- (a) Mumbai
- (b) Dehradun
- (c) Darjeeling
- (d) Chennai

2. Professor Amartya Sen received the Nobel Prize in this field.

- a) Literature
- b) Electronics
- c) Economics
- d) Geology

5. According to Swachh Survekshan 2017 ranking of the following city in India?

- 1) Mysuru
- 2) Bhopal
- 3) Indore
- 4) Visakhapatnam (Vizag)

3. First human heart transplant operation conducted by Dr. Christian Bernard on Louis Washkansky, was conducted in

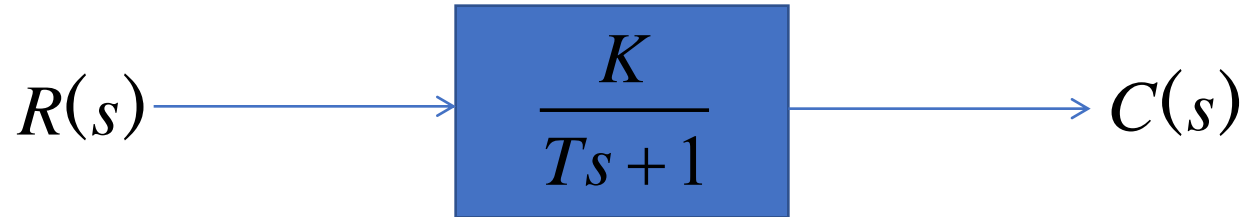
- A. 1958
- B. 1922
- C. 1967
- D. 1968



STEP RESPONSE OF 1ST ORDER SYSTEM



- Consider the following 1st order system



$$R(s) = U(s) = \frac{1}{s} \qquad C(s) = \frac{K}{s(Ts + 1)}$$

- In order to find out the inverse Laplace of the above equation, we need to break it into partial fraction expansion.

$$C(s) = \frac{K}{s} - \frac{KT}{Ts + 1}$$



STEP RESPONSE OF 1ST ORDER SYSTEM



$$C(s) = K \left(\frac{1}{s} - \frac{T}{Ts + 1} \right)$$

- Taking Inverse Laplace of above equation

$$c(t) = K \left(u(t) - e^{-t/T} \right)$$

- Where $u(t)=1$

$$c(t) = K \left(1 - e^{-t/T} \right)$$

- When $t=T$ (time constant)

