SNS COLLEGE OF TECHNOLOGY

Coimbatore-26
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT II – TIME RESPONSE ANALYSIS

TOPIC 5- STEADY STATE ERRORS

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS
- STEADY STATE ERROR
- CLASSIFICATION OF CONTROL SYSTEMS
- STEADY STATE ERROR OF UNITY FEEDBACK SYSTEMS
- STATIC ERROR CONSTANTS
- STATIC POSITION ERROR CONSTANT (K_P)
- ACTIVITY
- STATIC VELOCITY ERROR CONSTANT (K_V)
- STATIC ACCELERATION ERROR CONSTANT (K_A)
- EXAMPLE
- SUMMARY

STEADY STATE ERROR

- If the output of a control system at steady state does not exactly match with the input, the system is said to have steady state error

- Any physical control system inherently suffers steady-state error in response to certain types of inputs.

- A system may have no steady-state error to a step input, but the same system may exhibit nonzero steady-state error to a ramp input.

CLASSIFICATION OF CONTROL SYSTEMS

- Control systems may be classified according to their ability to follow
  - Step inputs,
  - Ramp inputs,
  - Parabolic inputs, and so on.

- The magnitudes of the steady-state errors due to these individual inputs are indicative of the goodness of the system.

CLASSIFICATION OF CONTROL SYSTEMS

- Consider the unity-feedback control system with the following open-loop transfer function

$$G(s) = \frac{K(T_a s + 1)(T_b s + 1)\cdots(T_m s + 1)}{s^N(T_1 s + 1)(T_2 s + 1)\cdots(T_p s + 1)}$$

- It involves the term s^N in the denominator, representing N poles at the origin.

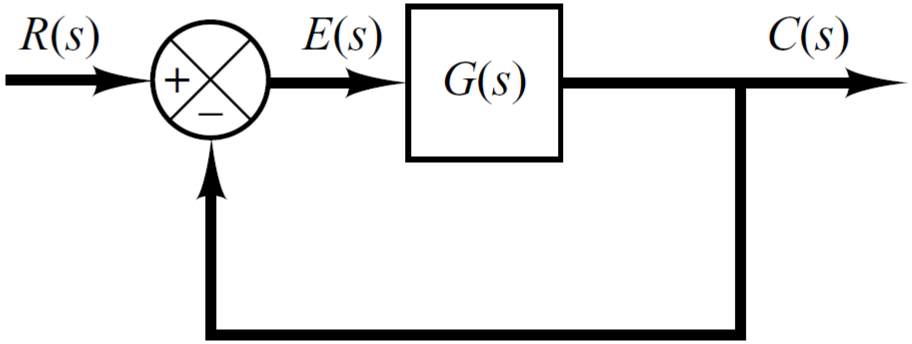
- A system is called type 0, type 1, type 2, ... , if N=0, N=1, N=2, ... , respectively.

CLASSIFICATION OF CONTROL SYSTEMS

- As the type number is increased, accuracy is improved.
- However, increasing the type number aggravates the stability problem.
- A compromise between steady-state accuracy and relative stability is always necessary.

STEADY STATE ERROR OF UNITY FEEDBACK SYSTEMS

- Consider the system shown in following figure.



- The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)} \qquad G(s) = \frac{K(T_a s + 1)(T_b s + 1)\cdots(T_m s + 1)}{s^N(T_1 s + 1)(T_2 s + 1)\cdots(T_p s + 1)}$$

STEADY STATE ERROR OF UNITY FEEDBACK SYSTEMS

- Steady state error is defined as the error between the input signal and the output signal when $t \to \infty$.

- The transfer function between the error signal E(s) and the input signal R(s) is $\dfrac{E(s)}{R(s)} = \dfrac{1}{1+G(s)}$
- The final-value theorem provides a convenient way to find the steady-state performance of a stable system.
- Since E(s) is $E(s) = \dfrac{1}{1+G(s)} R(s)$
- The steady state error is

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{sR(s)}{1+G(s)}$$

STATIC ERROR CONSTANTS

- The static error constants are figures of merit of control systems. The higher the constants, the smaller the steady-state error.

