SNS COLLEGE OF TECHNOLOGY

Coimbatore-22
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT I – CONTROL SYSTEM MODELING

TOPIC 8- SIGNAL FLOW GRAPH

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS
- INTRODUCTION
- BASIC ELEMENTS OF SIGNAL FLOW GRAPH
- 6 -EXAMPLE-SFG
- ACTIVITY
- CONVERSION OF BLOCK DIAGRAMS INTO SIGNAL FLOW GRAPHS
- EXAMPLE
- MASON'S GAIN FORMULA
- SUMMARY

INTRODUCTION

- Signal flow graph is a graphical representation of algebraic equations
- The block diagram reduction process takes more time for complicated system
- So, to overcome this drawback, use signal flow graphs (representation) is done where the calculation of transfer function is just by using a Mason's gain formula without doing any reduction process.

BASIC ELEMENTS OF SIGNAL FLOW GRAPH

Nodes and branches are the basic elements of signal flow graph.

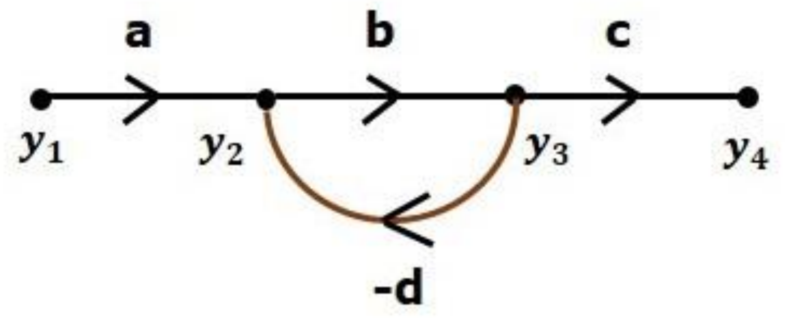
Node

Node is a point which represents either a variable or a signal. There are three types of nodes — input node, output node and mixed node.

Input Node – It is a node, which has only outgoing branches.

Output Node – It is a node, which has only incoming branches.

Mixed Node – It is a node, which has both incoming and outgoing branches.



BASIC ELEMENTS OF SIGNAL FLOW GRAPH…

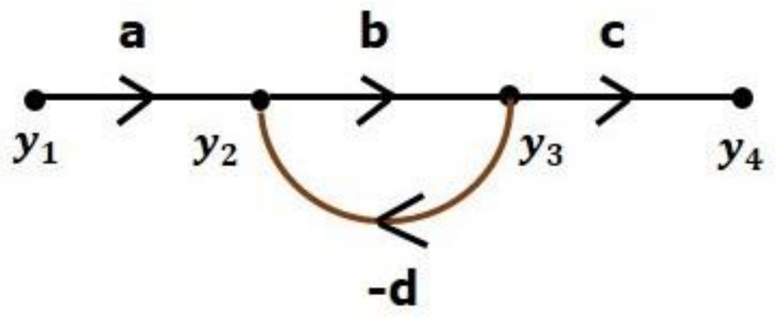
- **Forward path**: It is a path from an input node to an output node that does not cross any node more than once.
- **Individual loop**: It is a closed path starting from one node and after passing through the graph arrives at the same node without crossing any node more than once.
- **Non-touching loops**: If a loop does not have a common node then they are said to be non-touching loops

EXAMPLE

Let us consider the following signal flow graph to identify these nodes.

- The **nodes** present in this signal flow graph are $\mathbf{y_1}$, $\mathbf{y_2}$, $\mathbf{y_3}$ and $\mathbf{y_4}$.
- $\mathbf{y_1}$ and $\mathbf{y_4}$ are the **input node** and **output node** respectively.
- $\mathbf{y_2}$ and $\mathbf{y_3}$ are **mixed nodes**.

Branch is a line segment which joins two nodes. It has both **gain** and **direction**. For example, there are four branches in the above signal flow graph. These branches have **gains** of **a, b, c** and **-d**.



