

CAMS

INTRODUCTION

A cam is a mechanical device used to transmit motion to a follower by direct contact. The driver is called the cam and the driven member is called the follower. In a cam follower pair, the cam normally rotates while the follower may translate or oscillate. A familiar example is the camshaft of an automobile engine, where the cams drive the push rods (the followers) to open and close the valves in synchronization with the motion of the pistons.

Types of cams

Cams can be classified based on their physical shape.

a) Disk or plate cam (Fig. 6.1a and b): The disk (or plate) cam has an irregular contour to impart a specific motion to the follower. The follower moves in a plane perpendicular to the axis of rotation of the camshaft and is held in contact with the cam by springs or gravity.

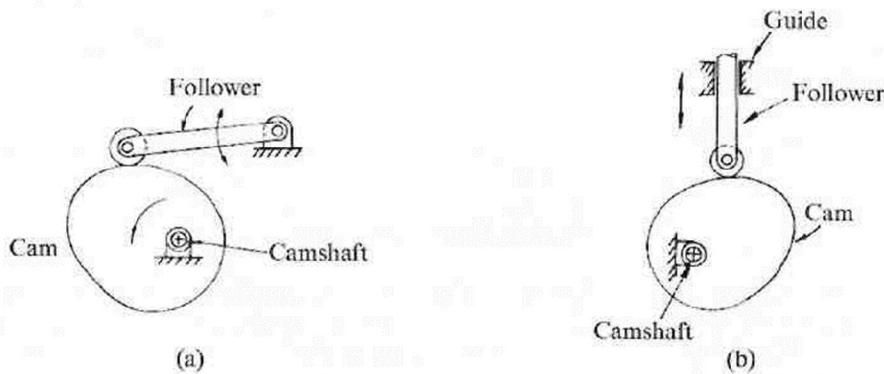


Fig. 6.1 Plate or disk cam.

b) Cylindrical cam (Fig. 6.2): The cylindrical cam has a groove cut along its cylindrical surface. The roller follows the groove, and the follower moves in a plane parallel to the axis of rotation of the cylinder.

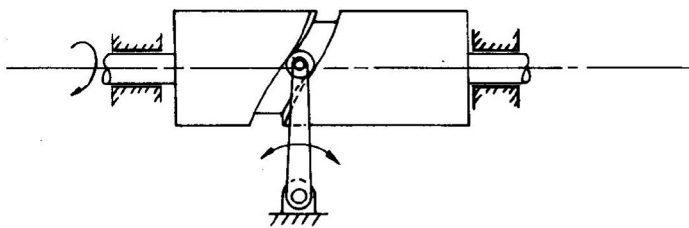


Fig. 6.2 Cylindrical cam.

c) Translating cam (Fig. 6.3a and b). The translating cam is a contoured or grooved plate sliding on a guiding surface(s). The follower may oscillate (Fig. 6.3a) or reciprocate (Fig. 6.3b). The contour or the shape of the groove is determined by the specified motion of the follower.

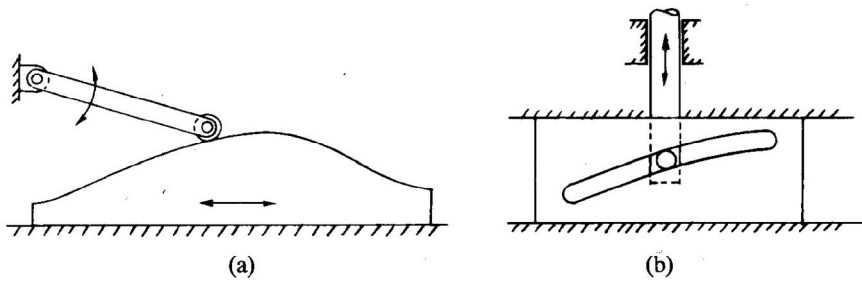


Fig. 6.3 Translating cam

Types of followers:

(i) Based on surface in contact. **(Fig.6.4)**

- (a) Knife edge follower
- (b) Roller follower
- (c) Flat faced follower
- (d) Spherical follower

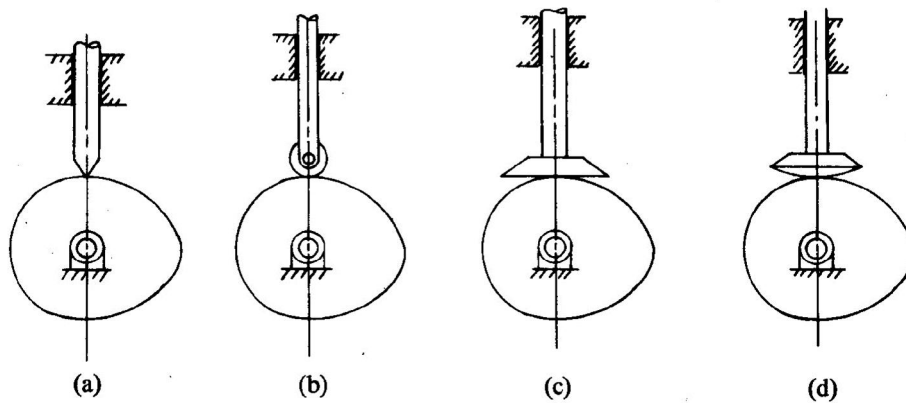


Fig. 6.4 Types of followers

(ii) Based on type of motion: **(Fig.6.5)**

- (a) Oscillating follower
- (b) Translating follower

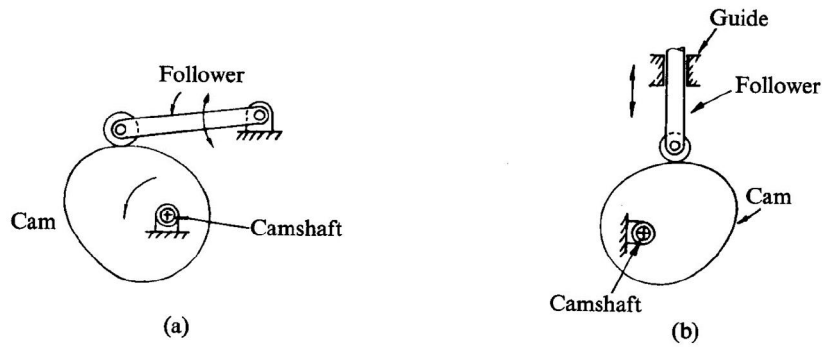


Fig.6.5

(iii) Based on line of motion:

- (a) Radial follower: The lines of movement of in-line cam followers pass through the centers of the camshafts (Fig. 6.4a, b, c, and d).
- (b) Off-set follower: For this type, the lines of movement are offset from the centers of the camshafts (Fig. 6.6a, b, c, and d).

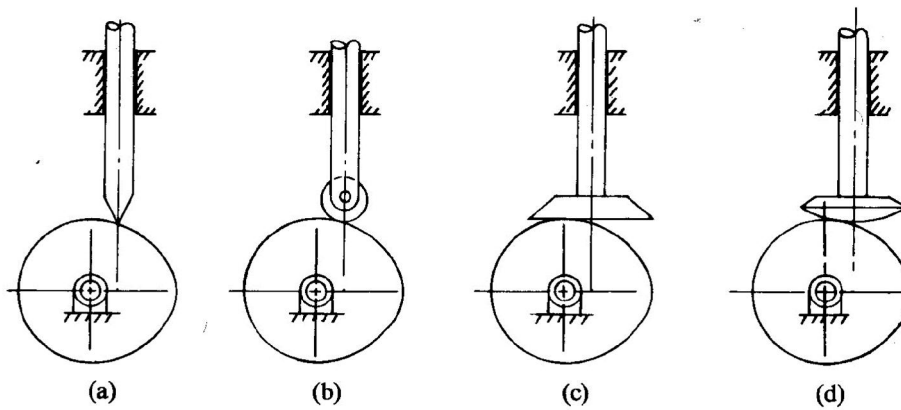


Fig.6.6 Off set followers

Cam nomenclature (Fig. 6.7):

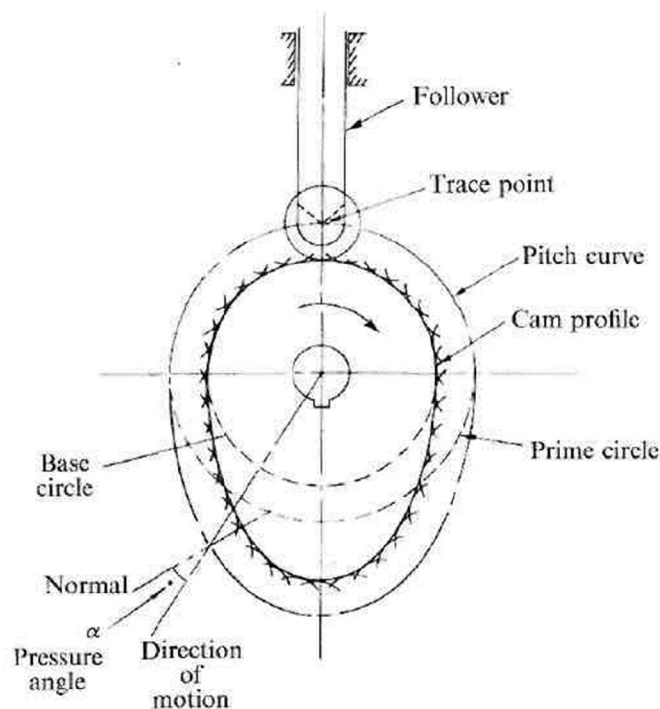


Fig.6.7

Cam Profile The contour of the working surface of the cam.

Tracer Point The point at the knife edge of a follower, or the center of a roller, or the center of a spherical face.

Pitch Curve The path of the tracer point.

Base Circle The smallest circle drawn, tangential to the cam profile, with its center on the axis of the camshaft. The size of the base circle determines the size of the cam.

Prime Circle The smallest circle drawn, tangential to the pitch curve, with its center on the axis of the camshaft.

Pressure Angle The angle between the normal to the pitch curve and the direction of motion of the follower at the point of contact.

Types of follower motion:

Cam follower systems are designed to achieve a desired oscillatory motion. Appropriate displacement patterns are to be selected for this purpose, before designing the cam surface. The cam is assumed to rotate at a constant speed and the follower raises, dwells, returns to its original position and dwells again through specified angles of rotation of the cam, during each revolution of the cam.

Some of the standard follower motions are as follows:

They are, follower motion with,

- (a) Uniform velocity

- (b) Modified uniform velocity
- (c) Uniform acceleration and deceleration
- (d) Simple harmonic motion
- (e) Cycloidal motion

Displacement diagrams: In a cam follower system, the motion of the follower is very important. Its displacement can be plotted against the angular displacement θ of the cam and it is called as the displacement diagram. The displacement of the follower is plotted along the y-axis and angular displacement θ of the cam is plotted along x-axis. From the displacement diagram, velocity and acceleration of the follower can also be plotted for different angular displacements θ of the cam. The displacement, velocity and acceleration diagrams are plotted for one cycle of operation i.e., one rotation of the cam. Displacement diagrams are basic requirements for the construction of cam profiles. Construction of displacement diagrams and calculation of velocities and accelerations of followers with different types of motions are discussed in the following sections.