- In a given system, the output may be the position, velocity, pressure, temperature, or the like.

- Therefore, in what follows, we shall call the output "position," the rate of change of the output "velocity," and so on.

- This means that in a temperature control system "position" represents the output temperature, "velocity" represents the rate of change of the output temperature, and so on.

STATIC POSITION ERROR CONSTANT (K_P)

- The steady-state error of the system for a unit-step input is

$$e_{ss} = \lim_{s \to 0} \frac{\cancel{s}}{1 + G(s)} \frac{1}{\cancel{s}}$$

$$= \frac{1}{1 + G(0)}$$

- The static position error constant K_p is defined by

$$K_p = \lim_{s \to 0} G(s) = G(0)$$

- Thus, the steady-state error in terms of the static position error constant K_p is given by

$$e_{ss} = \frac{1}{1 + K_p}$$

STATIC POSITION ERROR CONSTANT (K_P)

- For a Type 0 system

$$K_p = \lim_{s \to 0} \frac{K(T_a s + 1)(T_b s + 1)\cdots}{(T_1 s + 1)(T_2 s + 1)\cdots} = K$$

- For Type 1 or higher order systems

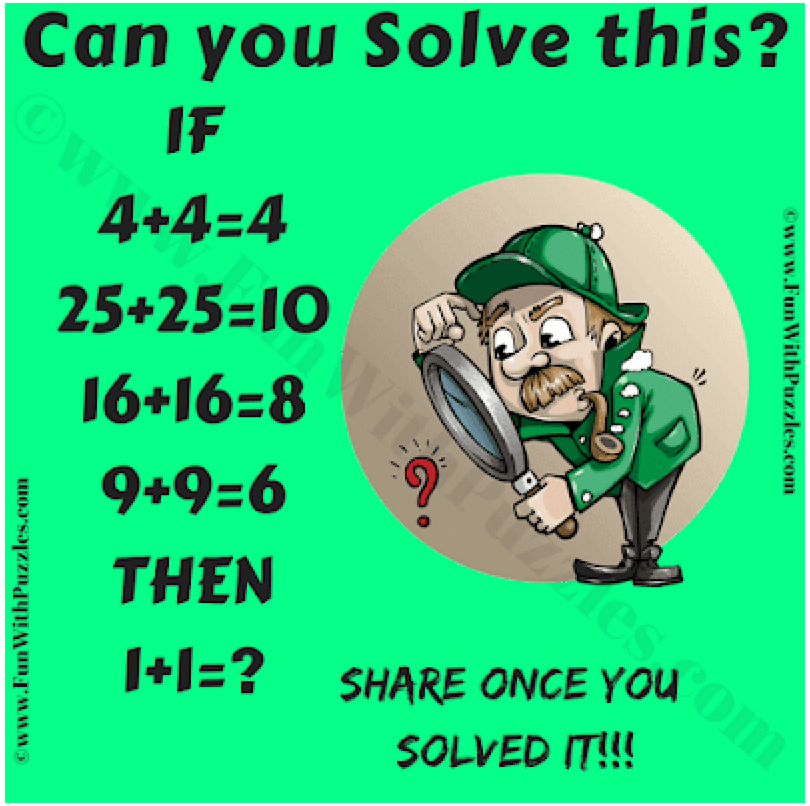
$$K_p = \lim_{s \to 0} \frac{K(T_a s + 1)(T_b s + 1)\cdots}{s^N(T_1 s + 1)(T_2 s + 1)\cdots} = \infty, \qquad \text{for } N \geq 1$$