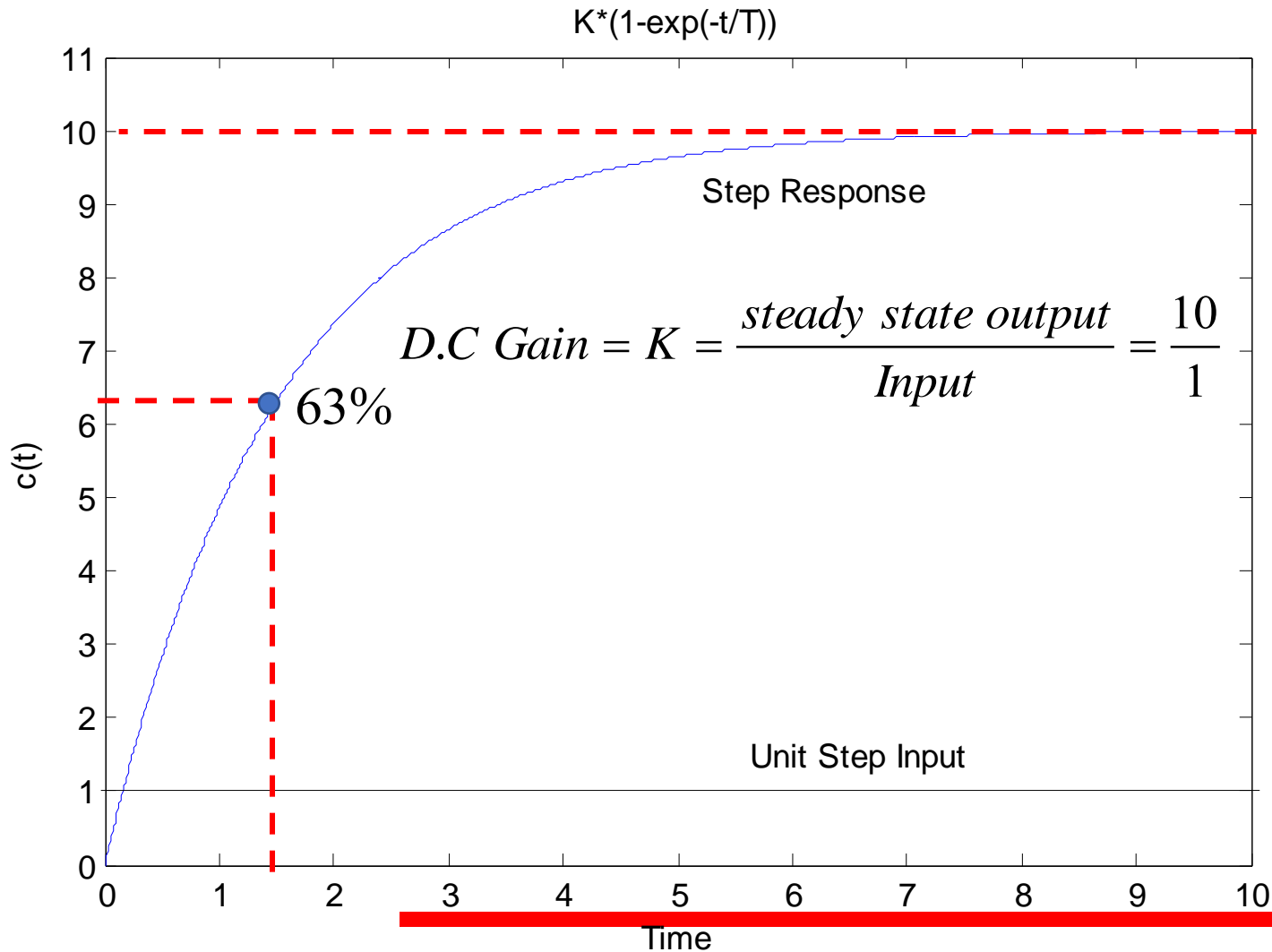
$$c(t) = K \left(1 - e^{-1} \right) = 0.632K$$