(a) Follower motion with Uniform velocity:

Fig.6.8 shows the displacement, velocity and acceleration patterns of a follower having uniform velocity type of motion. Since the follower moves with constant velocity, during rise and fall, the displacement varies linearly with θ . Also, since the velocity changes from zero to a finite value, within no time, theoretically, the acceleration becomes infinite at the beginning and end of rise and fall.

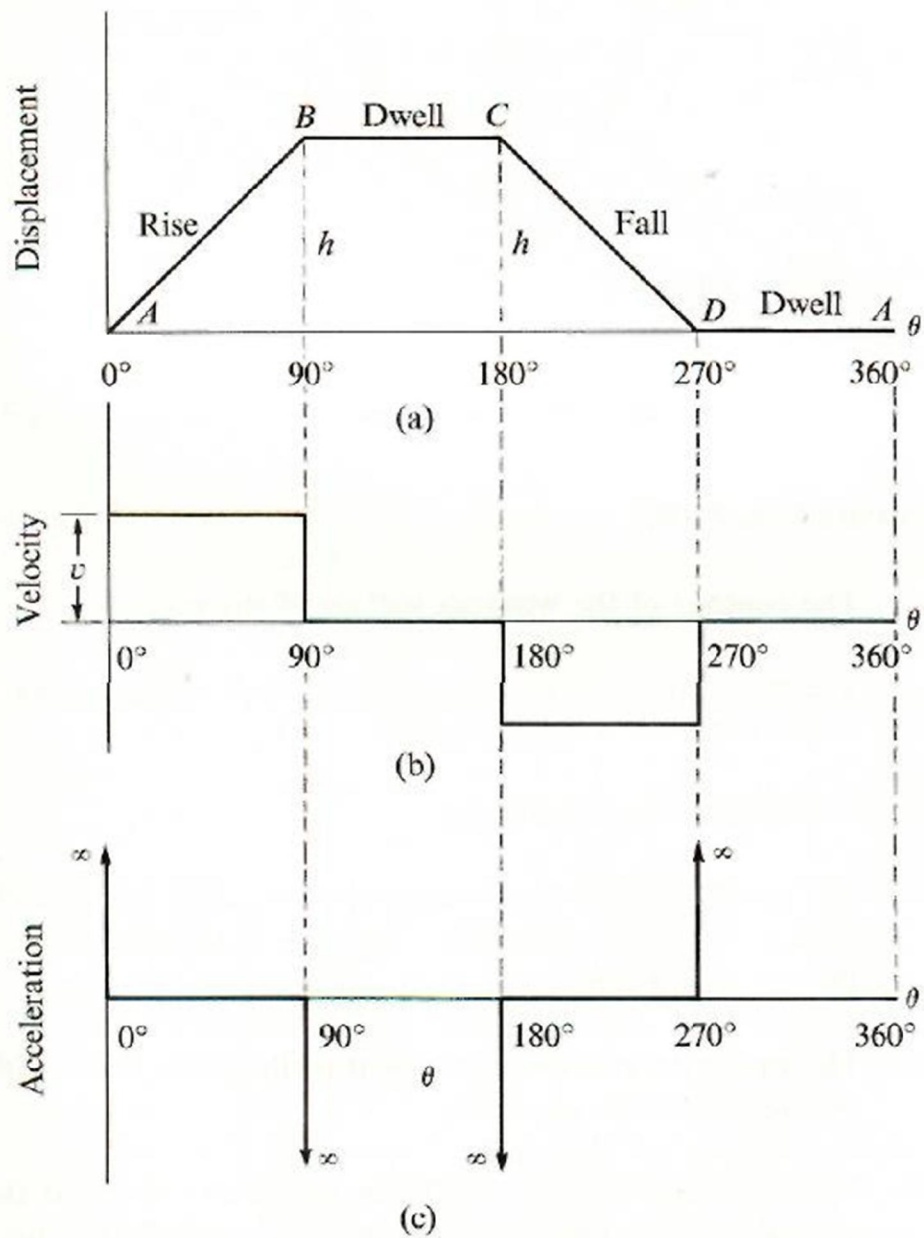


Fig.6.8

(b) Follower motion with modified uniform velocity:

It is observed in the displacement diagrams of the follower with uniform velocity that the acceleration of the follower becomes infinite at the beginning and ending of rise and return strokes. In order to prevent this, the displacement diagrams are slightly modified. In the modified form, the velocity of the follower changes uniformly during the beginning and end of each stroke. Accordingly, the displacement of the follower varies parabolically during these periods. With this modification, the acceleration becomes constant during these periods, instead of being

infinite as in the uniform velocity type of motion. The displacement, velocity and acceleration patterns are shown in **fig.6.9**.

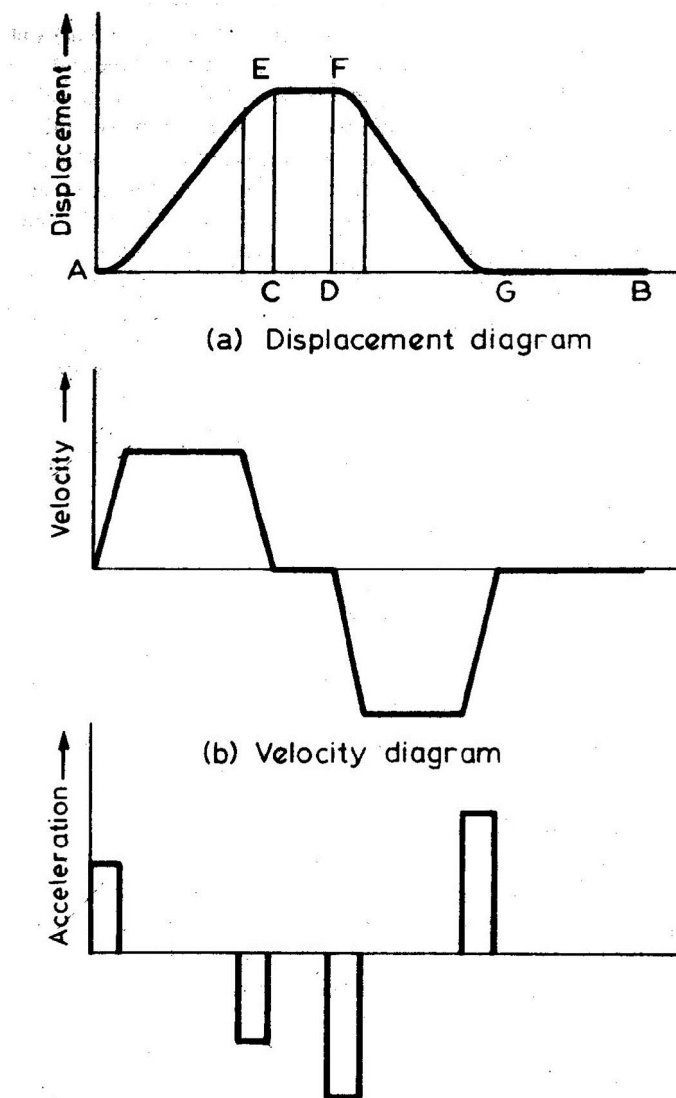
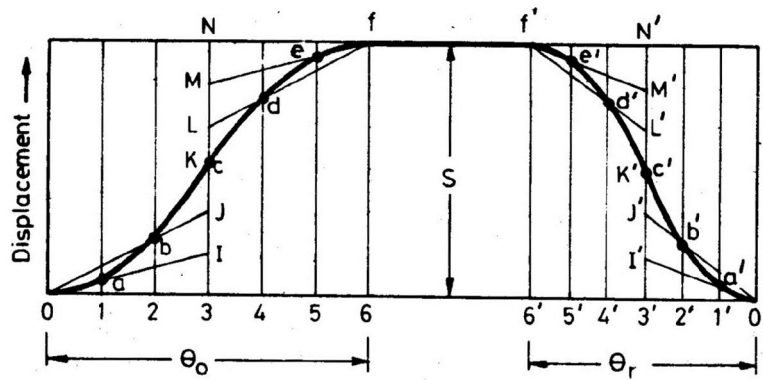


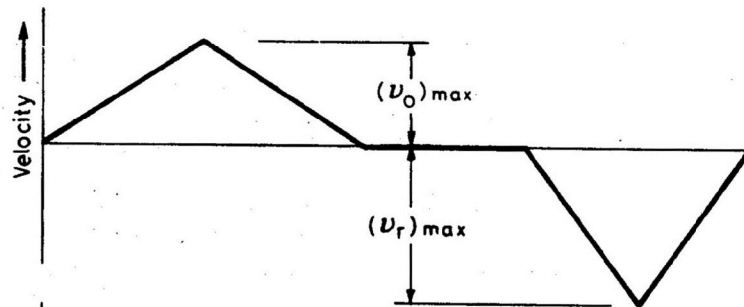
fig.6.9

(c) Follower motion with uniform acceleration and retardation (UARM):

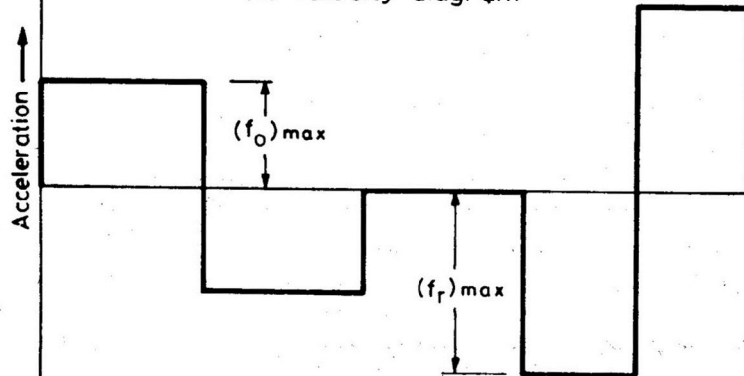
Here, the displacement of the follower varies parabolically with respect to angular displacement of cam. Accordingly, the velocity of the follower varies uniformly with respect to angular displacement of cam. The acceleration/retardation of the follower becomes constant accordingly. The displacement, velocity and acceleration patterns are shown in **fig. 6.10**.



(a) Displacement diagram



(b) Velocity diagram



(c) Acceleration diagram

Fig.6.10

s = Stroke of the follower

θ_o and θ_r = Angular displacement of the cam during outstroke and return stroke.