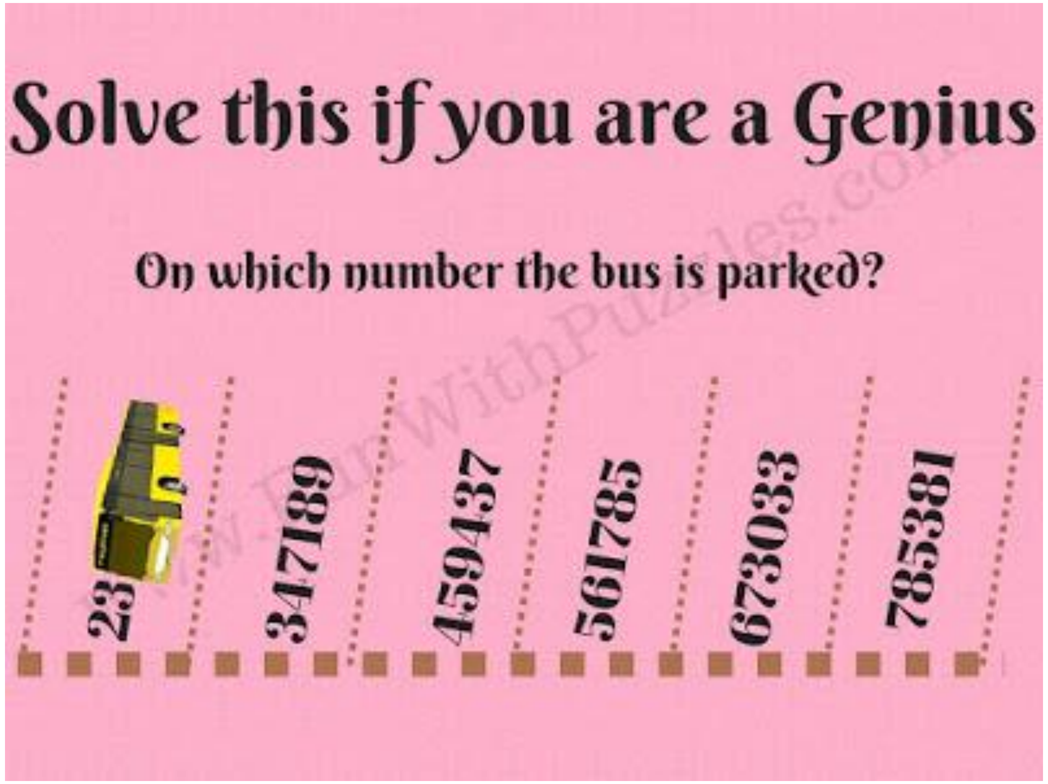
- For a unit step input the steady state error **e_{ss}** is

$$e_{ss} = \frac{1}{1 + K}, \qquad \text{for type 0 systems}$$

$$e_{ss} = 0, \qquad \text{for type 1 or higher systems}$$

ACTIVITY-PUZZLES





STATIC VELOCITY ERROR CONSTANT (K_V)

- The steady-state error of the system for a unit-ramp input is

$$e_{ss} = \lim_{s \to 0} \frac{s}{1 + G(s)} \frac{1}{s^2}$$

$$= \lim_{s \to 0} \frac{1}{sG(s)}$$

- The static velocity error constant K_v is defined by

$$K_v = \lim_{s \to 0} sG(s)$$

- Thus, the steady-state error in terms of the static velocity error constant K_v is given by

$$e_{ss} = \frac{1}{K_v}$$

STATIC VELOCITY ERROR CONSTANT (K_V)

- For a Type 0 system

$$K_v = \lim_{s\to 0} \frac{sK(T_a s + 1)(T_b s + 1)\cdots}{(T_1 s + 1)(T_2 s + 1)\cdots} = 0$$

- For Type 1 systems

$$K_v = \lim_{s\to 0} \frac{sK(T_a s + 1)(T_b s + 1)\cdots}{s(T_1 s + 1)(T_2 s + 1)\cdots} = K$$

- For type 2 or higher order systems

$$K_v = \lim_{s\to 0} \frac{sK(T_a s + 1)(T_b s + 1)\cdots}{s^N(T_1 s + 1)(T_2 s + 1)\cdots} = \infty, \qquad \text{for } N \geq 2$$

STATIC VELOCITY ERROR CONSTANT (K_V)