EXAMPLE

Let us construct a signal flow graph by considering the following algebraic eqns. –

$$y_2 = a_{12}y_1 + a_{42}y_4$$

$$y_3 = a_{23}y_2 + a_{53}y_5$$

$$y_4 = a_{34}y_3$$

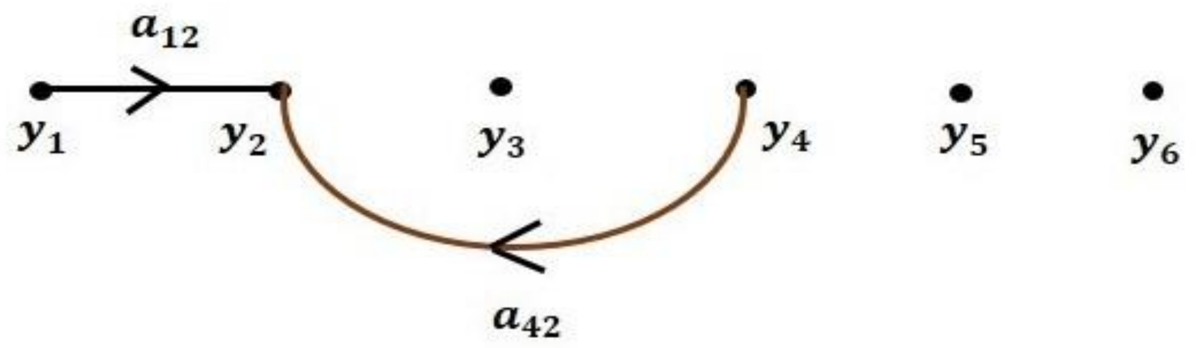
$$y_5 = a_{45}y_4 + a_{35}y_3$$

$$y_6 = a_{56}y_5$$

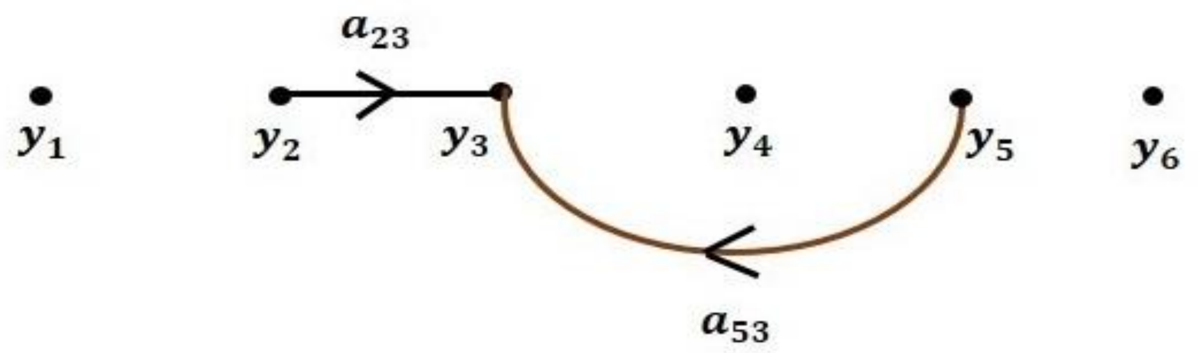
There will be six **nodes** (y_1, y_2, y_3, y_4, y_5 and y_6) and eight **branches** in this signal flow graph. The gains of the branches are a_{12}, a_{23}, a_{34}, a_{45}, a_{56}, a_{42}, a_{53} and a_{35}.

To get the overall signal flow graph, draw the signal flow graph for each equation, then combine all these signal flow graphs and then follow the steps given below –

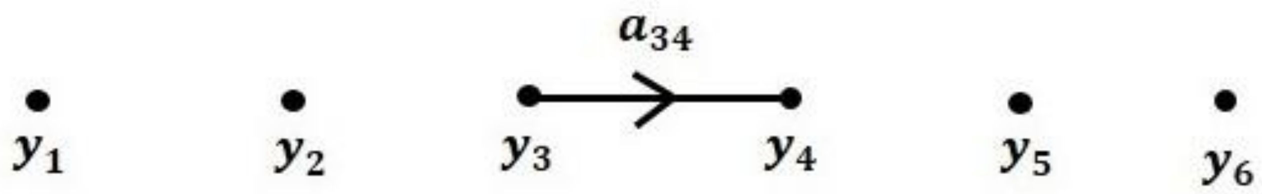
Step 1 – Signal flow graph for $y_2 = a_{13}y_1 + a_{42}y_4$ is shown in the following figure.



