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POISSON DISTRIBUTION

Definition: A random voniable X is said to follow Poisson distribution if it assumes only non-negative Values and its probability mass function is given by,

$$P(X = \chi) = p(\chi) = \begin{cases} \frac{e^{-\lambda} \lambda^{\chi}}{\chi!}, \chi = 0, 1, 2, \dots, \infty \\ \frac{\pi!}{\chi!}, \text{ otherwise} \end{cases}$$

Moment Generating function -

$$\begin{split} M_{\chi}(t) &= \pounds E(e^{t\chi}) \\ &= \frac{\infty}{\chi_{=0}} e^{t\chi} \dot{p}(\chi) \\ &= \frac{\infty}{\chi_{=0}} \frac{e^{-\lambda} \lambda^{\chi}}{\chi!} e^{t\chi} \\ &= e^{-\lambda} \frac{\infty}{\chi_{=0}} \frac{(\lambda e^{t})^{\chi}}{\chi!} \\ &= e^{-\lambda} \left[1 + \frac{\lambda e^{t}}{1!} + \frac{(\lambda e^{t})^{2}}{\chi!} + \cdots \right] \\ &= e^{-\lambda} \left[1 + \frac{\lambda e^{t}}{1!} + \frac{(\lambda e^{t})^{2}}{\chi!} + \cdots \right] \\ &= e^{-\lambda} e^{\lambda e^{t}} \qquad (\ \because e^{\chi} = 1 + \frac{\chi}{1!} + \frac{\chi^{2}}{\chi!} + \cdots \right] \\ &= M_{\chi}(t) = e^{\lambda(e^{t} - 1)} \end{split}$$

Mean and Variance:

$$\mu_{i}' = E(x)$$

$$= \frac{\infty}{x = 0} \times p(z)$$

$$= \frac{\infty}{x = 0} \times \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$= \overline{e^{\lambda}} \left[0 + 1 \cdot \frac{\lambda}{1!} + \frac{2 \cdot \lambda^{2}}{2!} + 3 \frac{\lambda^{3}}{3!} + \cdots \right]$$

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$$\begin{aligned} \mu_{i} &= \lambda c \qquad \left[1 + \frac{\lambda}{i!} + \frac{\lambda}{k!} + \frac{\lambda^{-}}{3!} + \cdots \right] \\ &= \lambda e^{-\lambda} e^{\lambda} \end{aligned}$$

$$\begin{aligned} \mu_{i}' &= \operatorname{Mean} &= \lambda \end{aligned}$$

$$\begin{aligned} \mu_{i}' &= E(x^{2}) \\ &= \frac{\infty}{2} x^{2} \cdot p(x) \\ &= \frac{\infty}{2} x^{2} \cdot \frac{e^{-\lambda} \lambda^{x}}{x!} \\ &= \frac{\infty}{x = 0} \left[x(x-i) + x \right] \frac{e^{\lambda} \lambda^{x}}{x!} \\ &= \frac{\infty}{x = 0} \pi(x-i) \frac{e^{-\lambda} \lambda^{x}}{x!} + \frac{x}{x = 0} x \frac{e^{-\lambda} \lambda^{x}}{x!} \\ &= \frac{\infty}{x = 0} \frac{\pi(x+i) e^{-\lambda} \lambda^{x-\lambda} \lambda^{\lambda}}{\pi(x+i) (x-\lambda) \cdots i} + \lambda \quad (\cdots \mu_{i}' = \lambda) \end{aligned}$$

$$\begin{aligned} &= e^{-\lambda} \lambda^{2} \frac{\infty}{x = 0} \frac{\lambda^{x-2}}{(x-2)!} + \lambda \\ &= e^{-\lambda} \lambda^{2} \frac{\infty}{x = 0} \frac{\lambda^{x-2}}{(x-2)!} + \lambda \\ &= e^{-\lambda} \lambda^{2} \sum_{x=0}^{n} \frac{\lambda^{x-2}}{(x-2)!} + \lambda \end{aligned}$$

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(or)

Mean:

