SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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DEPARTMENT OF AUTOMOBILE ENGINEERING

19AU303 – Finite Element Methods and Analysis

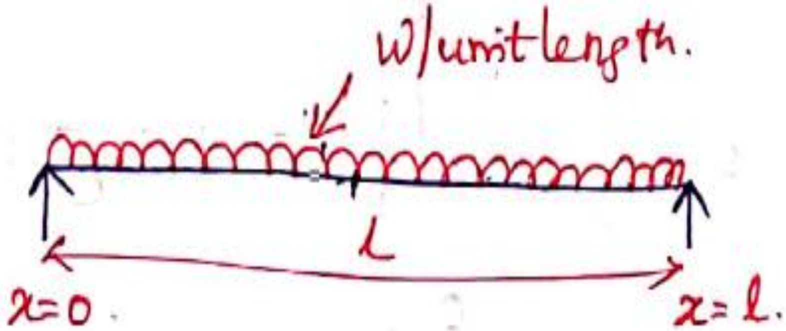
III YEAR / VI SEM

UNIT – 1 - Introduction

Topic – Rayleigh Ritz Method

Rayleigh Ritz Method- Problem

A Simply Supported beam Subjected to udl over entire Span. Determine the bending moment and deflection at midspan by using Rayleigh - Ritz method & Compare with exact Solutions



To Find:

(i) Deflection & Bending moment

(ii) Compare with exact Soln.

Solution

Soln:-

For SSB, the fourier series,

$$Y = \sum_{n=1,3}^{\infty} a \sin \frac{n\pi x}{l} \quad \text{(approximating fn.)}$$

Let us consider

$$\text{deflection},\ y = a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{3\pi x}{l}. \quad [a_1, a_2 - \text{Ritz parameters}]$$

Total potential Energy of the beam, $\pi = U - H$

Strain Energy of the beam due to bending,

$$U = \frac{EI}{2} \int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx.$$

U – Strain Energy

H – Work done by ext. force

$$y = a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{3\pi x}{l}$$

$$\frac{dy}{dx} = a_1 \cos \frac{\pi x}{l} \left(\frac{\pi}{l}\right) + a_2 \cos \frac{3\pi x}{l} \cdot \left(\frac{3\pi}{l}\right)$$

$$= \frac{a_1 \pi}{l} \cdot \cos \frac{\pi x}{l} + \frac{a_2 3\pi}{l} \cdot \cos \frac{3\pi x}{l}.$$

$$\frac{d^2y}{dx^2} = -\frac{a_1 \pi}{l} \sin \frac{\pi x}{l} \times \frac{\pi}{l} - a_2 \frac{3\pi}{l} \sin \frac{3\pi x}{l} \times \frac{3\pi}{l}.$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-\pi^2 a_1}{l^2} \sin \frac{\pi x}{l} - a_2 \frac{9\pi^2}{l^2} \sin \frac{3\pi x}{l}} \dots \to (1).$$

$$U = \frac{EI}{2} \int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx \quad \rightarrow \text{Sub. } (1)$$

$$= \frac{EI}{2} \int_0^l \left[-\frac{a_1 \pi^2}{l^2} \sin \frac{\pi x}{l} - \frac{a_2 9\pi^2}{l^2} \sin \frac{3\pi x}{l} \right]^2 dx.$$

$$= \frac{EI}{2} \times \frac{\pi^4}{l^4} \int_0^l \left[a_1 \sin \frac{\pi x}{l} + 9 a_2 \sin \frac{3\pi x}{l} \right]^2 dx$$

$$\boxed{(a+b)^2 = a^2+b^2+2ab.}$$

$$U = \frac{EI}{2} \times \frac{\pi^4}{l^4} \cdot \int_0^l \left[a_1^2 \sin^2\frac{\pi x}{l} + 81 a_2^2 \sin^2\frac{3\pi x}{l} + 2\left(a_1 \sin\frac{\pi x}{l}\right)\left(9 a_2 \sin\frac{3\pi x}{l}\right)\right] dx$$