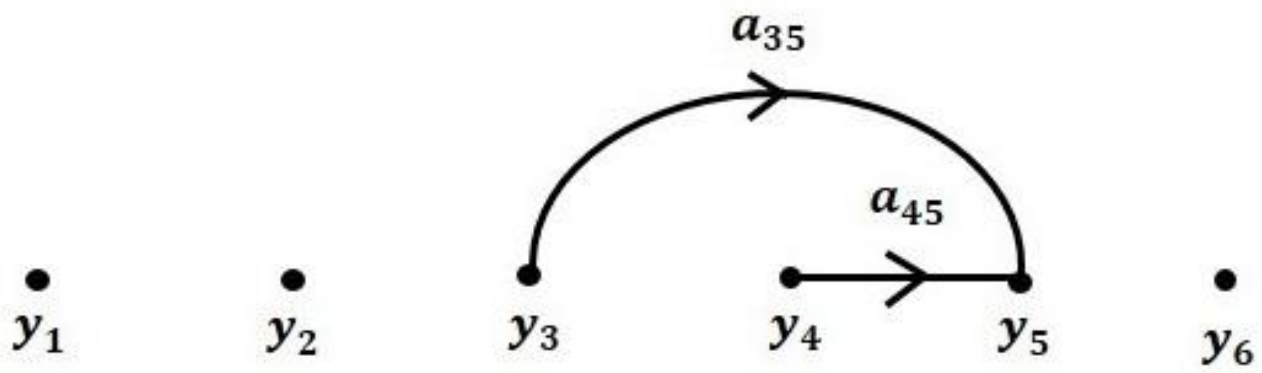
Step 2 – Signal flow graph for $y_3 = a_{23}y_2 + a_{53}y_5$ is shown in the following figure.



Step 3 – Signal flow graph for $y_4 = a_{34}y_3$ is shown in the following figure.

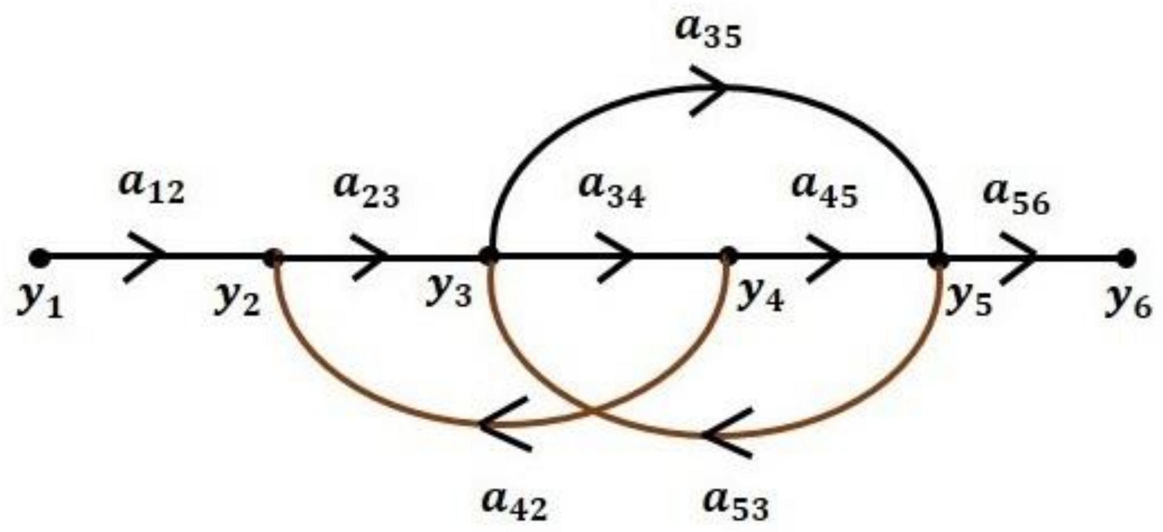


Step 4 – Signal flow graph for $y_5 = a_{45}y_4 + a_{35}y_3$ is shown in the following figure.



Step 5 – Signal flow graph for $y_6 = a_{56}y_5$ is shown in the following figure.

Step 6 – Signal flow graph of overall system is shown in the following figure.



ACTIVITY

ODD MAN OUT WITH REASON..?

108,120,132,144,156,164

Options :

A - 156

B - 164

C - 120

D - 108

Three of the following four are same in a certain way and hence form a group. Find out the one which does not belong to that group.