STEP RESPONSE OF 1ST ORDER SYSTEM



- If $K=10$ and $T=1.5s$ then $c(t) = K(1 - e^{-t/T})$

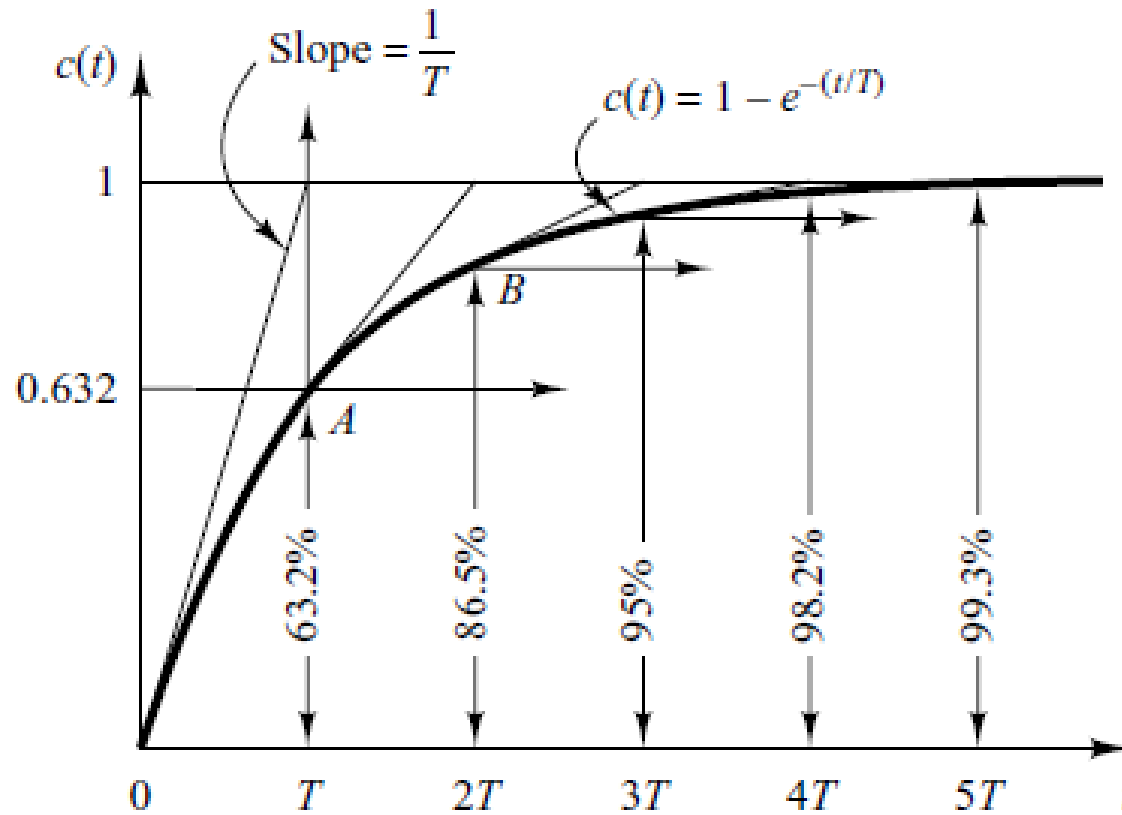




STEP RESPONSE OF 1ST ORDER SYSTEM



- System takes five time constants to reach its final value.

