SNS COLLEGE OF TECHNOLOGY

Coimbatore-14
An Autonomous Institution

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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT I – CONTROL SYSTEM MODELING

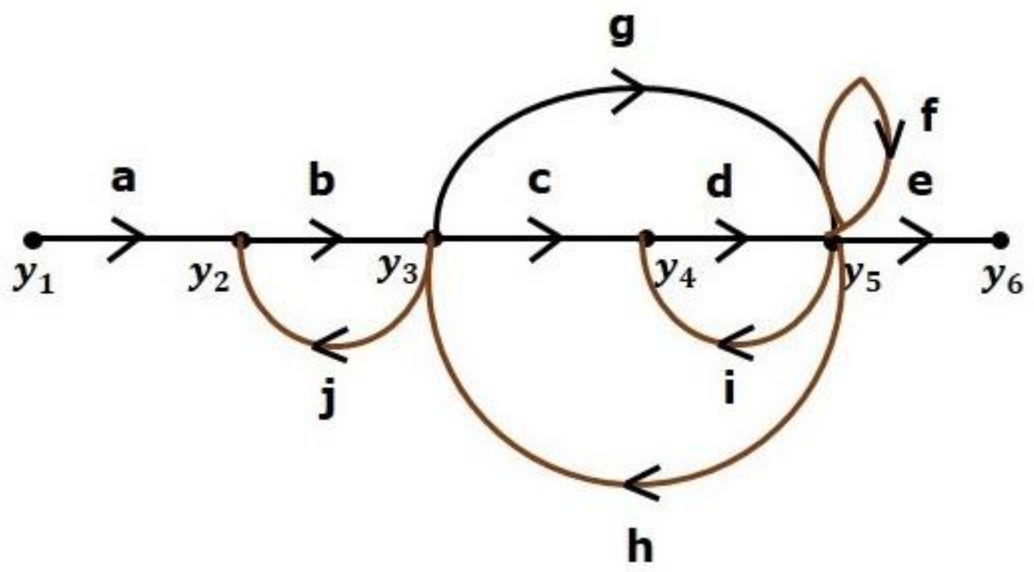
TOPIC 9- SIGNAL FLOW GRAPH

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS
- TERMINOLOGY OF SIGNAL FLOW GRAPH
- PATH, FORWARD PATH, FORWARD PATH GAIN
- LOOP, LOOP GAIN, NON-TOUCHING LOOPS
- ACTIVITY
- MASON’S GAIN FORMULA
- CALCULATION OF TRANSFER FUNCTION USING MASON’S GAIN FORMULA
- EXAMPLE
- SUMMARY

SIGNAL FLOW GRAPH- TERMINOLOGY

Consider the following signal flow graph in order to understand the basic terminology involved here.



SIGNAL FLOW GRAPH-TERMINOLOGY

Path
It is a traversal of branches from one node to any other node in the direction of branch arrows. It should not traverse any node more than once.
Examples – y2→y3→y4→y5 and y5→y3→y2

Forward Path
The path that exists from the input node to the output node is known as **forward path**.
Examples – y1→y2→y3→y4→y5→y6 and y1→y2→y3→y5→y6

Forward Path Gain
It is obtained by calculating the product of all branch gains of the forward path.
Examples – abcde is the forward path gain of y1→y2→y3→y4→y5→y6y1→y2→y3→y4→y5→y6 and abge is the forward path gain of y1→y2→y3→y5→y6y1→y2→y3→y5→y6

SIGNAL FLOW GRAPH- TERMINOLOGY

Loop

The path that starts from one node and ends at the same node is known as **loop**. Hence, it is a closed path.

Examples – $y2 \rightarrow y3 \rightarrow y2$ and $y3 \rightarrow y5 \rightarrow y3$

Loop Gain

It is obtained by calculating the product of all branch gains of a loop.

Examples – b_j is the loop gain of $y2 \rightarrow y3 \rightarrow y2$ and
g_h is the loop gain of $y3 \rightarrow y5 \rightarrow y3$.

Non-touching Loops

These are the loops, which should not have any common node.