Options :

A - Violet

B - Black

C - Green

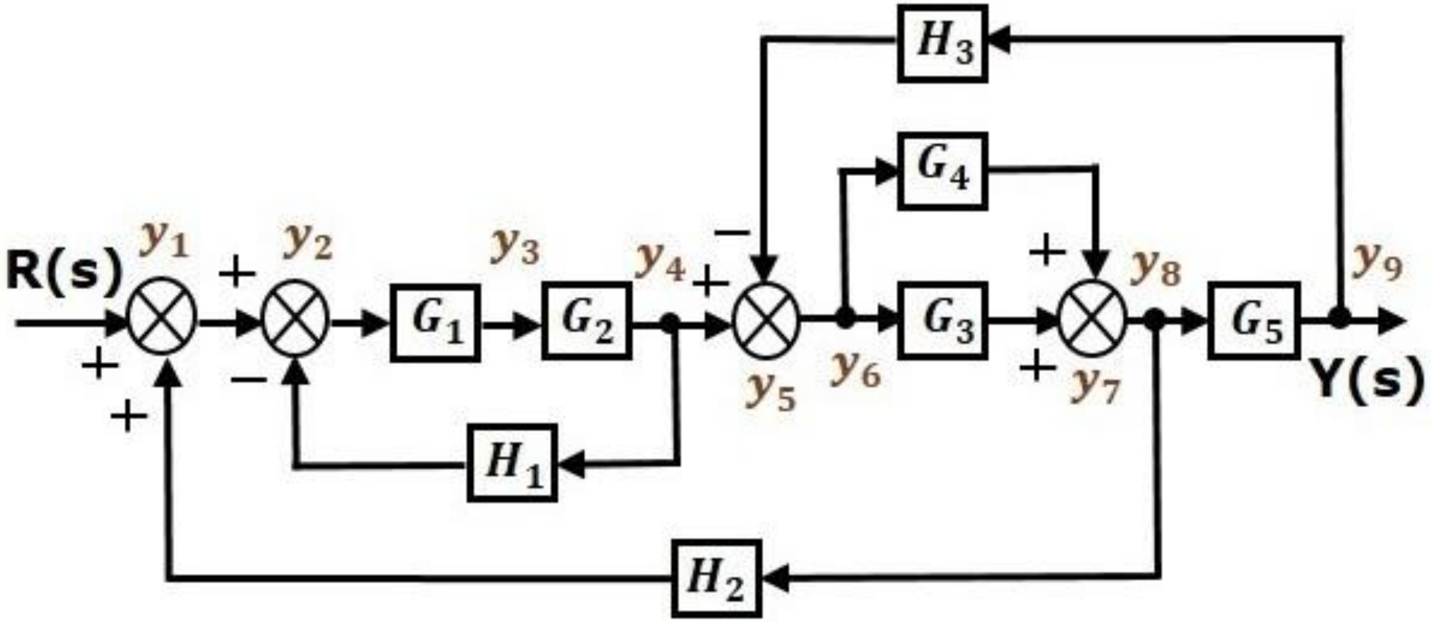
D - Red

CONVERSION OF BLOCK DIAGRAMS INTO SIGNAL FLOW GRAPHS

- Represent all the signals, variables, summing points and take-off points of block diagram as **nodes** in signal flow graph.
- Represent the blocks of block diagram as **branches** in signal flow graph.
- Represent the transfer functions inside the blocks of block diagram as **gains** of the branches in signal flow graph.
- Connect the nodes as per the block diagram.

EXAMPLE

Let us convert the following block diagram into its equivalent signal flow graph.



