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#### **DEPARTMENT OF MATHEMATICS**

### RANDOM VARIABLES

Definition: A real variable 'x' whose value is determined by the outcome of a random experiment is called a random variable.

a coin. Consider the random variable which is the number of heads (0,1 or 2).

Outcome : HH HT TH TT

Value of x : 2 1 1 0

Types of random Variables:

There are two types of random variables:

- 1. Discrete random vaniable
- 2 Continuous random Variable.

Discrete random Variable:

A random Variable which can assume only a Countable number of real Values is called a discrete random Variable.

## Examples :

- 1. Number of telephone calls per unit time
- 2. Marks obtained in a test.

Distribution function (or) Cumulative Distribution function of the random Variable X:

The C.D.F of a random variable x is defined

$$F(x) = P(x \le x) = \sum_{x \le x} P(x_i)$$

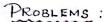
in (-0,0)



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1) Find the Constant 'k' from the following probability distribution of a discrete random Variable X.

Values of $X = x$	1	2	3	4	5	Total
(x)	0.1	0.2	K	аĸ	0.1	1

Solution: We know that,

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

$$\sum_{i=1}^{5} p(x_i) = 1$$

$$p(1) + p(2) + p(3) + p(4) + p(5) = 1$$

$$0.1 + 0.2 + k + 2k + 0.1 = 1$$

$$3k + 0.4 = 1$$

$$3k = 1 - 0.4 = 0.6$$

$$3k = 0.6$$

$$k = 0.6$$

$$k = 0.6$$

$$1 + 0.2$$

- (2) For the following probability distribution,
  - (1) Find the distribution function of X,
  - (ii) What is the Smallest Value of 'x' for which  $P(x \le x) > 0.5$

### Solution:

(i) The distribution function of X is given by,



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X = X	$F(x) = P(x \leq x)$
0	$F(0) = P(x \le 0) = P(x = 0) = \frac{1}{4} = 0.25$
1	$F(1) = P(X \le 1) = P(X = 0) + P(X = 1)$ $= \frac{1}{4} + \frac{2}{4} = \frac{3}{4} = 0.75$
2	$F(2) = P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$ $= \frac{1}{4} + \frac{2}{4} + \frac{1}{4} = 1$

- (ii) The smallest value of 'x' for which P(x = x) > 0.5
- A random Variable 'X' has the following probability function Value of  $X = \alpha$ K P(x) 2K ak 3K 2K2
  - (i) Find K (ii) Evaluate P(X < b),  $P(X \ge 6)$  and P(0 < X < 5)
  - (iii) If  $P(X \le K) > \frac{1}{2}$ , find the minimum value of K and determine the distribution function of X.

# Solution :

(i) We know that,

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

$$\sum_{i=0}^{7} p(x_i) = 1$$

$$p(0) + p(1) + p(2) + \dots + p(7) = 1$$
  
 $0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$ 



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$$|0 \times -1 \times -1 \times$$

$$K = \frac{\lambda}{20}, \frac{-20}{20}$$

$$K = \frac{1}{10} \quad \text{or} \quad K = -1$$

K = -1 is not possible, Since probability cannot be a negative Value.

$$\therefore K = \frac{1}{10}$$

subs K = 1 in the given table,

(ii) 
$$P(X \ge 6) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

$$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100}$$

$$= \frac{10 + 20 + 20 + 30 + 1}{100}$$

$$P(X < 6) = \frac{81}{100}$$

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$$P(X \ge 6) = 1 - P(X \ge 6)$$
  
=  $1 - \frac{81}{100} = \frac{100 - 81}{100}$ 

$$P(x \ge 6) = \frac{19}{100}$$

$$P(0 \le X \le 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$$

$$= \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10}$$

$$= \frac{8}{10} = \frac{4}{5}$$

$$P(0 < x < 5) = 4$$

$$P(x \le 3) = P(x = 0) + \cdots + P(x = 3)$$

$$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10}$$

$$P(x \leq \underline{+}) = P(x = 0) + \cdots$$

$$X = x F(x) = P(x \le x)$$

$$0 F(0) = P(x \le 0) = P(x = 0) = 0$$

$$1 F(1) = P(x \le 1) = P(x = 0) + P(x = 1)$$

$$= 0 + \frac{1}{10} = \frac{1}{10} = \sqrt{10}$$

2 
$$F(2) = P(x \le 2) = P(x = 0) + P(x = 1) + P(x = 2)$$
  
=  $0 + \frac{1}{10} + \frac{2}{10} = \frac{3}{10}$ 

3 
$$F(3) = P(X \le 3) = P(X = 0) + P(X = 1) + P(X = 2)$$
  
+  $P(X = 3)$   
=  $0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} = \frac{5}{10}$ 



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