- For a ramp input the steady state error $\mathbf{e_{ss}}$ is

$$e_{ss} = \frac{1}{K_v} = \infty, \qquad \text{for type 0 systems}$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K}, \qquad \text{for type 1 systems}$$

$$e_{ss} = \frac{1}{K_v} = 0, \qquad \text{for type 2 or higher systems}$$

STATIC ACCELERATION ERROR CONSTANT (K_A)

- The steady-state error of the system for parabolic input is

$$e_{ss} = \lim_{s \to 0} \frac{s}{1 + G(s)} \frac{1}{s^3}$$

$$= \frac{1}{\lim_{s \to 0} s^2 G(s)}$$

- The static acceleration error constant K_a is defined by

$$K_a = \lim_{s \to 0} s^2 G(s)$$

- Thus, the steady-state error in terms of the static acceleration error constant K_a is given by

$$e_{ss} = \frac{1}{K_a}$$

STATIC ACCELERATION ERROR CONSTANT (K_A)

- For a Type 0 system

$$K_a = \lim_{s \to 0} \frac{s^2 K(T_a s + 1)(T_b s + 1)\cdots}{(T_1 s + 1)(T_2 s + 1)\cdots} = 0$$

- For Type 1 systems

$$K_a = \lim_{s \to 0} \frac{s^2 K(T_a s + 1)(T_b s + 1)\cdots}{s(T_1 s + 1)(T_2 s + 1)\cdots} = 0$$

- For type 2 systems

$$K_a = \lim_{s \to 0} \frac{s^2 K(T_a s + 1)(T_b s + 1)\cdots}{s^2(T_1 s + 1)(T_2 s + 1)\cdots} = K$$

- For type 3 or higher order systems

$$K_a = \lim_{s \to 0} \frac{s^2 K(T_a s + 1)(T_b s + 1)\cdots}{s^N(T_1 s + 1)(T_2 s + 1)\cdots} = \infty, \qquad \text{for } N \geq 3$$

STATIC ACCELERATION ERROR CONSTANT (K_A)

- For a parabolic input the steady state error **e_{ss}** is

$$e_{ss} = \infty, \quad \text{for type 0 and type 1 systems}$$

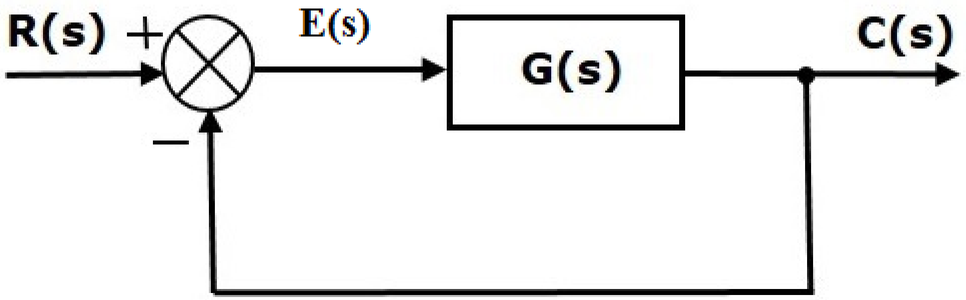
$$e_{ss} = \frac{1}{K}, \quad \text{for type 2 systems}$$

$$e_{ss} = 0, \quad \text{for type 3 or higher systems}$$

STEADY STATE ERROR

- The deviation of the output of control system from desired response during steady state is known as steady state error. It is represented as e_{ss}

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} E(s)$$



STEADY STATE ERROR

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$\Rightarrow C(s) = \frac{R(s)G(s)}{1+G(s)}$$

$$E(s) = R(s) - C(s)$$

$$\Rightarrow E(s) = \frac{R(s)}{1+G(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1+G(s)}$$