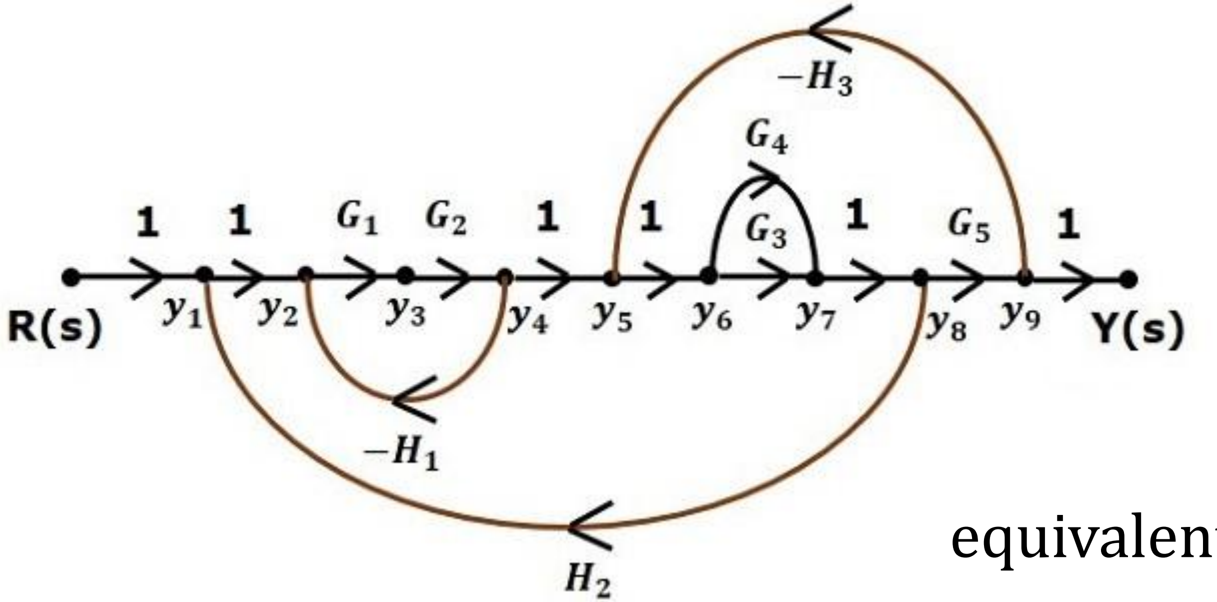
EXAMPLE ...

Represent the input signal R(s)R(s) and output signal C(s)C(s) of block diagram as input node R(s)R(s) and output node C(s)C(s) of signal flow graph.

Just for reference, the remaining nodes (y_1 to y_9) are labelled in the block diagram. There are nine nodes other than input and output nodes.

That is four nodes for four summing points, four nodes for four take-off points and one node for the variable between blocks G1G1 and G2G2.
The following figure shows the equivalent signal flow graph.

EXAMPLE …



equivalent signal flow graph

With the help of **Mason’s gain formula** you can calculate the transfer function of this signal flow graph.

This is the advantage of signal flow graphs.

To simplify (reduce) the signal flow graphs for calculating the transfer function.

MASON'S GAIN FORMULA:-

Mason's gain formula states that,

Overall gain, $T = \frac{1}{\Delta} \sum_{k} P_k \Delta_k$.

where, $T = T(s)$ = Transfer fn of the system.

P_k = Forward path gain of k^{th} forward path.

Δ = 1 − (Sum of individual loop gains)

+ [Sum of gain products of all possible combinations of two non-touching loops]

− [Sum of gain products of all possible combinations of three non-touching loops]

+ . . . . . .

Δ_k = Δ for that part of the graph which is not touching k^{th} forward path.

MASON'S GAIN FORMULA

Let us now discuss the Mason's Gain Formula. Suppose there are 'N' forward paths in a signal flow graph.

The gain between the input and the output nodes of a signal flow graph is nothing but the **transfer function** of the system. It can be calculated by using Mason's gain formula.

MASON’S GAIN FORMULA

- Represent all the signals, variables, summing points and take-off points of block diagram as **nodes** in signal flow graph.
- Represent the blocks of block diagram as **branches** in signal flow graph.
- Represent the transfer functions inside the blocks of block diagram as **gains** of the branches in signal flow graph.
- Connect the nodes as per the block diagram.

MASON'S GAIN FORMULA

$$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{N} P_i \Delta_i}{\Delta}$$

Where,

- **C(s)** is the output node
- **R(s)** is the input node
- **T** is the transfer function or gain between $R(s)$ and $C(s)$
- $\mathbf{P_i}$ is the i^{th} forward path gain

$$\Delta = 1 - (sum\ of\ all\ individual\ loop\ gains)$$

$$+(sum\ of\ gain\ products\ of\ all\ possible\ two\ nontouching\ loops)$$

$$-(sum\ of\ gain\ products\ of\ all\ possible\ three\ nontouching\ loops) + \ldots$$

Δ_i is obtained from Δ by removing the loops which are touching the i^{th} forward path.

Consider the following signal flow graph in order to understand the basic terminology involved here.

SUMMARY