ω = Angular velocity of cam.

$$\text{Time required for follower outstroke} = t_o = \frac{\theta_o}{\omega}$$

$$\text{Time required for follower return stroke} = t_r = \frac{\theta_r}{\omega}$$

$$\text{Average velocity of follower} = \frac{s}{t}$$

$$\text{Average velocity of follower during outstroke} = \frac{s/2}{t_o/2} = \frac{s}{t_o} = \frac{v_{o_{\min}} + v_{o_{\max}}}{2}$$

$$v_{o_{\min}} = 0$$

$$\therefore v_{o_{\max}} = \frac{2s}{t_o} = \frac{2\omega s}{\theta_o} = \text{Max. velocity during outstroke.}$$

$$\text{Average velocity of follower during return stroke} = \frac{s/2}{t_r/2} = \frac{s}{t_r} = \frac{v_{r_{\min}} + v_{r_{\max}}}{2}$$

$$v_{r_{\min}} = 0$$

$$\therefore v_{r_{\max}} = \frac{2s}{t_r} = \frac{2\omega s}{\theta_r} = \text{Max. velocity during return stroke.}$$

$$\text{Acceleration of the follower during outstroke} = a_o = \frac{v_{o_{\max}}}{t_o/2} = \frac{4\omega^2 s}{\theta_o^2}$$

$$\text{Similarly acceleration of the follower during return stroke} = a_r = \frac{4\omega^2 s}{\theta_r^2}$$

(d) Simple Harmonic Motion: In fig.6.11, the motion executed by point P¹, which is the projection of point P on the vertical diameter is called simple harmonic motion. Here, P moves with uniform angular velocity ω_p , along a circle of radius r ($r = s/2$).

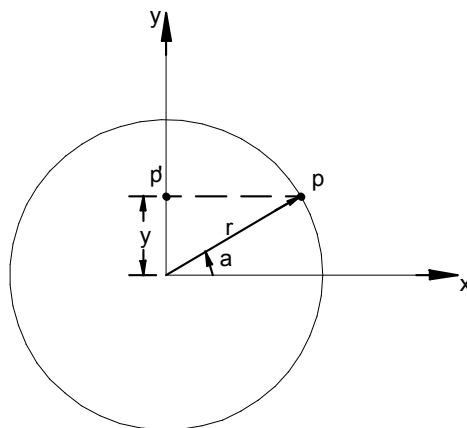


Fig.6.11

$$\text{Displacement} = y = r \sin \alpha = r \sin \omega_p t ; y_{\max} = r \quad [\text{d1}]$$

$$\text{Velocity} = \dot{y} = \omega_p r \cos \omega_p t ; \dot{y}_{\max} = r \omega_p \quad [\text{d2}]$$

$$\text{Acceleration} = \ddot{y} = -\omega_p^2 r \sin \omega_p t = -\omega_p^2 y ; \ddot{y}_{\max} = -r \omega_p^2 \quad [\text{d3}]$$

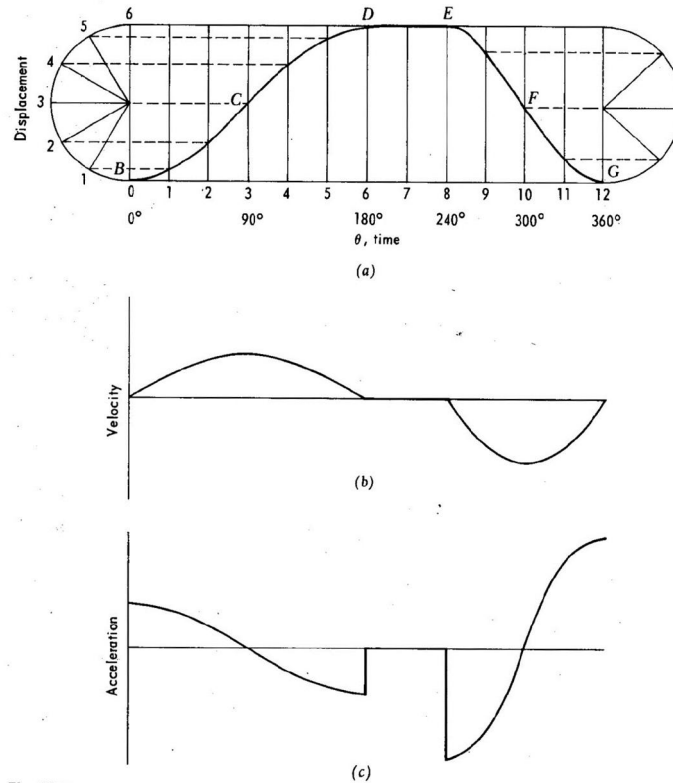


Fig.6.11

s = Stroke or displacement of the follower.

θ_o = Angular displacement during outstroke.

θ_r = Angular displacement during return stroke

ω = Angular velocity of cam.

$$t_o = \text{Time taken for outstroke} = \frac{\theta_o}{\omega}$$

$$t_r = \text{Time taken for return stroke} = \frac{\theta_r}{\omega}$$

Max. velocity of follower during outstroke = $v_{o\max} = r\omega_p$ (from d2)

$$v_{o_{\max}} = \frac{s}{2} \frac{\pi}{t_o} = \frac{\pi \omega s}{2\theta_o}$$

Similarly Max. velocity of follower during return stroke = , $v_{r_{\max}} = \frac{s}{2} \frac{\pi}{t_r} = \frac{\pi \omega s}{2\theta_r}$

Max. acceleration during outstroke = $a_{o_{\max}} = r\omega^2_p$ (from d3) = $\frac{s}{2} \left(\frac{\pi}{t_o} \right)^2 = \frac{\pi^2 \omega^2 s}{2\theta_o^2}$

Similarly, Max. acceleration during return stroke = $a_{r_{\max}} = \frac{s}{2} \left(\frac{\pi}{t_r} \right)^2 = \frac{\pi^2 \omega^2 s}{2\theta_r^2}$

(e) Cycloidal motion:

Cycloid is the path generated by a point on the circumference of a circle, as the circle rolls without slipping, on a straight/flat surface. The motion executed by the follower here, is similar to that of the projection of a point moving along a cycloidal curve on a vertical line as shown in figure 6.12.

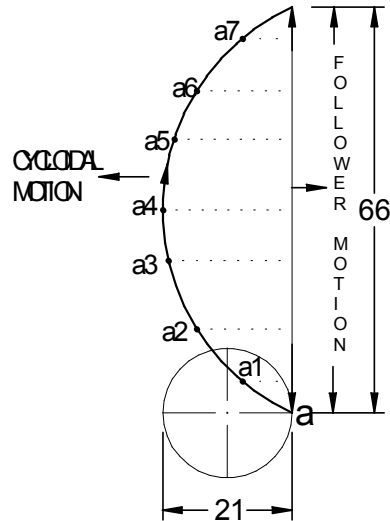


Fig.6.12

The construction of displacement diagram and the standard patterns of velocity and acceleration diagrams are shown in fig.6.13. Compared to all other follower motions, cycloidal motion results in smooth operation of the follower.

The expressions for maximum values of velocity and acceleration of the follower are shown below.

s = Stroke or displacement of the follower.

$$d = \text{dia. of cycloid generating circle} = \frac{s}{\pi}$$

θ_o = Angular displacement during outstroke.

θ_r = Angular displacement during return stroke

ω = Angular velocity of cam.

$$t_o = \text{Time taken for outstroke} = \frac{\theta_o}{\omega}$$

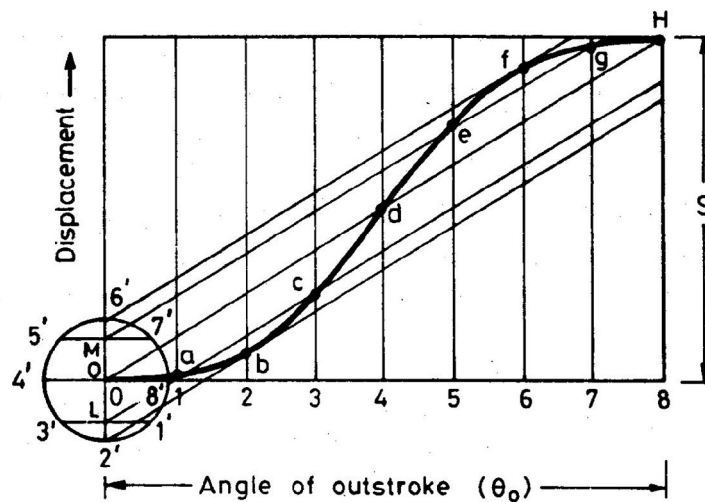
$$t_r = \text{Time taken for return stroke} = \frac{\theta_r}{\omega}$$

$$v_{o_{\max}} = \text{Max. velocity of follower during outstroke} = \frac{2\omega s}{\theta_o}$$

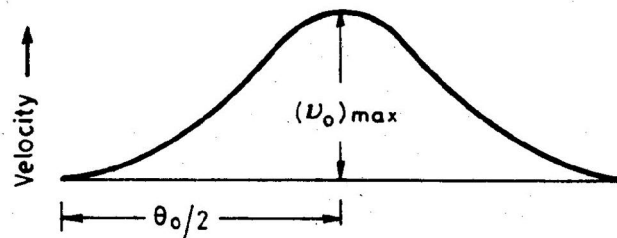
$$v_{r_{\max}} = \text{Max. velocity of follower during return stroke} = \frac{2\omega s}{\theta_r}$$

$$a_{o_{\max}} = \text{Max. acceleration during outstroke} = \frac{2\pi\omega^2 s}{\theta_o^2}$$

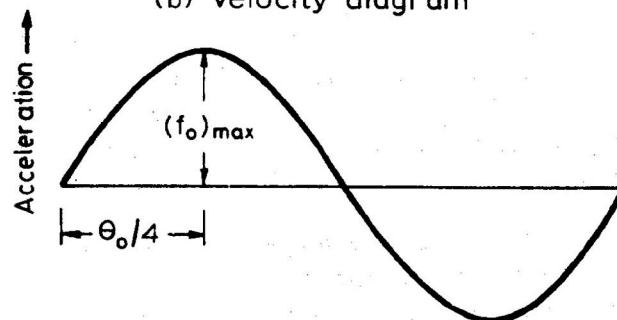
$$a_{r_{\max}} = \text{Max. acceleration during return stroke} = \frac{2\pi\omega^2 s}{\theta_r^2}$$