$$U = \frac{EI}{2} \frac{\pi^4}{l^4} \int_0^l \left[a_1^2 \sin^2\frac{\pi x}{l} + 81 a_2^2 \sin^2\frac{3\pi x}{l} + 18 a_1 a_2 \sin\frac{\pi x}{l} \sin\frac{3\pi x}{l}\right] dx$$

$$\int_0^l a_1^2 \sin^2\frac{\pi x}{l}\, dx = a_1^2 \int_0^l \frac{1}{2}\left(1-\cos\frac{2\pi x}{l}\right) dx.$$

$$\boxed{\sin^2 x = \frac{1-\cos 2x}{2}.}$$

$$= \frac{a_1^2}{2} \int_0^l \left(1 - \cos\frac{2\pi x}{l}\right) dx.$$

$$= \frac{a_1^2}{2} \left[\int_0^l dx - \int_0^l \cos\frac{2\pi x}{l} \cdot dx \right]$$

$$= \frac{a_1^2}{2} \left[(x)_0^l - \left(\frac{\sin\frac{2\pi x}{l}}{\frac{2\pi}{l}} \right)_0^l \right]$$

$$= \frac{a_1^2}{2} \left[l - 0 - \frac{1}{2\pi}\left(\sin\frac{2\pi l}{l} - \sin 0 \right) \right]$$

$$= \frac{a_1^2}{2}\left[l - 0 - \frac{1}{2\pi}\left(\sin\frac{2\pi l}{l} - \sin 0 \right) \right]$$

$$\left[\sin 2\pi = 0; \quad \sin 0 = 0 \right]$$

$$\frac{a_1^2}{2}\left[l - \frac{1}{2\pi}(0-0) \right]$$

$$\boxed{\int_0^l a_1^2 \sin^2\frac{\pi x}{l}\,dx = \frac{a_1^2 l}{2}}$$

$$= \frac{a_1^2 l}{2}.$$

$$\int_0^l 81 a_2^2 \sin^2\frac{3\pi x}{l} = 81 a_2^2 \int_0^l \frac{1}{2}\left(1 - \cos\frac{6\pi x}{l}\right)$$

$$\left[\sin^2 x = \frac{1-\cos 2x}{2} \right]$$

$$= \frac{81}{2} a_2^2 \left[(x)_0^l - \left(\frac{\sin\frac{6\pi x}{l}}{\frac{6\pi}{l}} \right)_0^l \right]$$

$$= \frac{81 a_2^2}{2}\left[l - 0 - \frac{1}{6\pi}\left(\sin\frac{6\pi l}{l} - \sin 0\right)\right]$$

$$= \frac{81 a_2^2}{2}\left[l - \frac{1}{6\pi}\left(\sin 6\pi - \sin 0\right)\right]$$

$$\boxed{\sin 6\pi = 0 \; ; \; \sin 0 = 0}$$

$$\frac{81 a_2^2}{2}\left[l - 0\right]$$

$$= \frac{81 a_2^2 l}{2}.$$

$$\int_0^l 18 a_1 a_2 \sin\frac{\pi x}{l}\sin\frac{3\pi x}{l}.$$

$$= 18 a_1 a_2 \int_0^l \sin\frac{\pi x}{l}\sin\frac{3\pi x}{l}. = 18 a_1 a_2 \int_0^l \sin\frac{3\pi x}{l}\sin\frac{\pi x}{l}.$$

$$= \frac{18 a_1 a_2}{2}\left[\int_0^l \cos\frac{2\pi x}{l}dx - \int_0^l \cos\frac{4\pi x}{l}dx\right] \quad \boxed{\sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}}$$

$$= \frac{\overset{9}{\cancel{18}}\, a_1 a_2}{\cancel{2}} \left[\left(\frac{\sin \frac{2\pi x}{l}}{\frac{2\pi}{l}} \right)_0^l - \left(\frac{\sin \frac{4\pi x}{l}}{\frac{4\pi}{l}} \right)_0^l \right]$$

$$\boxed{\begin{array}{l} \sin 2\pi = 0 \\ \sin 4\pi = 0 \\ \sin 0 = 0 \end{array}}$$

$$= 9 a_1 a_2 \left[0 - 0 \right].$$

$$= 0.$$

Thank You