(5) Consider a bar of (i) An axial load of 200 kN is applied at point p.

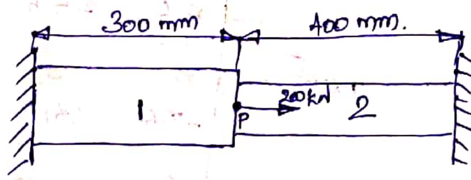
Take $A_1 = 2400\ mm^2$, $E_1 = 70\times10^9\ N/m^2$, $A_2 = 600\ mm^2$, $E_2 = 200\times10^9\ N/m^2$.

Calculate:

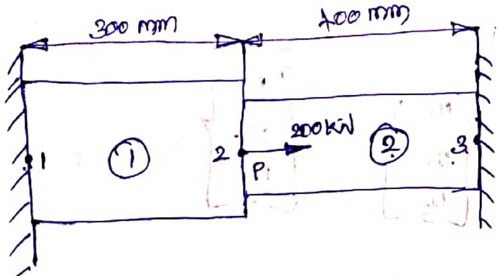
(a) The nodal displacement at point p.

(b) Stress in each material

(c) Reaction Force.



Given Data:-



$1\ m = 1000\ mm$

Area of element (1), $A_1 = 2400\ mm^2$

Young's Modulus (1), $E_1 = 70\times10^9\ N/m^2 = 70\times10^3\ N/mm^2$

Length of Element (1), $l_1 = 300\ mm$.

Point Load, $P = 200\ kN = 200\times10^3\ N$

Area of element (2), $A_2 = 600\ mm^2$

Young's Mod. (2), $E_2 = 200\times10^9\ N/m^2 = 200\times10^3\ N/mm^2$

Length of Element (2), $l_2 = 400\ mm$

To Find:-

(i) Nodal displacement at point (P), u_2

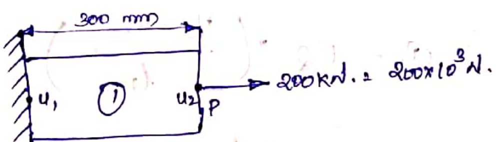
(ii) Stress in each material (σ), σ_1 & σ_2

(iii) Reaction Force R_1, R_2.

Solution:-

$$\{F\} = [K]\{u\}.$$

For element (1), (Nodes 1,2):



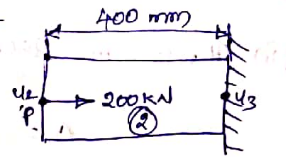
$$[K_1] = \frac{A_1E_1}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= \frac{2400\times70\times10^3}{300}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$[K_1] = 5.6\times10^5\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$\Rightarrow 1\times10^5 \begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \begin{bmatrix}5.6 & -5.6\\ -5.6 & 5.6\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix}.$$

For element (2), (nodes 2,3):-



$$\Rightarrow [K_2] = \frac{A_2E_2}{l_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= \frac{600\times200\times10^3}{400}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$[K_2] = 3\times10^5\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$\Rightarrow 1\times10^5 \begin{array}{cc} a_{22} & a_{23} \\ a_{32} & a_{33} \end{array} \begin{bmatrix}3 & -3\\ -3 & 3\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix} = \begin{Bmatrix}F_2\\ F_3\end{Bmatrix}$$

Assembling the finite elements ① & ②

$$\Rightarrow 1\times10^5 \begin{bmatrix} \overset{a_{11}}{5.6} & \overset{a_{12}}{-5.6} & \overset{a_{13}}{0} \\ \overset{a_{21}}{-5.6} & \overset{a_{22}}{5.6+3} & \overset{a_{23}}{-3} \\ \overset{a_{31}}{0} & \overset{a_{32}}{-3} & \overset{a_{33}}{3} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow 1\times10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

Applying boundary Conditions.

* Displacement at node 1, and node 3 is zero. $(u_1 = u_3 = 0)$
* Load of 200×10^3 N is acting at node 2, $F_2 = 200\times10^3$ N
* Self weight is neglected, $F_1 = F_3 = 0$

$$1\times10^5 \begin{bmatrix} \cancel{5.6} & \cancel{-5.6} & \cancel{0} \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 200\times10^3 \\ 0 \end{Bmatrix}$$

In the given problem the nodes 1 & 3 are fixed, so neglecting first row, first column & third row, third column of the [K] matrix.

$$1\times10^5 (8.6\, u_2) = 200\times10^3.$$

$$8.6\times10^5\, u_2 = 200\times10^3.$$

$$\boxed{u_2 = 0.2325 \text{ mm.}}$$

(ii) Stress in each element.

$$\sigma = E \frac{du}{dx}.$$

For Element ①, $\sigma_1 = E_1 \times \left(\frac{u_2 - u_1}{l_1}\right)$

$$= 70\times10^3 \times \frac{(0.2325 - 0)}{300}$$

$$\boxed{\sigma_1 = 54.25 \text{ N/mm}^2}$$

For Element ②

$$\sigma_2 = E_2 \times \left(\frac{u_3 - u_2}{l_2}\right)$$

$$= 200\times10^3 \frac{(0 - 0.2325)}{400}.$$

$$\boxed{\sigma_2 = -116.25 \text{ N/mm}^2}$$

Negative stress means that compressive stress is acting on element ②.

(iii) Reaction Force, $\{R\} = [K]\{u\} - \{F\}$.

$$\Rightarrow \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1\times10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} - \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1\times10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.2325 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 200\times10^3 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1\times10^5 \begin{bmatrix} 0-(5.6\times0.2325)+0 \\ 0+(8.6\times0.2325)+0 \\ 0-(3\times0.2325)+0 \end{bmatrix} - \begin{Bmatrix} 0 \\ 200\times10^3 \\ 0 \end{Bmatrix}$$

$$\underbrace{(3\times3)(3\times1)}_{} = (3\times1)$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1\times10^5 \begin{Bmatrix} -1.302 \\ 1.9995 \\ -0.6975 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 200\times10^3 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = \begin{Bmatrix} -1.302\times10^5 \\ -50\times\cancel{10^5} \\ -0.6975\times10^5 \end{Bmatrix}$$

Result:

(i) Nodal displacement at Point (P), $u_2 = 0.2325$ mm.

(ii) Stress in each material, $\sigma_1 = 54.25\ N/mm^2$ (Tensile)

$\sigma_2 = -116.25\ N/mm^2$ (Compressive)

(iii) Reaction Forces, $R_1 = -1.302\times10^5$ N.

$R_2 = -50$ N.

$R_3 = -0.6975\times10^5$ N.

x x