(a) Displacement diagram



(b) Velocity diagram



(c) Acceleration diagram

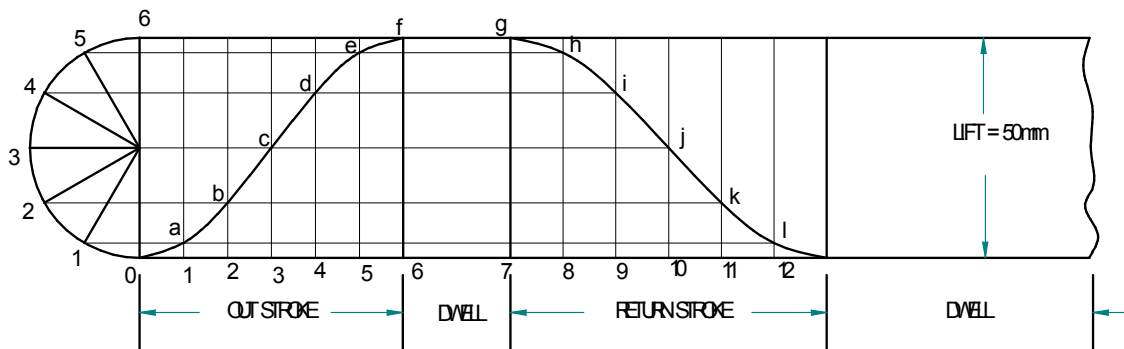
Fig. 6.13

Solved problems

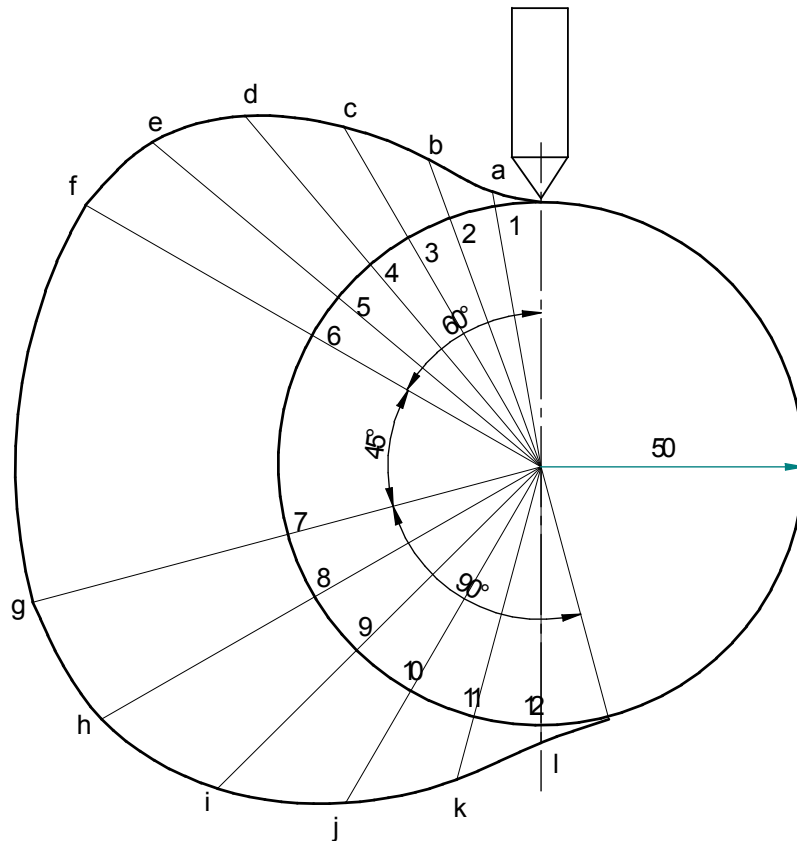
(1) Draw the cam profile for following conditions:

Follower type = Knife edged, in-line; lift = 50mm; base circle radius = 50mm; out stroke with SHM, for 60° cam rotation; dwell for 45° cam rotation; return stroke with SHM, for 90° cam rotation; dwell for the remaining period. Determine max. velocity and acceleration during out stroke and return stroke if the cam rotates at 1000 rpm in clockwise direction.

Displacement diagram:



Cam profile: Construct base circle. Mark points 1,2,3.....in direction opposite to the direction of cam rotation. Transfer points a,b,c.....l from displacement diagram to the cam profile and join them by a smooth free hand curve. This forms the required cam profile.



Calculations:

$$\text{Angular velocity of cam} = \omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 1000}{60} = \mathbf{104.76 \text{ rad/sec}}$$

$$\text{Max. velocity of follower during outstroke} = v_{o_{\max}} = \frac{\pi \omega s}{2\theta_o} =$$

$$= \frac{\pi \times 104.76 \times 50}{2 \times \frac{\pi}{3}} = 7857 \text{ mm/sec} = \mathbf{7.857 \text{ m/sec}}$$

$$\text{Similarly Max. velocity of follower during return stroke} = , v_{r_{\max}} = \frac{\pi \omega s}{2\theta_r} =$$

$$= \frac{\pi \times 104.76 \times 50}{2 \times \frac{\pi}{2}} = 5238 \text{ mm/sec} = \mathbf{5.238 \text{ m/sec}}$$

$$\text{Max. acceleration during outstroke} = a_{o_{\max}} = r\omega_p^2 \text{ (from d3)} = \frac{\pi^2 \omega^2 s}{2\theta_o^2} =$$

$$= \frac{\pi^2 \times (104.76)^2 \times 50}{2 \times \left(\frac{\pi}{3}\right)^2} = 2469297.96 \text{mm/sec}^2 = \mathbf{2469.3 \text{m/sec}^2}$$

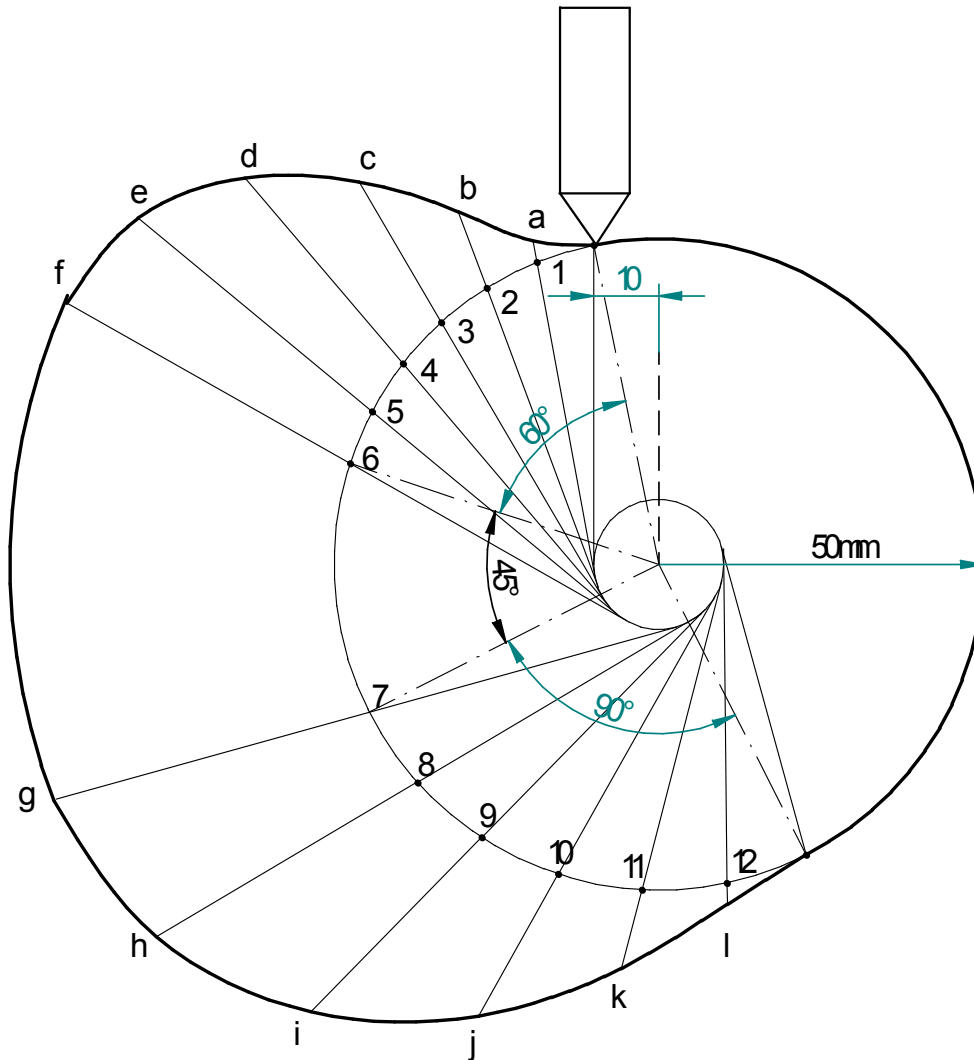
Similarly, Max. acceleration during return stroke = $a_{r_{\max}} = \frac{\pi^2 \omega^2 s}{2\theta_r^2} =$

$$= \frac{\pi^2 \times (104.76)^2 \times 50}{2 \times \left(\frac{\pi}{2}\right)^2} = 1097465.76 \text{mm/sec}^2 = \mathbf{1097.5 \text{m/sec}^2}$$

(2) Draw the cam profile for the same operating conditions of problem (1), with the follower off set by 10 mm to the left of cam center.

Displacement diagram: Same as previous case.

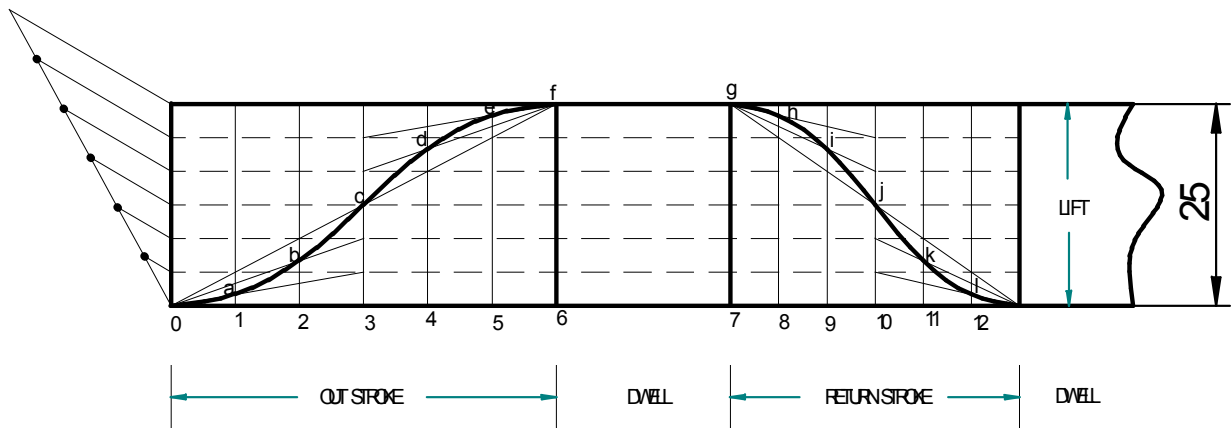
Cam profile: Construction is same as previous case, except that the lines drawn from 1,2,3.... are tangential to the offset circle of 10mm dia. as shown in the fig.



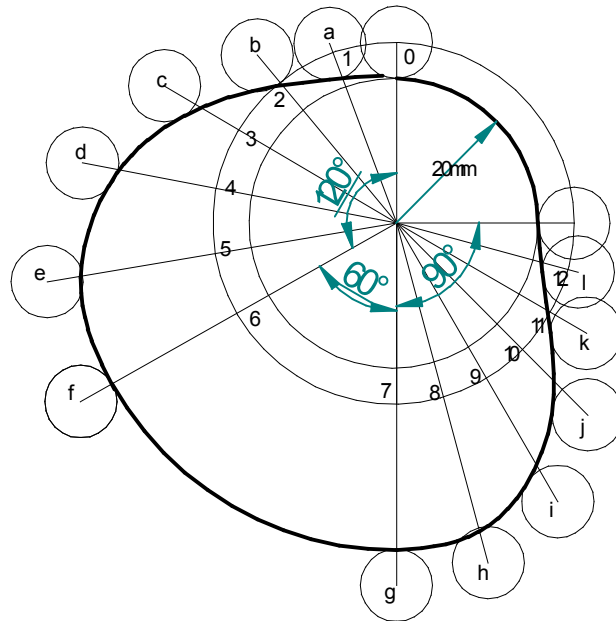
(3) Draw the cam profile for following conditions:

Follower type = roller follower, in-line; lift = 25mm; base circle radius = 20mm; roller radius = 5mm; out stroke with UARM, for 120° cam rotation; dwell for 60° cam rotation; return stroke with UARM, for 90° cam rotation; dwell for the remaining period. Determine max. velocity and acceleration during out stroke and return stroke if the cam rotates at 1200 rpm in clockwise direction.

