1. Consider the differential eqn. for the problem Lecture - 2 - 17/09/2020.
3 - 19/09/2020

$$\frac{d^2y}{dx^2} + 300x^2 = 0, \quad 0 \le x \le 1$$

With the boundary condition $y(0) = 0$, $y(1) = 0$. Find the soln of the problem using the trial function $y = a_1 x(1-x^3)$. Use (i) Point collocation mtd (ii) Sub-domain collocation mtd, (iii) Least squares mtd, (iv) Galerkin's mtd.

Soln:-

Trial function $y = a_1 x(1-x^3)$

For $x = 0$, $y = a_1(0)(1-0) = 0$.

$x = 1$, $y = a_1(1)(1-1^3) = 0$.

(i) Point collocation method.

$$\frac{dy}{dx} = a_1 x(1-x^3) \longrightarrow a_1(x - x^4)$$

$$= a_1(1-3x^2) \longrightarrow a_1(1-4x^3)$$

nx^{n-1}

$$\frac{d^2y}{dx^2} = a_1(0-12x^2)$$

$$\boxed{d^2y/dx^2 = -12a_1x^2}$$

$$\frac{d^2y}{dx^2} + 300x^2 = 0, \quad \boxed{\frac{d^2y}{dx^2} = -12a_1x^2}$$

Residue $(R) = -12a_1x^2 + 300x^2 = 0$.

Boundary Condn: $0 \le x \le 1$.

Taking $x = \frac{1}{2}$.

$$= -12a_1(1/2)^2 + 300(1/2)^2 = 0.$$

$$= 3a_1 = 75$$

$$\Rightarrow \boxed{a_1 = 25}$$

Given trial fn } $Y = a_1 x(1-x^3)$

$$\boxed{Y = 25x(1-x^3)}$$ Ans//

(ii) Subdomain Collocation Method:

$$\int_0^1 R\,dx = 0.$$

$$\int_0^1 (-12a_1 x^2 + 300x^2)\,dx = 0.$$

$$\left[-\overset{4}{\cancel{12}}a_1 \frac{x^3}{\cancel{3}} + \overset{100}{\cancel{300}}\frac{x^3}{\cancel{3}}\right]_0^1 = 0.$$

$$\left[-4a_1 x^3 + 100x^3\right]_0^1 = 0.$$

$$\left[-4a_1 (1)^3 + 100(1)^3\right] - [0] = 0.$$

$$-4a_1 + 100 = 0.$$

$$-4a_1 = -100.$$

$$\boxed{a_1 = 25}$$

$Y = 25x(1-x^3)$ Ans

(iii) Least Squares method:-

$$I = \int_0^1 R^2\,dx$$

$$= \int_0^1 (-12a_1x^2 + 300x^2)^2\,dx.$$

$$= \int_0^1 (144a_1^2x^4 + 90000x^4 - 7200a_1x^4)\,dx$$

$$= \left[144a_1^2\frac{x^5}{5} + 90000\frac{x^5}{5} - 7200a_1\frac{x^5}{5}\right]_0^1$$

$$= \left[\frac{144a_1^2}{5} + \frac{90000}{5} - \frac{7200a_1}{5}\right]$$

$$\frac{\partial I}{\partial a_1} = 0 \Rightarrow \frac{288}{5}a_1 = \frac{7200}{5}$$

$$\frac{288}{\cancel{5}}a_1 = \frac{\cancel{5}}{7200}.$$

$$288a_1 = 7200 \Rightarrow a_1 = \frac{7200}{288}$$

$$\boxed{a_1 = 25}$$

$Y = 25x(1-x^3)$ Ans.

(iv) Galerkin Method:

$$\int_0^1 (y \cdot R)\,dx = 0.$$

$$\int_0^1 a_1x(1-x^3)(-12a_1x^2 + 300x^2)\,dx = 0.$$

$$\int_0^1 (a_1x - a_1x^4)(-12a_1x^2 + 300x^2)\,dx = 0$$

$$\int_0^1 (-12a_1^2x^3 + 300a_1x^3 + 12a_1^2x^6 - 300a_1x^6)\,dx = 0.$$

$$\left[-12a_1^2\frac{x^4}{4} + 300a_1\frac{x^4}{4} + 12a_1^2\frac{x^7}{7} - 300a_1\frac{x^7}{7}\right]_0^1 = 0.$$

$$-\frac{12a_1^2}{4} + \frac{300}{4}a_1 + \frac{12a_1^2}{7} - \frac{300a_1}{7} = 0 \dashrightarrow \text{dividing by } (-12a_1)$$

$$a_1/4 - \frac{25}{4} - \frac{a_1}{7} + \frac{25}{7} = 0.$$

$$\frac{7a_1 - 4a_1}{28} \Rightarrow \boxed{a_1 = 25}$$

$Y = 25x(1-x^3)$ Ans.