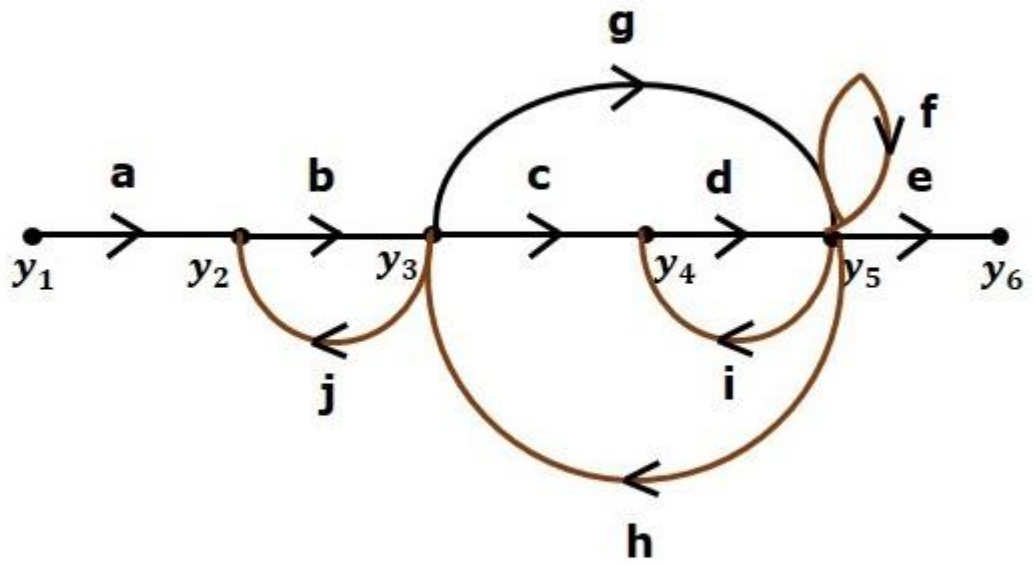
Examples – The loops, $y2 \rightarrow y3 \rightarrow y2$ and $y4 \rightarrow y5 \rightarrow y4$ are non-touching.

ACTIVITY -BLOOD RELATION TEST

1. Pointing to a photograph of a boy Suresh said, "He is the son of the only son of my mother." How is Suresh related to that boy?
   A. Brother
   B. Uncle
   C. Cousin
   D. Father

CALCULATION OF TRANSFER FUNCTION USING MASON’S GAIN FORMULA

Let us consider the same signal flow graph for finding transfer function.



Answer: Option **D**

Explanation:

The boy in the photograph is the only son of the son of Suresh's mother i.e., the son of Suresh. Hence, Suresh is the father of boy.

CALCULATION OF TRANSFER FUNCTION …

- Number of forward paths, N = 2.
- First forward path is - y1→y2→y3→y4→y5→y6.
- First forward path gain, p1=abcde
- Second forward path is - y1→y2→y3→y5→y6
- Second forward path gain, p2=abge
- Number of individual loops, L = 5.

CALCULATION OF TRANSFER FUNCTION …

- Loops are - $y_2 \to y_3 \to y_2$, $y_3 \to y_5 \to y_3$, $y_3 \to y_4 \to y_5 \to y_3$

  $y_4 \to y_5 \to y_4$ and $y_5 \to y_5$.

- Loop gains are - $l_1 = bj$, $l_2 = gh$, $l_3 = cdh$, $l_4 = di$ and $l_5 = f$.
- Number of two non-touching loops = 2.
- First non-touching loops pair is - $y_2 \to y_3 \to y_2$, $y_4 \to y_5 \to y_4$.
- Gain product of first non-touching loops pair, $l_1 l_4 = bjdi$
- Second non-touching loops pair is - $y_2 \to y_3 \to y_2$, $y_5 \to y_5$.
- Gain product of second non-touching loops pair is - $l_1 l_5 = bjf$

CALCULATION OF TRANSFER FUNCTION…

Higher number of (more than two) non-touching loops are not present in this signal flow graph.

We know,

$$\Delta = 1 - (\text{sum of all individual loop gains})$$
$$+ (\text{sum of gain products of all possible two non touching loops})$$
$$- (\text{sum of gain products of all possible three non touching loops}) + \dots$$

CALCULATION OF TRANSFER FUNCTION …

Substitute the values in the above equation,

$$\Delta=1-(bj+gh+cdh+di+f)+(bjdi+bjf)-(0)$$

$$\Delta=1-(bj+gh+cdh+di+f)+(bjdi+bjf)$$

$$\Rightarrow\Delta=1-(bj+gh+cdh+di+f)+bjdi+bjf$$

There is no loop which is non-touching to the first forward path.

So, $\Delta_1=1$ Similarly, $\Delta_2=1$

Since, no loop which is non-touching to the second forward path.

Substitute, N = 2 in Mason’s gain formula

MASON'S GAIN FORMULA

$$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{N} P_i \Delta_i}{\Delta}$$

Where,

- **C(s)** is the output node
- **R(s)** is the input node
- **T** is the transfer function or gain between $R(s)$ and $C(s)$
- $\mathbf{P_i}$ is the i^{th} forward path gain

$$\Delta = 1 - (sum\ of\ all\ individual\ loop\ gains)$$

$$+(sum\ of\ gain\ products\ of\ all\ possible\ two\ nontouching\ loops)$$

$$-(sum\ of\ gain\ products\ of\ all\ possible\ three\ nontouching\ loops) + \ldots$$

Δ_i is obtained from Δ by removing the loops which are touching the i^{th} forward path.

Consider the following signal flow graph in order to understand the basic terminology involved here.

CALCULATION OF TRANSFER FUNCTION …

Substitute, N = 2 in Mason's gain formula

$$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{2} P_i \Delta_i}{\Delta}$$

$$T = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

Substitute all the necessary values in the above equation.

$$T = \frac{C(s)}{R(s)} = \frac{(abcde)1 + (abge)1}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

$$\Rightarrow T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

Therefore, the transfer function is -

$$T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

SUMMARY