Displacement diagram:



Cam profile: Construct base circle and prime circle (25mm radius). Mark points 1,2,3.....in direction opposite to the direction of cam rotation, on prime circle. Transfer points a,b,c.....l from displacement diagram. At each of these points a,b,c... draw circles of 5mm radius, representing rollers. Starting from the first point of contact between roller and base circle, draw a smooth free hand curve, tangential to all successive roller positions. This forms the required cam profile.



Calculations:

$$\text{Angular velocity of the cam} = \omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 1200}{60} = \mathbf{125.71 \text{ rad/sec}}$$

$$\begin{aligned} \text{Max. velocity during outstroke} &= v_{o_{\max}} = \frac{2s}{t_o} = \frac{2\omega s}{\theta_o} = \\ &= \frac{2 \times 125.71 \times 25}{2 \times \pi / 3} = 2999.9 \text{ mm/sec} = \mathbf{2.999 \text{ m/sec}} \end{aligned}$$

$$\begin{aligned} \text{Max. velocity during return stroke} &= v_{r_{\max}} = \frac{2s}{t_r} = \frac{2\omega s}{\theta_r} = \frac{2 \times 125.71 \times 25}{\pi / 2} = \\ &= 3999.86 \text{ mm/sec} = \mathbf{3.999 \text{ m/sec}} \end{aligned}$$

$$\begin{aligned} \text{Acceleration of the follower during outstroke} &= a_o = \frac{v_{o_{\max}}}{t_o / 2} = \frac{4\omega^2 s}{\theta_o^2} = \\ &= \frac{4 \times (125.71)^2 \times 25}{(2 \times \pi / 3)^2} = 359975 \text{ mm/sec}^2 = \mathbf{359.975 \text{ m/sec}^2} \end{aligned}$$

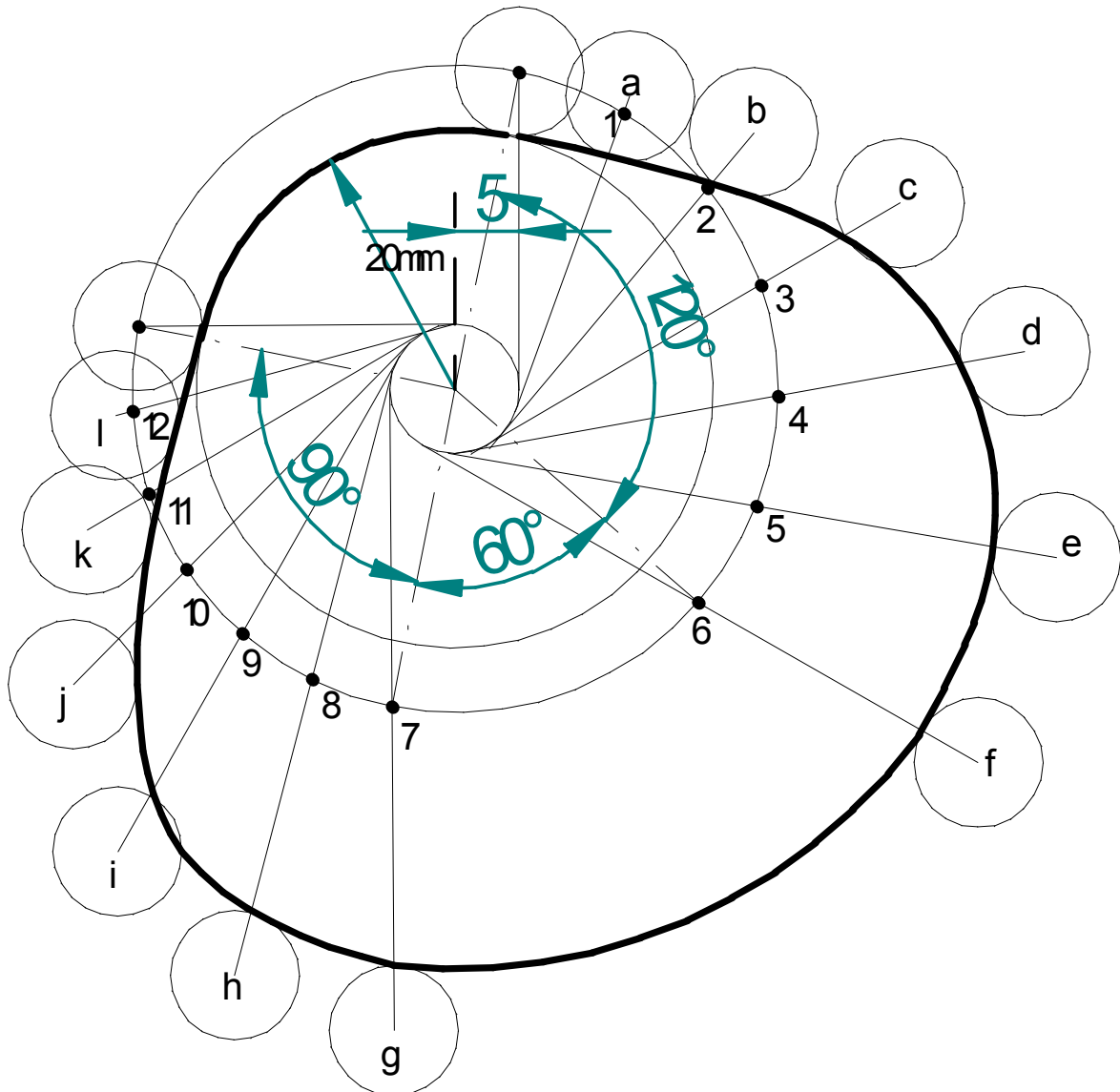
$$\text{Similarly acceleration of the follower during return stroke} = a_r = \frac{4\omega^2 s}{\theta_r^2} =$$

$$= \frac{4 \times (125.71)^2 \times 25}{\left(\frac{\pi}{2}\right)^2} = 639956 \text{mm/sec}^2 = \mathbf{639.956 \text{m/sec}^2}$$

(4) Draw the cam profile for conditions same as in (3), with follower off set to right of cam center by 5mm and cam rotating counter clockwise.

Displacement diagram: Same as previous case.

Cam profile: Construction is same as previous case, except that the lines drawn from 1,2,3.... are tangential to the offset circle of 10mm dia. as shown in the fig.



(5) Draw the cam profile for following conditions:

Follower type = roller follower, off set to the right of cam axis by 18mm; lift = 35mm; base circle radius = 50mm; roller radius = 14mm; out stroke with SHM in 0.05sec; dwell for 0.0125sec; return stroke with UARM, during 0.125sec; dwell for the remaining period. During return stroke, acceleration is 3/5 times retardation. Determine max. velocity and acceleration during out stroke and return stroke if the cam rotates at 240 rpm.

Calculations:

Cam speed = 240rpm. Therefore, time for one rotation = $\frac{60}{240} = 0.25$ sec

$$\text{Angle of out stroke} = \theta_o = \frac{0.05}{0.25} \times 360 = 72^\circ$$

$$\text{Angle of first dwell} = \theta_{w1} = \frac{0.0125}{0.25} \times 360 = 18^\circ$$

$$\text{Angle of return stroke} = \theta_r = \frac{0.125}{0.25} \times 360 = 180^\circ$$

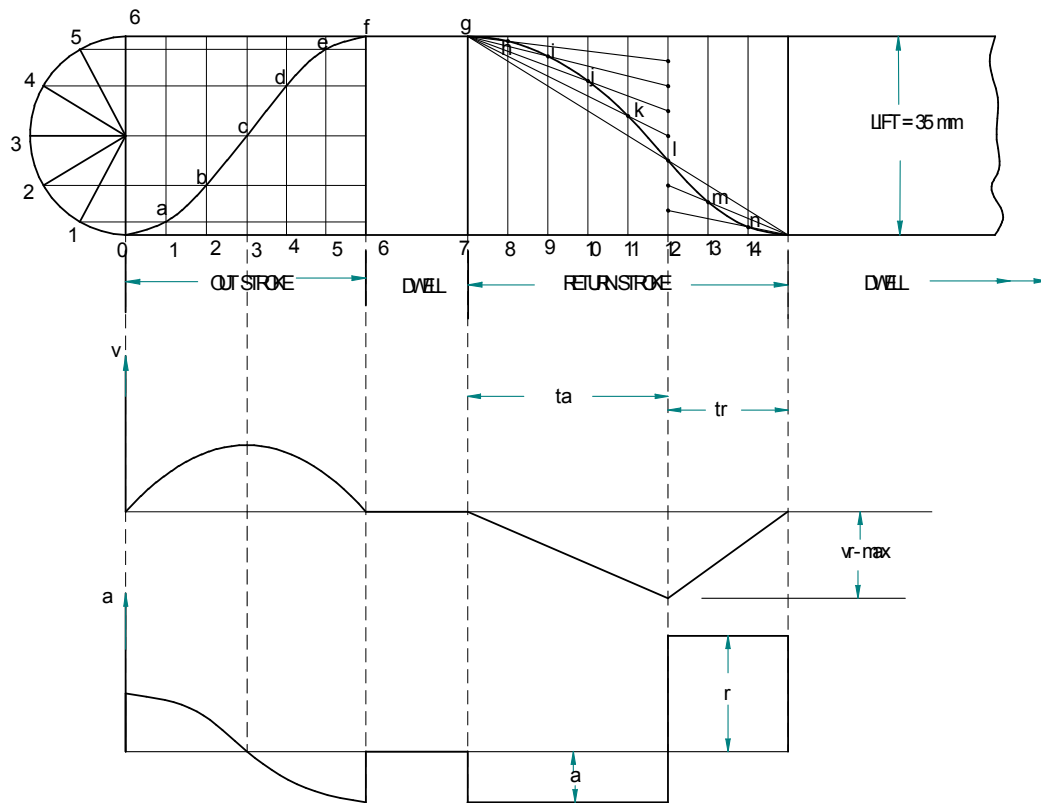
$$\text{Angle of second dwell} = \theta_{w2} = 90^\circ$$

Since acceleration is 3/5 times retardation during return stroke,

$$a = \frac{3}{5}r \text{ (from acceleration diagram)} \therefore \frac{a}{r} = \frac{3}{5}$$

$$\text{But } a = \frac{v_{\max}}{t_a}; r = \frac{v_{\max}}{t_r} \therefore \frac{a}{r} = \frac{t_r}{t_a} = \frac{3}{5}$$

Displacement diagram is constructed by selecting t_a and t_r accordingly.