STEADY STATE ERROR

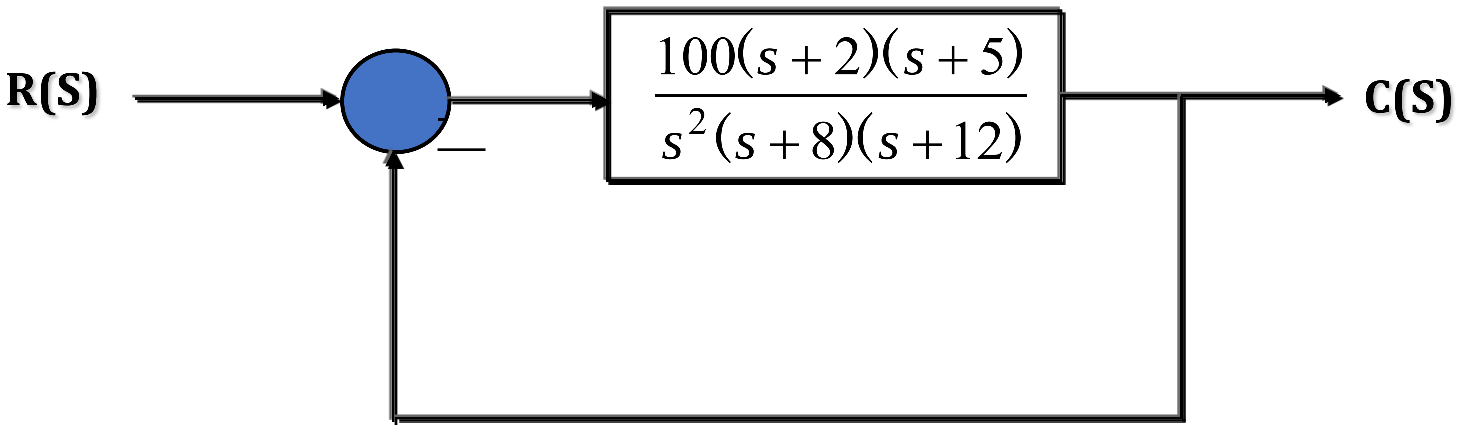
- The following table shows the steady state errors and the error constants for standard input signals like unit step, unit ramp & unit parabolic signals.

Input signal	Steady state error e_{ss}	Error constant
unit step signal	$\frac{1}{1+k_p}$	$K_p = \lim_{s \to 0} G(s)$
unit ramp signal	$\frac{1}{K_v}$	$K_v = \lim_{s \to 0} sG(s)$
unit parabolic signal	$\frac{1}{K_a}$	$K_a = \lim_{s \to 0} s^2 G(s)$

- Where Kp, Kv, Ka are the position error constant, velocity error constant and acceleration error constant respectively.

EXAMPLE

- For the system shown in figure below evaluate the static error constants and find the expected steady state errors for the standard step, ramp and parabolic inputs.



EXAMPLE

$$G(s) = \frac{100(s+2)(s+5)}{s^2(s+8)(s+12)}$$

$$K_p = \lim_{s\to 0} G(s)$$

$$K_p = \lim_{s\to 0}\left(\frac{100(s+2)(s+5)}{s^2(s+8)(s+12)}\right)$$

$$K_p = \infty$$

$$K_v = \lim_{s\to 0} sG(s)$$

$$K_v = \lim_{s\to 0}\left(\frac{100s(s+2)(s+5)}{s^2(s+8)(s+12)}\right)$$

$$K_v = \infty$$

$$K_a = \lim_{s\to 0} s^2G(s)$$

$$K_a = \lim_{s\to 0}\left(\frac{100s^2(s+2)(s+5)}{s^2(s+8)(s+12)}\right)$$

$$K_a = \left(\frac{100(0+2)(0+5)}{(0+8)(0+12)}\right) = 10.4$$

EXAMPLE

$$K_p = \infty \qquad K_v = \infty \qquad K_a = 10.4$$

$$e_{ss} = \frac{1}{1 + K_p} = 0$$

$$e_{ss} = \frac{1}{K_v} = 0$$

$$e_{ss} = \frac{1}{K_a} = 0.09$$

SUMMARY

	Step Input $r(t) = 1$	Ramp Input $r(t) = t$	Acceleration Input $r(t) = \frac{1}{2}t^2$
Type 0 system	$\frac{1}{1+K}$	∞	∞
Type 1 system	0	$\frac{1}{K}$	∞
Type 2 system	0	0	$\frac{1}{K}$

SUMMARY