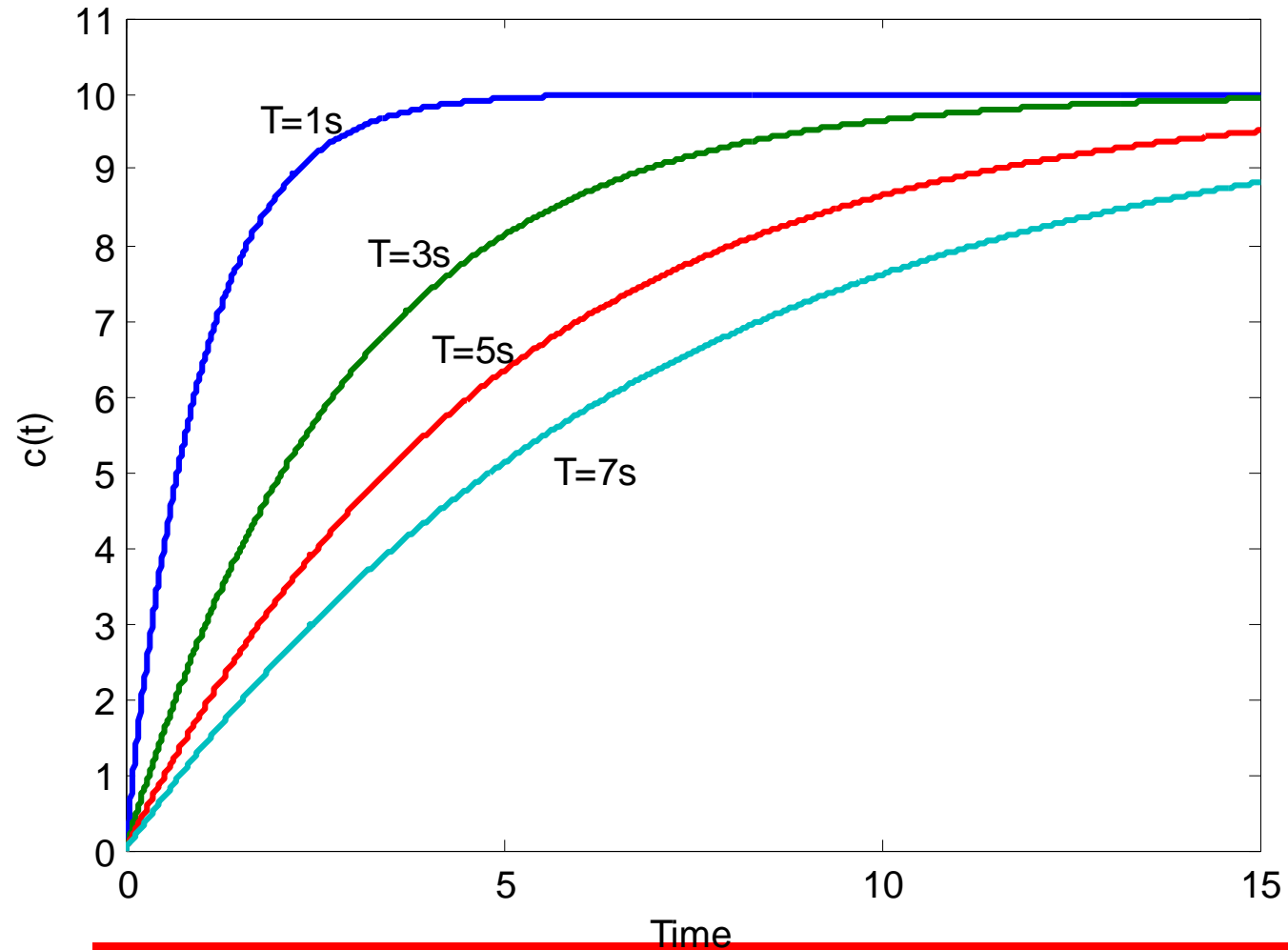




STEP RESPONSE OF 1ST ORDER SYSTEM



- If $K=10$ and $T=1, 3, 5, 7$ $c(t) = K(1 - e^{-t/T})$
 $K*(1-\exp(-t/T))$

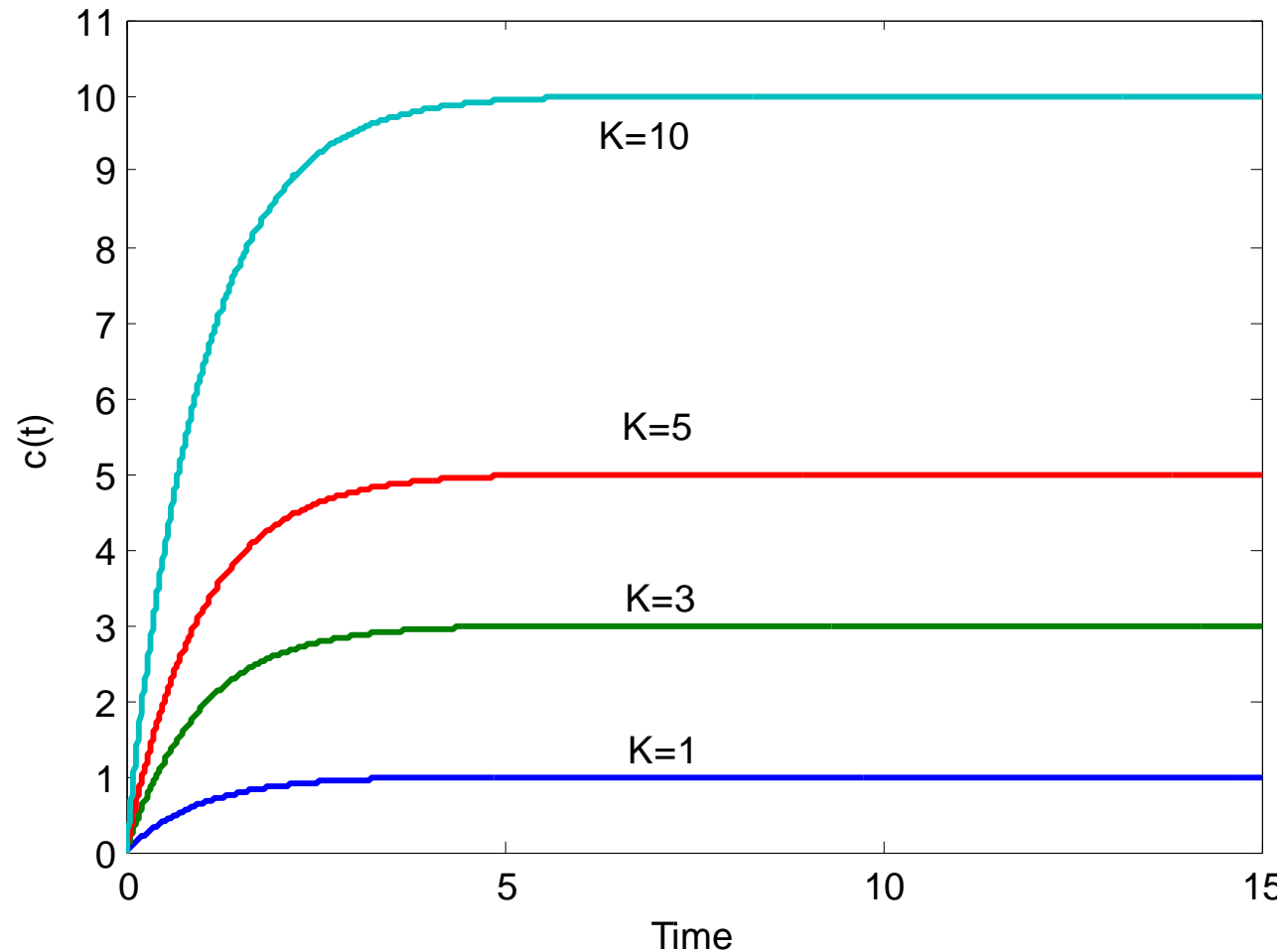




STEP RESPONSE OF 1ST ORDER SYSTEM



- If $K=1, 3, 5, 10$ and $T=1$ $c(t) = K(1 - e^{-t/T})$
 $K*(1-\exp(-t/T))$





RELATION BETWEEN STEP AND IMPULSE RESPONSE



- The step response of the first order system is

$$c(t) = K(1 - e^{-t/T}) = K - Ke^{-t/T}$$

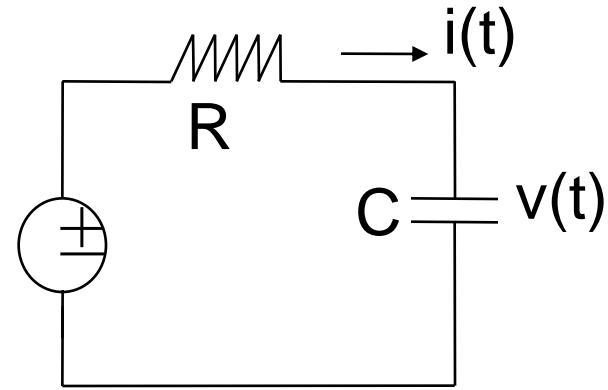
- Differentiating $c(t)$ with respect to t yields