 $M_x(t) = e^{\lambda(e^t - i)} = e^{\lambda e^t} e^{-\lambda}$ $M_{x}'(t) = \lambda e^{t} e^{\lambda e^{t}} e^{-\lambda}$ $M_{x}'(o) = \lambda \cdot e^{\lambda} \cdot e^{-\lambda}$ $M_{x}'(o) = \lambda$ $Mean = E(x) = M_{x}'(o) = \lambda$ $M_{x}''(t) = E(x^{2}) = (\lambda e^{t})^{2} e^{\lambda e^{t}} e^{-\lambda} + \lambda e^{\lambda e^{t}} e^{-\lambda}$ $M_{x}''(0) = \lambda^{2} e^{\lambda} e^{-\lambda} + \lambda e^{\lambda} e^{-\lambda}$ $E(x^{2}) = M_{x}''(o) = \lambda^{2} + \lambda$ Variance = $E(x^2) - [E(x)]^2$ $= \lambda^2 - \lambda^2 + \lambda$ Variance $= \lambda$



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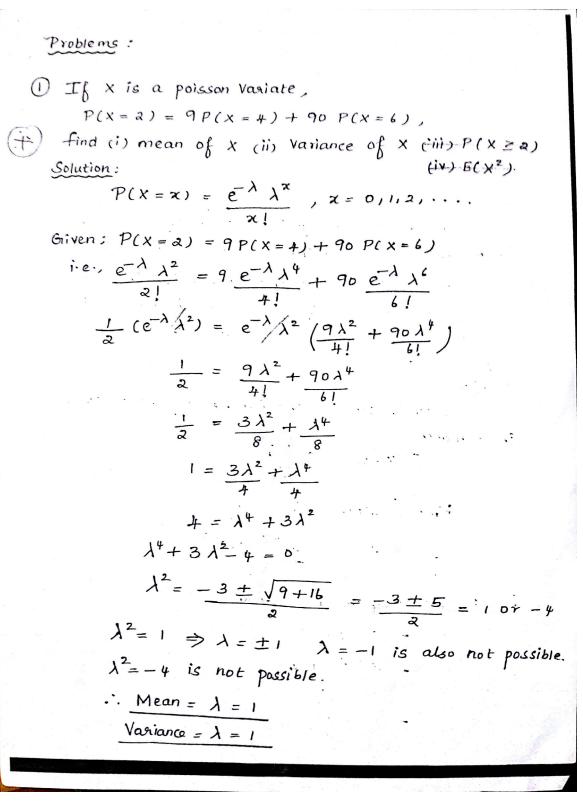


Prove that poisson distribution is the limiting case of binomial distribution. Suppose in a binomial distribution, 1. The number of trials is indefinitely large i.e., n -> 00 a. p is very small i.e. p -> 0 3. $np = \lambda$ is finite. Now $P(X = x) = nC_x p^x q^{n-x}$, x = 0, 1, 2, ... n $= \frac{n(n-1)(n-2)\cdots(n-x+1)}{x!} p^{x} q^{n-1}$ $= \frac{n(n-1)(n-2)\cdots(n-x+1)}{x!} \left(\frac{\lambda}{n}\right)^{x} \binom{1-\lambda}{n}$ $=\frac{\lambda^{\alpha}}{\alpha !}\int \left(\left(\frac{1-1}{n}\right) \left(\frac{1-2}{n}\right) \cdots \left(\left(\frac{1-2}{n}\right) \left(\frac{1-\lambda}{n}\right) \right)^{n} \left(\frac{1-\lambda}{n}\right)$ Taking limit as n -> 00, $\frac{1t}{n \to \infty} p(x) = \frac{\lambda^{\chi}}{x!} e^{-\lambda} \text{ for } \chi = 0, 1, 2, \dots$ $\left(\begin{array}{c} \cdot \cdot \cdot \pm t \\ \cdot \cdot \cdot - \lambda \end{array}\right)^{n} = e^{-t}$ which is the p.m.f of the poisson distribution.



