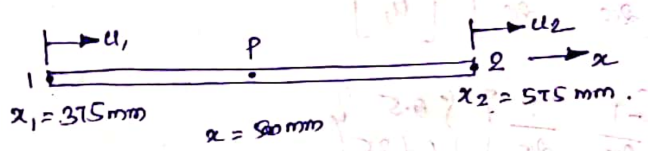
(3) Consider a bar shown below, cross sectional area of the bar is $750\ mm^2$ & Young's Modulus is $2\times10^5\ N/mm^2$. If $u_1 = 0.5mm$ and $u_2 = 0.625mm$, calculate (i) Displacement at point, P (ii) Strain, e (iii) Stress, σ (iv) Element stiffness matrix [k] (v) Strain Energy, U



Given Data:-

Area, $A = 750\ mm^2$

Young's Modulus, $E = 2\times10^5\ N/mm^2$

Displacements, $u_1 = 0.5mm$; $u_2 = 0.625mm$

Distance, $x_1 = 375mm$; $x_2 = 575mm$; $x = 500mm$

To Find:-

(i) u at P (ii) e (iii) σ (iv) [K] (v) U.

Soln:-

(i) Displacement at Point, 'p'

Actual length of the bar, $l = x_2 - x_1$

$= 575 - 375$

$l = 200\ mm$

The distance between 1 & Point P

$x = 500 - 375$

$x = 125\ mm$

W.K.T

Displacement function for two noded bar element of (u) $= N_1 u_1 + N_2 u_2$.

$N_1 = \frac{l-x}{l}$

$N_2 = \frac{x}{l}$.

$N_1 = \frac{200-125}{200}$.

$N_1 = 0.375$

$N_2 = \frac{125}{200}$.

$N_2 = 0.625$

$u = (0.375)(0.5) + (0.625)(0.625)$

(i) Displacement at Point P, $u = 0.578\ mm$

(ii) Strain, e

$e = [B]\{u\}$

[B] - Strain displacement matrix.

$[B] = \begin{bmatrix} \frac{-1}{l} & \frac{1}{l} \end{bmatrix}$

$e = \begin{bmatrix} \frac{-1}{200} & \frac{1}{200} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

$= \begin{bmatrix} \frac{-1}{200} & \frac{1}{200} \end{bmatrix} \begin{Bmatrix} 0.5 \\ 0.625 \end{Bmatrix}$

$= \begin{bmatrix} \frac{-1}{200} \times 0.5 + \frac{1}{200} \times 0.625 \end{bmatrix}$

$\boxed{e = 6.25 \times 10^{-4}}$

(iii) Stress, σ

$\sigma = E e$

$= (2\times10^5)(6.25\times10^{-4})$

$\boxed{\sigma = 125 \text{ N/mm}^2}$

(iv) Stiffness matrix [K]

$[K] = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$= \frac{750 \times 2\times10^5}{200} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$[K] = 7.5\times10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ //.

(v) Strain Energy, U

$U = \frac{1}{2} \{u^*\}^T [K] \{u\}$.

$\{u\} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

$\{u\}^T = [u_1 \quad u_2]$

$= \frac{1}{2} [0.5 \quad 0.625] \times 7.5\times10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.5 \\ 0.625 \end{Bmatrix}$

$= \frac{1}{2} [0.5 \quad 0.625] \cdot 7.5\times10^5 \begin{Bmatrix} +0.5 & -0.625 \\ -0.5 & +0.625 \end{Bmatrix}$

$(2\times2)\times(2\times1) = 2\times1$

$= \frac{1}{2} \times 7.5\times10^5 \begin{bmatrix} 0.5 & 0.625 \end{bmatrix} \begin{Bmatrix} -0.125 \\ 0.125 \end{Bmatrix}$

$$= \frac{1}{2} \times 7.5\times10^5 \left[0.5(-0.125) + 0.625(0.125)\right]$$

Strain Energy, $U = 5859.37$ N-mm.

Result

$u = 0.578$ mm

$e = 6.25 \times 10^{-4}$

$\sigma = 125$ N/mm^2

$U = 5859.37$ N-mm

$$[K] = 7.5\times10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$