$$\frac{dc(t)}{dt} = \frac{d}{dt} (K - Ke^{-t/T})$$

$$\frac{dc(t)}{dt} = \frac{K}{T} e^{-t/T}$$



ANALYSIS OF SIMPLE RC CIRCUIT



$$R \cdot i(t) + v(t) = v_T(t)$$

$$i(t) = \frac{d(Cv(t))}{dt} = C \frac{dv(t)}{dt}$$

$$\Rightarrow RC \frac{dv(t)}{dt} + v(t) = v_T(t)$$

state
variable

Input
waveform



ANALYSIS OF SIMPLE RC CIRCUIT



Step-input response:

$$RC \frac{dv(t)}{dt} + v(t) = v_0 u(t)$$

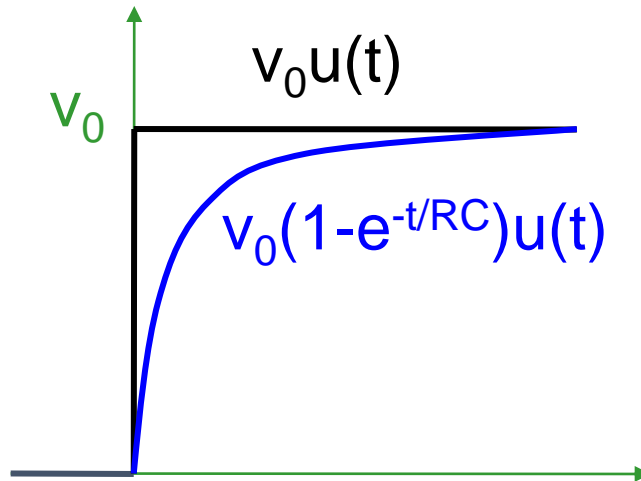
$$v(t) = K e^{-t/RC} + v_0 u(t)$$

match initial state:

$$v(0) = 0 \Rightarrow K + v_0 u(t) = 0 \Rightarrow K + v_0 = 0$$

output response for step-input:

