SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF AUTOMOBILE ENGINEERING

19AUB303 – Finite Element Methods and Analysis

III YEAR / VI SEM

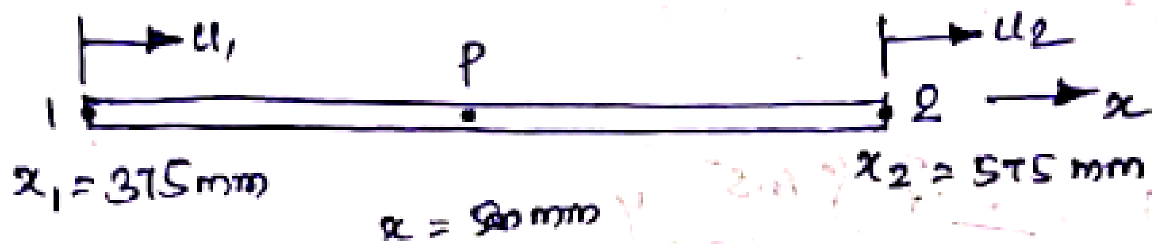
UNIT – 2 General Procedures of FEM

Topic – 5 – Problems on Formulation of Element Characteristic Matrices

(3) Consider a bar shown below, cross sectional area of the bar is $750\ mm^2$ & Young's Modulus is $2 \times 10^5\ N/mm^2$. If $u_1 = 0.5mm$ and $u_2 = 0.625mm$, calculate

(i) Displacement at point, P (ii) Strain, e (iii) Stress, σ (iv) Element Stiffness matrix [k]

(v) Strain Energy, U



To Find

To Find:-

(i) u at P (ii) e (iii) σ (iv) $[K]$ (v) U.

Soln:-

(i) Displacement at Point, 'P'

Actual length of the bar, $l = x_2 - x_1$

$= 575 - 375$

$l = 200\ mm$

The distance between 1 & Point P

$x = 500 - 375$

$x = 125\ mm$

W.K.T

Displacement function for two noded two elements $\}$ $(u) = N_1 u_1 + N_2 u_2$.

$N_1 = \frac{l - x}{l}$

$N_2 = \frac{x}{l}$.

$N_1 = \frac{200 - 125}{200}$.

$\boxed{N_1 = 0.375}$

$N_2 = \frac{125}{200}$.

$\boxed{N_2 = 0.625}$

$$u = (0.375)(0.5) + (0.625)(0.625)$$

(i) Displacement at Point P, $u = 0.578$ mm

(ii) Strain, e

$$e = [B]\{u\}$$

[B] - Strain displacement matrix.

$$[B] = \begin{bmatrix} \frac{-1}{l} & \frac{1}{l} \end{bmatrix}$$

$$e = \begin{bmatrix} \frac{-1}{200} & \frac{1}{200} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{200} & \frac{1}{200} \end{bmatrix} \begin{Bmatrix} 0.5 \\ 0.625 \end{Bmatrix}$$

$$= \left[\frac{-1}{200} \times 0.5 + \frac{1}{200} \times 0.625 \right]$$

$$\boxed{e = 6.25 \times 10^{-4}}$$

(iii) Stress, σ

$$\sigma = E e$$

$$= (2 \times 10^5)(6.25 \times 10^{-4})$$

$$\boxed{\sigma = 125 \ N/mm^2}$$

(iv) Stiffness matrix [K]

$$[K] = \frac{AE}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{750 \times 2\times10^5}{200}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K] = 7.5\times10^5\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

(v) Strain Energy, U

$$U = \frac{1}{2}\{u^*\}^T [k] \{u\}.$$

$$= \frac{1}{2}\begin{bmatrix}0.5 & 0.625\end{bmatrix} \times 7.5\times10^5 \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \begin{Bmatrix}0.5\\ 0.625\end{Bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}0.5 & 0.625\end{bmatrix} \cdot 7.5\times10^5 \begin{Bmatrix}+0.5 & -0.625\\ -0.5 & +0.625\end{Bmatrix}$$

$$= \frac{1}{2} \times 7.5\times10^5 \begin{bmatrix}0.5 & 0.625\end{bmatrix} \begin{Bmatrix}-0.125\\ 0.125\end{Bmatrix}$$

$$= \frac{1}{2} \times 7.5 \times 10^5 \left[0.5 (-0.125) + 0.625 (0.125) \right]$$

Strain Energy, $U = 5859.37$ N-mm.

Result

$u = 0.578$ mm

$e = 6.25 \times 10^{-4}$

$\sigma = 125$ N/mm^2

$U = 5859.37$ N-mm

$$[K] = 7.5 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Thank You