$$\text{Angular velocity of cam} = \omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 240}{60} = \mathbf{25.14 \text{ rad/sec}}$$

$$\begin{aligned} \text{Max. velocity of follower during outstroke} &= v_{o_{\max}} = \frac{\pi \omega s}{2\theta_o} = \\ &= \frac{\pi \times 25.14 \times 35}{2 \times (2 \times \pi/5)} = 1099.87 \text{ mm/sec} = \mathbf{1.1 \text{ m/sec}} \end{aligned}$$

$$\begin{aligned} \text{Similarly Max. velocity during return stroke} &= v_{r_{\max}} = \frac{2\omega s}{\theta_r} = \frac{2 \times 25.14 \times 35}{\pi} = \\ &= 559.9 \text{ mm/sec} = \mathbf{0.56 \text{ m/sec}} \end{aligned}$$

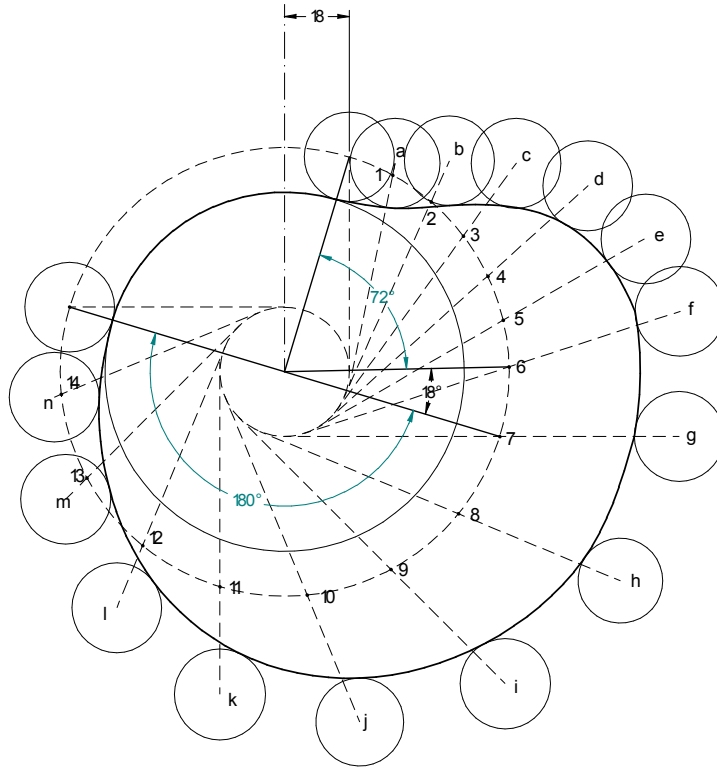
$$\begin{aligned} \text{Max. acceleration during outstroke} &= a_{o_{\max}} = r\omega_p^2 \text{ (from d3)} = \frac{\pi^2 \omega^2 s}{2\theta_o^2} = \\ &= \frac{\pi^2 \times (25.14)^2 \times 35}{2 \times (2 \times \pi/5)^2} = 69127.14 \text{ mm/sec}^2 = \mathbf{69.13 \text{ m/sec}^2} \end{aligned}$$

acceleration of the follower during return stroke =

$$a_r = \frac{v_{r_{\max}}}{t_a} = \frac{2\omega s / \theta_r}{5 \times \pi / 8 \times \omega} = \frac{16 \times \omega^2 \times s}{5 \times \pi \times \theta_r} = \frac{16 \times (25.14)^2 \times 35}{5 \times \pi \times \pi} = 7166.37 \text{ mm/sec}^2 = \mathbf{7.17 \text{ m/sec}^2}$$

similarly retardation of the follower during return stroke =

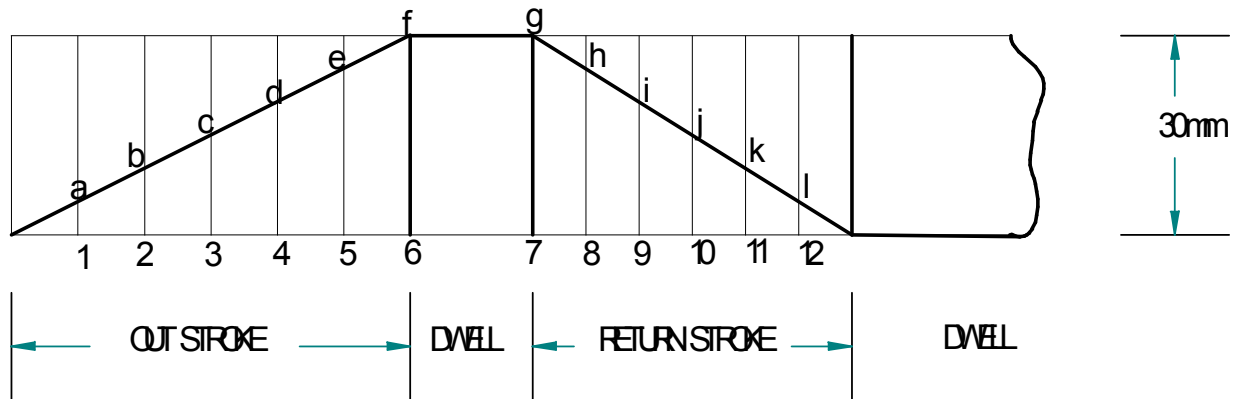
$$r_r = \frac{vr_{\max}}{t_r} = \frac{2\omega s / \theta_r}{3 \times \pi / 8 \times \omega} = \frac{16 \times \omega^2 \times s}{3 \times \pi \times \theta_r} = \frac{16 \times (25.14)^2 \times 35}{3 \times \pi \times \pi} = 11943.9 \text{ mm/sec}^2 = \mathbf{11.94 \text{ m/sec}^2}$$



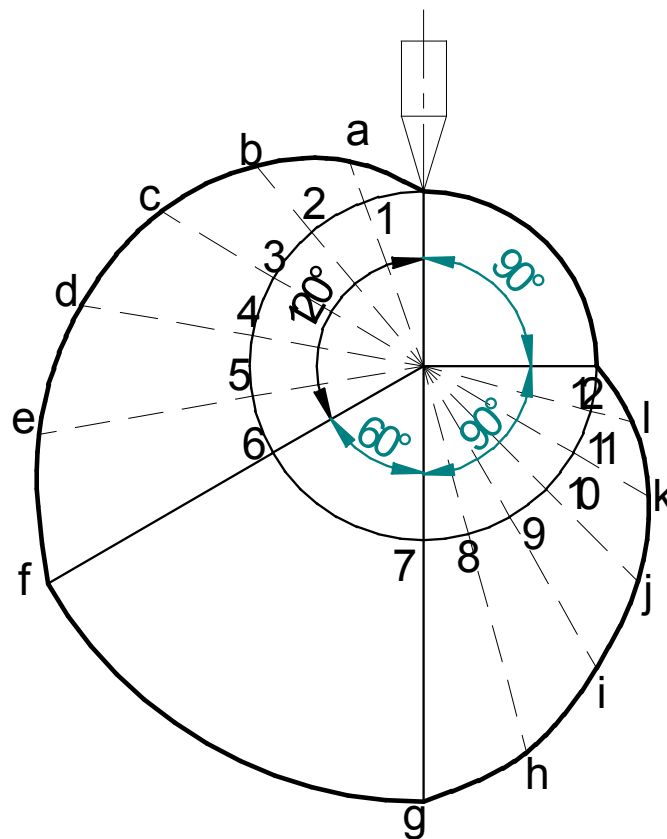
(6) Draw the cam profile for following conditions:

Follower type = knife edged follower, in line; lift = 30mm; base circle radius = 20mm; out stroke with uniform velocity in 120° of cam rotation; dwell for 60° ; return stroke with uniform velocity, during 90° of cam rotation; dwell for the remaining period.

Displacement diagram:

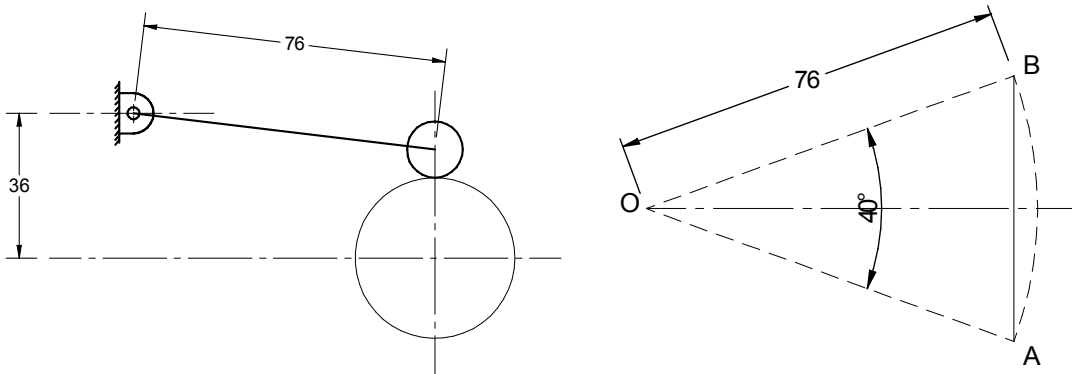


Cam profile:



(7) Draw the cam profile for following conditions:

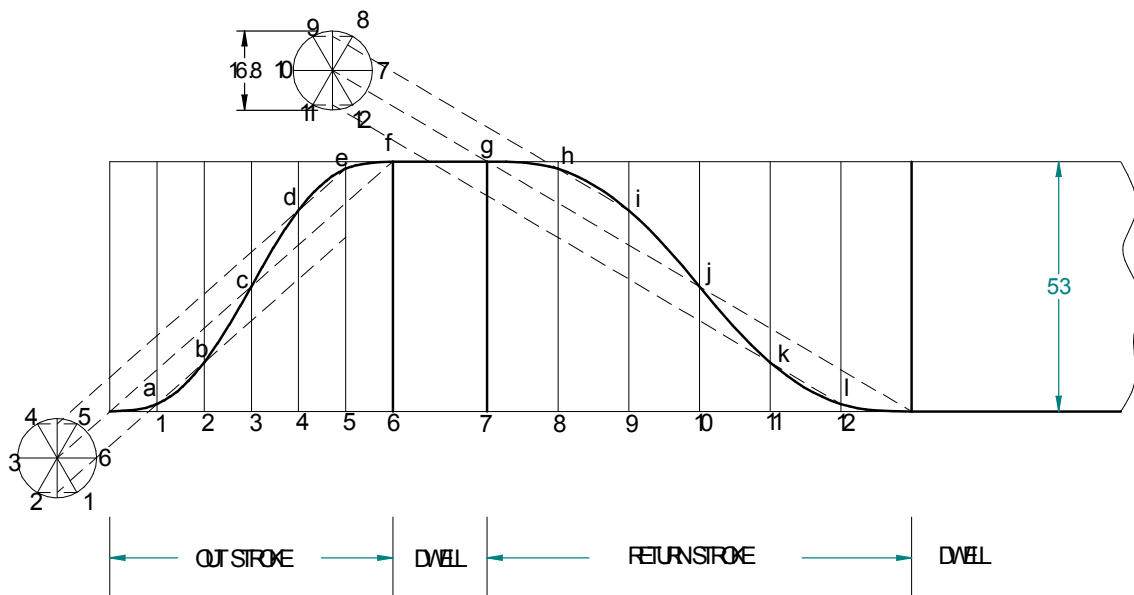
Follower type = oscillating follower with roller as shown in fig.; base circle radius = 20mm; roller radius = 7mm; follower to rise through 40° during 90° of cam rotation with cycloidal motion; dwell for 30° ; return stroke with cycloidal motion during 120° of cam rotation; dwell for the remaining period. Also determine the max. velocity and acceleration during outstroke and return stroke, if the cam rotates at 600 rpm.