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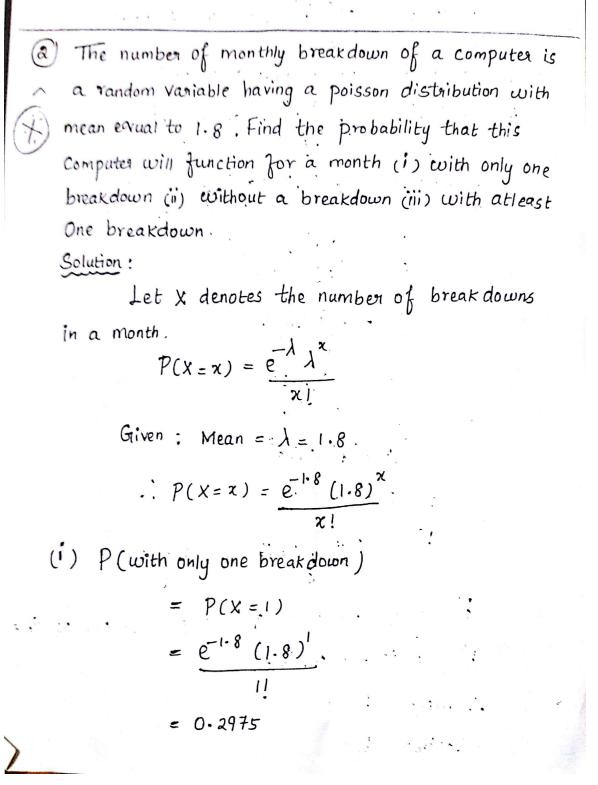
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(ii)
$$P(\text{ without a break down}) = P(X = 0)$$

$$= \frac{e^{-1.8}(1.8)}{0!} = e^{-1.8}$$

$$= 0.1653$$
(i) $P(\text{Atleast one breakdown}) = P(X \ge 1)$

$$= 1 - P(X \ge 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - 0.1653$$

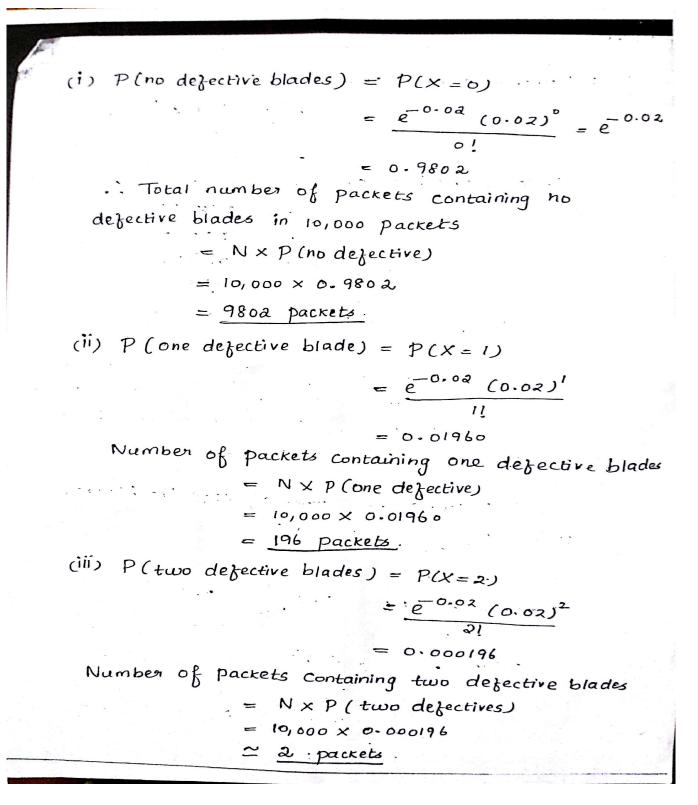
$$= 0.8347$$
(i) The certain hactory turning vazar blades there is a small chance of 1/500 for any blade to be defective:
The blades are in packets of 10. Use poisson disbibution to calculate the approximate number of packets containing (i) no defective (ii) One defective (iii) a defective blades respectively in a consignment of 10,000 packets.
Solution:
Let X denote the number of defective blades.
Given: $P = \frac{1}{500}$, $n = 10$.
 $N = 10,000$
Mean = $\lambda = np = 10 \times \frac{1}{500} = 0.02$
 $\lambda = 0.02$
 $P(X = X) = \frac{e^{-\lambda} X^{X}}{X!}$
 $= \frac{e^{-0.02}}{(0.02)^{X}}$