$$v(t) = v_0 (1 - e^{-t/RC}) u(t)$$

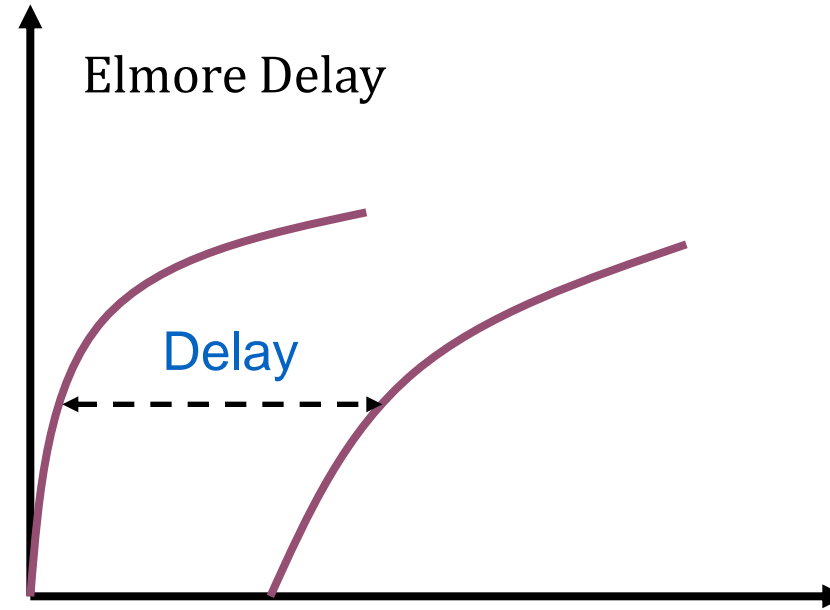




ANALYSIS OF SIMPLE RC CIRCUIT

RC Circuit

- $v(t) = v_0(1 - e^{-t/RC})$ -- waveform under step input $v_0u(t)$
- $v(t)=0.5v_0 \Rightarrow t = 0.69RC$
 - i.e., delay = $0.69RC$ (50% delay)
- $v(t)=0.1v_0 \Rightarrow t = 0.1RC$
- $v(t)=0.9v_0 \Rightarrow t = 2.3RC$
 - i.e., rise time = $2.2RC$ (if defined as time from 10% to 90% of V_{dd})
- For simplicity, industry uses $T_D = RC$ (= Elmore delay)



1. 50%-50% point delay
2. Delay= $0.69RC$



EXAMPLE 1



- Impulse response of a 1st order system is given below. $c(t) = 3e^{-0.5t}$
- Find out: Time constant T, D.C Gain K, Transfer Function ,Step Response
- The Laplace Transform of Impulse response of a system is actually the transfer function of the system.
- Therefore taking Laplace Transform of the impulse response given by following equation.

$$C(s) = \frac{3}{S + 0.5} \times 1 = \frac{3}{S + 0.5} \times \delta(s)$$

$$\frac{C(s)}{\delta(s)} = \frac{C(s)}{R(s)} = \frac{3}{S + 0.5} \qquad \frac{C(s)}{R(s)} = \frac{6}{2S + 1}$$



EXAMPLE 1



- Impulse response of a 1st order system is given below.

$$c(t) = 3e^{-0.5t}$$

- Find out
 - Time constant **T=2**
 - D.C Gain **K=6**
 - Transfer Function
 - Step Response



EXAMPLE 1



- For step response integrate impulse response

$$c(t) = 3e^{-0.5t}$$

$$\int c(t)dt = 3 \int e^{-0.5t} dt$$

$$c_s(t) = -6e^{-0.5t} + C$$

- We can find out C if initial condition is known e.g. $c_s(0)=0$

$$0 = -6e^{-0.5 \times 0} + C$$

$$C = 6$$

$$c_s(t) = 6 - 6e^{-0.5t}$$



EXAMPLE 1

- If initial conditions are not known then partial fraction expansion is a better choice

$$\frac{C(s)}{R(s)} = \frac{6}{2S + 1}$$

since $R(s)$ is a step input, $R(s) = \frac{1}{s}$

$$C(s) = \frac{6}{s(2S + 1)}$$

$$\frac{6}{s(2S + 1)} = \frac{A}{s} + \frac{B}{2s + 1}$$

$$\frac{6}{s(2S + 1)} = \frac{6}{s} - \frac{6}{s + 0.5}$$

$$c(t) = 6 - 6e^{-0.5t}$$



SUMMARY