$$\text{Lift of the follower} = S = \text{length } AB \approx \text{arc } AB = OA \times \theta = 76 \times 40 \times \frac{\pi}{180} = 53 \text{ mm.}$$

$$\text{Radius of cycloid generating circle} = \frac{53}{2 \times \pi} = 8.4 \text{ mm}$$

Displacement diagram;



$$\text{Angular velocity of cam} = \omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 600}{60} = 62.86 \text{ rad/sec}$$

$$v_{o_{\max}} = \text{Max. velocity of follower during outstroke} = \frac{2\omega s}{\theta_o} = \frac{2 \times 62.86 \times 53}{\pi/2} = 4240.2 \text{ mm/sec}$$

$$v_{r_{\max}} = \text{Max. velocity of follower during return stroke} = \frac{2\omega s}{\theta_r} = \frac{2 \times 62.86 \times 53}{2 \times \pi/3} = 3180 \text{ mm/sec}$$

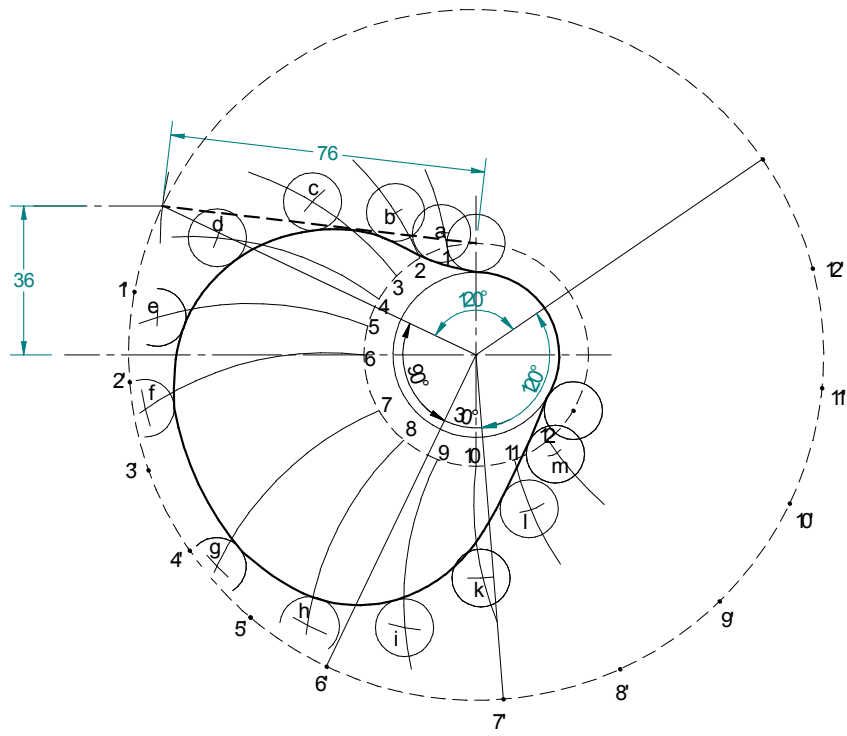
$$a_{o_{\max}} = \text{Max. acceleration during outstroke} = \frac{2\pi\omega^2 s}{\theta_o^2} = \frac{2 \times \pi \times (62.86)^2 \times 53}{(\pi/2)^2} = 533077 \text{ mm/sec}^2$$

$$= 533.1 \text{ m/sec}^2.$$

$$a_{r_{\max}} = \text{Max. acceleration during return stroke} = \frac{2\pi\omega^2 s}{\theta_r^2} = \frac{2 \times \pi \times (62.86)^2 \times 53}{(2 \times \pi/3)^2} =$$

$$= 299855.8 \text{ mm/sec}^2 = 299.8 \text{ m/sec}^2.$$

Cam profile: Draw base circle and prime circle. Draw another circle of radius equal to the distance between cam center and follower pivot point. Take the line joining cam center and pivot point as reference and draw lines indicating successive angular displacements of cam. Divide these into same number of divisions as in the displacement diagram. Show points 1', 2', 3'... on the outer circle. With these points as centers and radius equal to length of follower arm, draw arcs, cutting the prime circle at 1,2,3.... Transfer points a,b,c.. on to these arcs from displacement diagram. At each of these points a,b,c... draw circles of 7mm radius, representing rollers. Starting from the first point of contact between roller and base circle, draw a smooth free hand curve, tangential to all successive roller positions. This forms the required cam profile.



(8) Draw the cam profile for following conditions:

Follower type = knife edged follower, in line; follower rises by 24mm with SHM in 1/4 rotation, dwells for 1/8 rotation and then raises again by 24mm with UARM in 1/4 rotation and dwells for 1/16 rotation before returning with SHM. Base circle radius = 30mm.

$$\text{Angle of out stroke (1)} = \theta_{01} = \frac{1}{4} \times 360^\circ = 90^\circ$$

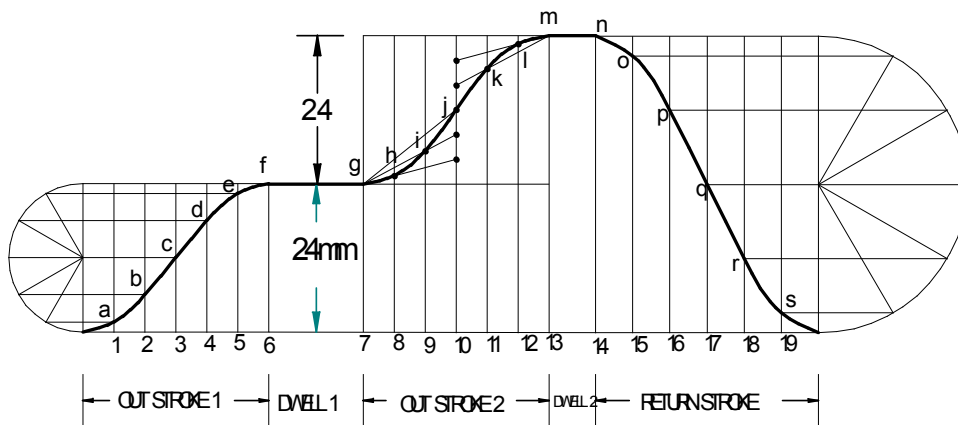
$$\text{Angle of dwell (1)} = \frac{1}{8} \times 360^\circ = 45^\circ$$

$$\text{Angle of out stroke (2)} = \theta_{02} = \frac{1}{4} \times 360^\circ = 90^\circ$$

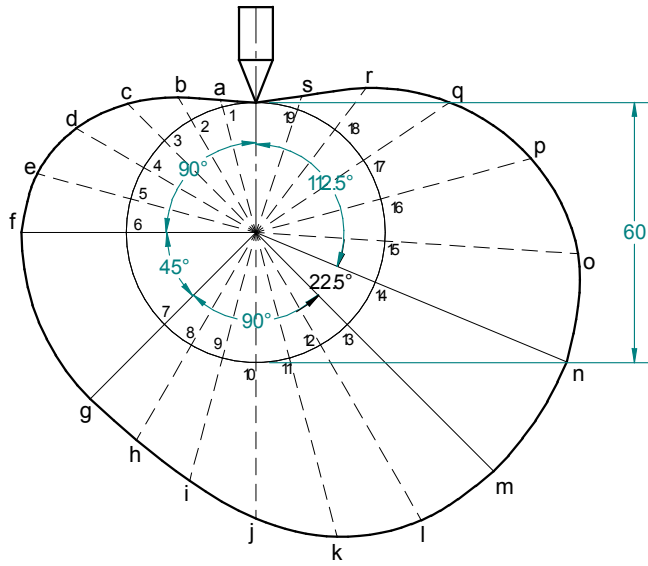
$$\text{Angle of dwell (2)} = \frac{1}{16} \times 360^\circ = 22.5^\circ$$

$$\text{Angle of return stroke} = \theta_r = \left[1 - \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{4} + \frac{1}{16} \right) \right] \times 360 = \frac{5}{16} \times 360^\circ = 112.5^\circ$$

Displacement diagram:



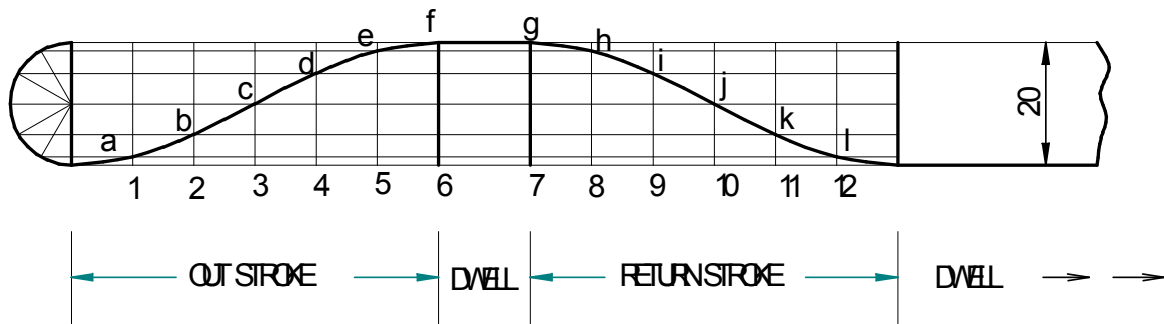
Cam profile:



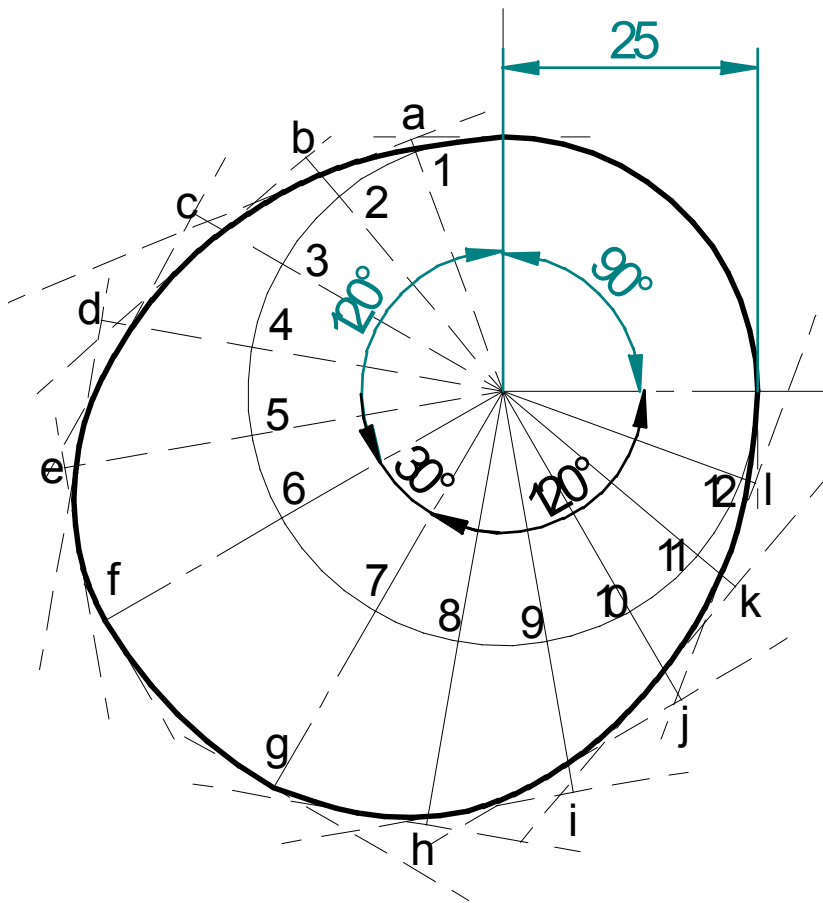
(9) Draw the cam profile for following conditions:

Follower type = flat faced follower, in line; follower rises by 20mm with SHM in 120° of cam rotation, dwells for 30° of cam rotation; returns with SHM in 120° of cam rotation and dwells during the remaining period. Base circle radius = 25mm.