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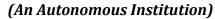
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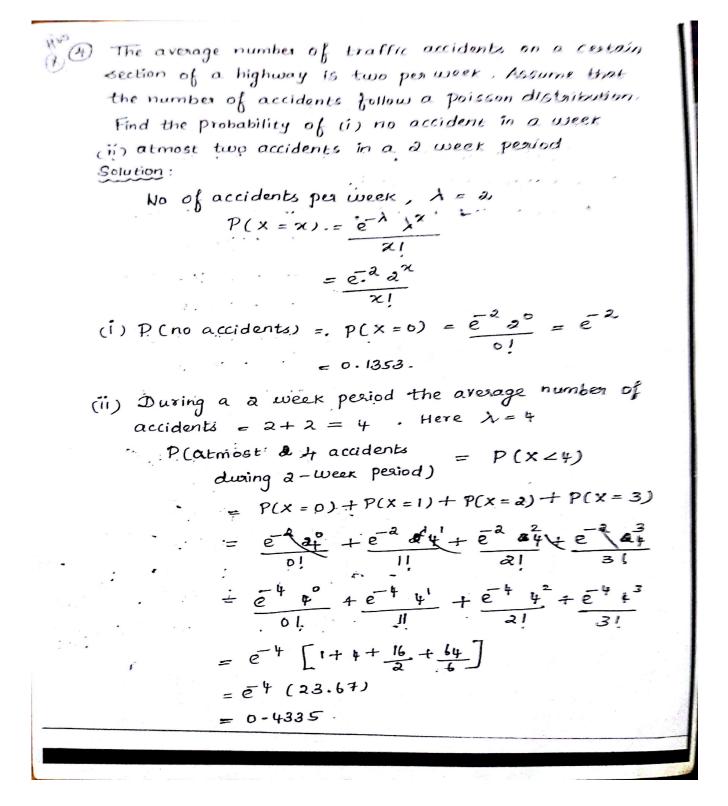


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If x and y are independent random Variables. Show that the conditional distribution of X given X+Y is a binomial distribution. Solution : Let X and Y be independent random variables with parameters λ_1 & λ_d . $P(x = r / x + y = 4) = P(x = r \cap x + y = 4)$ $\frac{P(x=r \cap Y = \sqrt{e}-r)}{P(x+Y = \sqrt{e})}$ $\frac{P(x=r) \cdot P(Y=s-r)}{P(x+Y=s)}$ $= \underbrace{\frac{e^{-\lambda_{1}}}{r_{1}}}_{e^{-\lambda_{2}}} \underbrace{\frac{e^{-\lambda_{2}}}{c_{2}}}_{\frac{\lambda_{2}}{r_{2}}} \underbrace{\frac{e^{-\lambda_{2}}}{c_{2}}}_{\frac{\lambda_{2}}{r_{2}}}$ $= \frac{\mathscr{B}(n)}{r!(\mathscr{B}(-r))!} \frac{\lambda_{i}^{r} \lambda_{2}^{\mathscr{B}-r}}{(\lambda_{i}+\lambda_{2})^{\mathscr{B}-r+r}}$ = $hc_{r} h c_{r} \left(\frac{\lambda_{i}}{\lambda_{i}+\lambda_{2}}\right)^{*} \left(\frac{\lambda_{2}}{\lambda_{i}+\lambda_{2}}\right)^{*}$ = $hc_{r} p^{r} q^{n-r}$ where $p = \frac{\lambda_{i}}{\lambda_{i}+\lambda_{2}}$ $q = \frac{\lambda_{2}}{\lambda_{i}+\lambda_{2}}$