Displacement diagram:



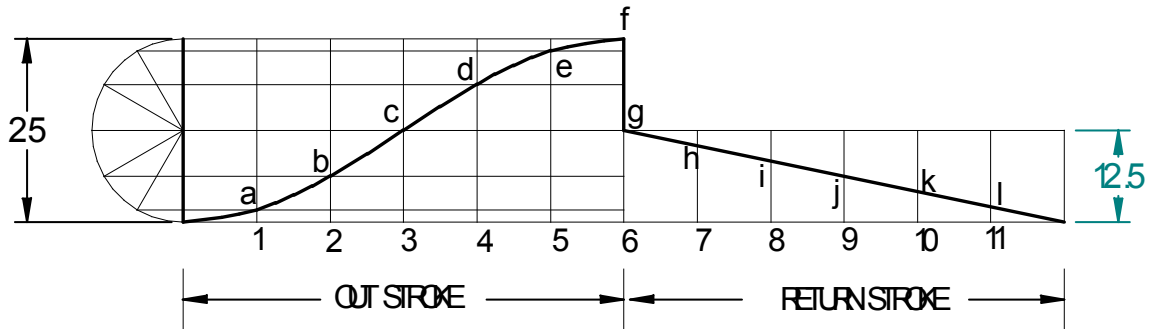
Cam profile: Construct base circle. Mark points 1,2,3.....in direction opposite to the direction of cam rotation, on prime circle. Transfer points a,b,c.....l from displacement diagram. At each of these points a,b,c... draw perpendicular lines to the radials, representing flat faced followers. Starting from the first point of contact between follower and base circle, draw a smooth free hand curve, tangential to all successive follower positions. This forms the required cam profile.



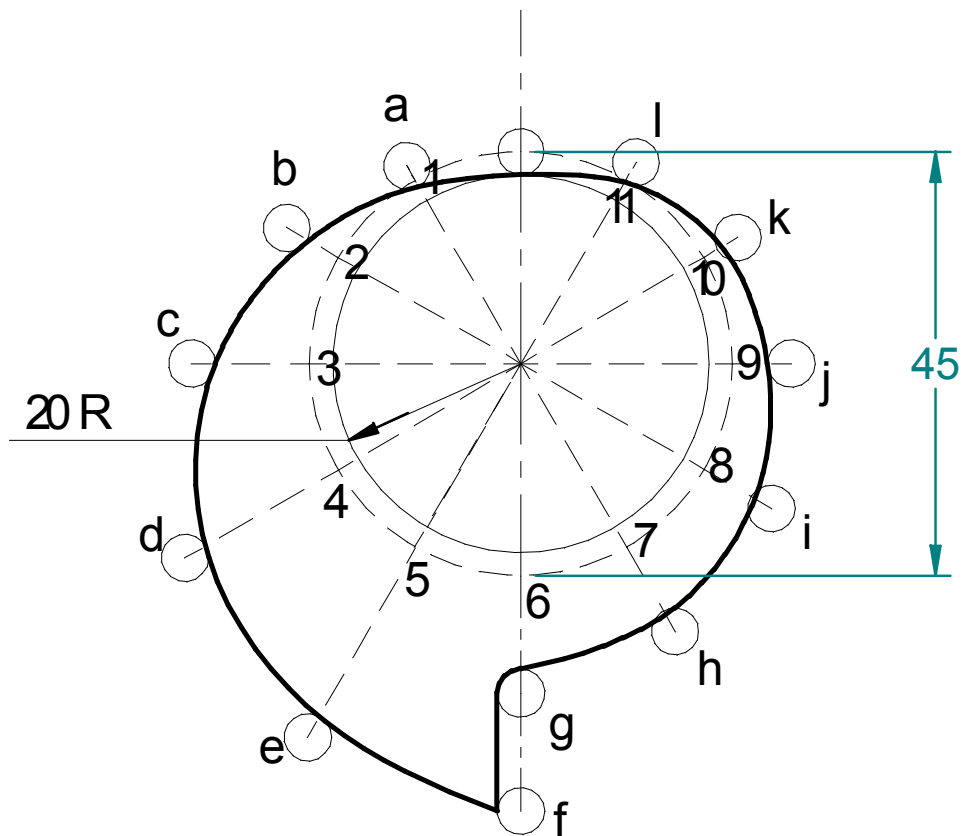
(10) Draw the cam profile for following conditions:

Follower type = roller follower, in line; roller dia. = 5mm; follower rises by 25mm with SHM in 180° of cam rotation, falls by half the distance instantaneously; returns with Uniform velocity in 180° of cam rotation. Base circle radius = 20mm.

Displacement diagram:



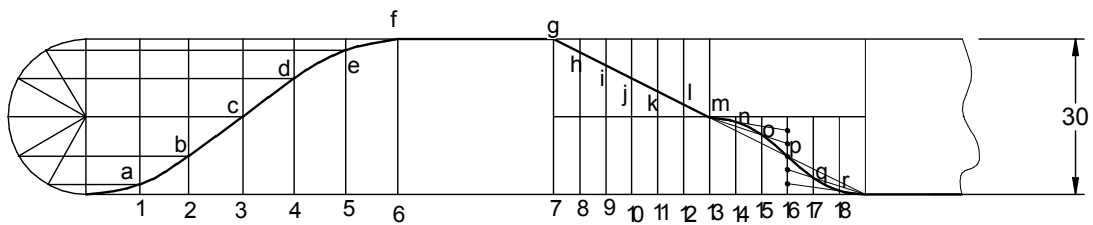
Cam profile:



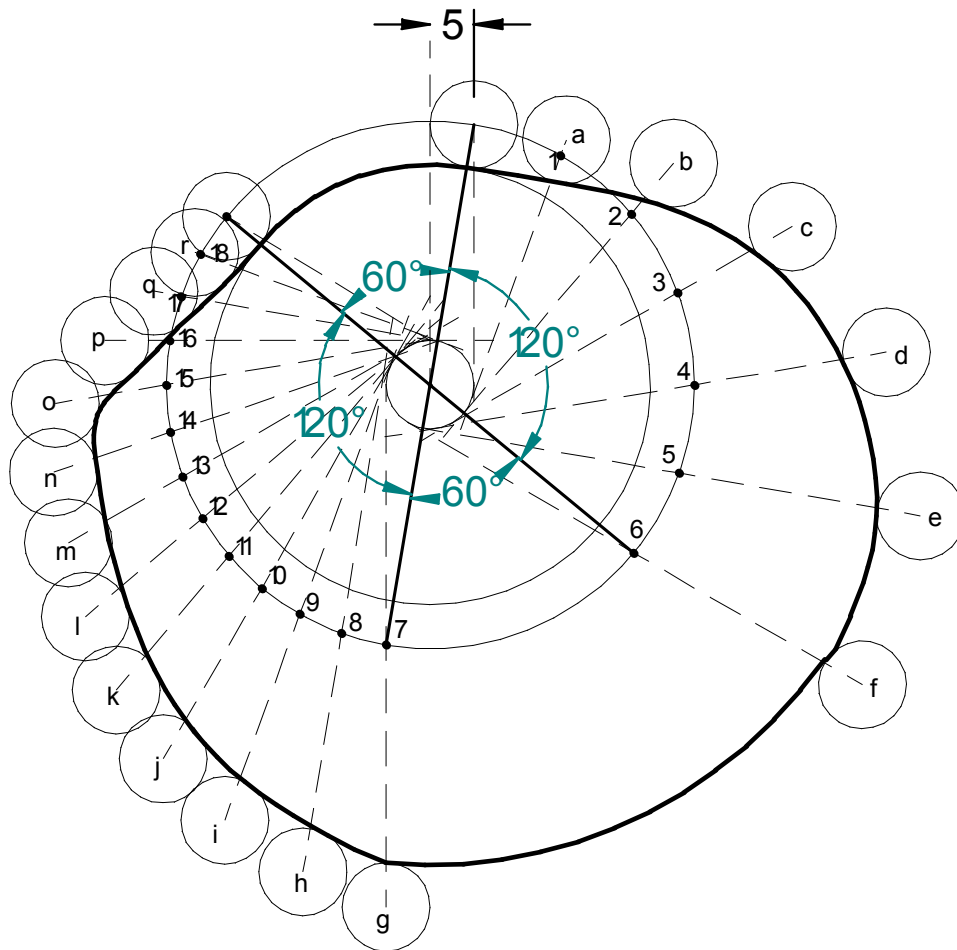
(11) Draw the cam profile for following conditions:

Follower type = roller follower, off-set to the right by 5mm; lift = 30mm; base circle radius = 25mm; roller radius = 5mm; out stroke with SHM, for 120° cam rotation; dwell for 60° cam rotation; return stroke during 120° cam rotation; first half of return stroke with Uniform velocity and second half with UARM; dwell for the remaining period.

Displacement diagram:

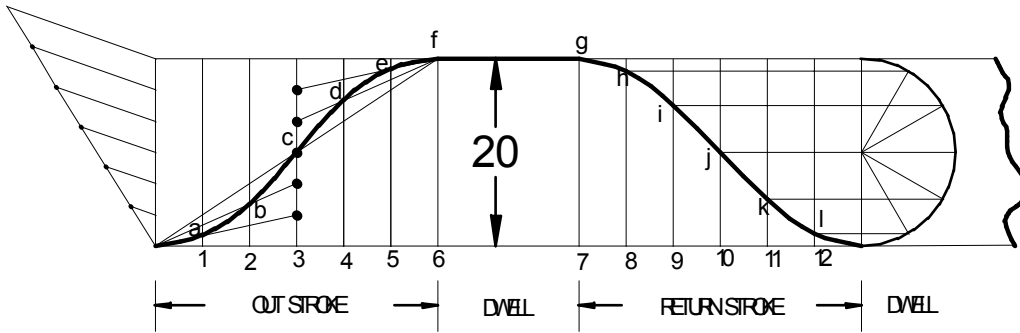


Cam profile:

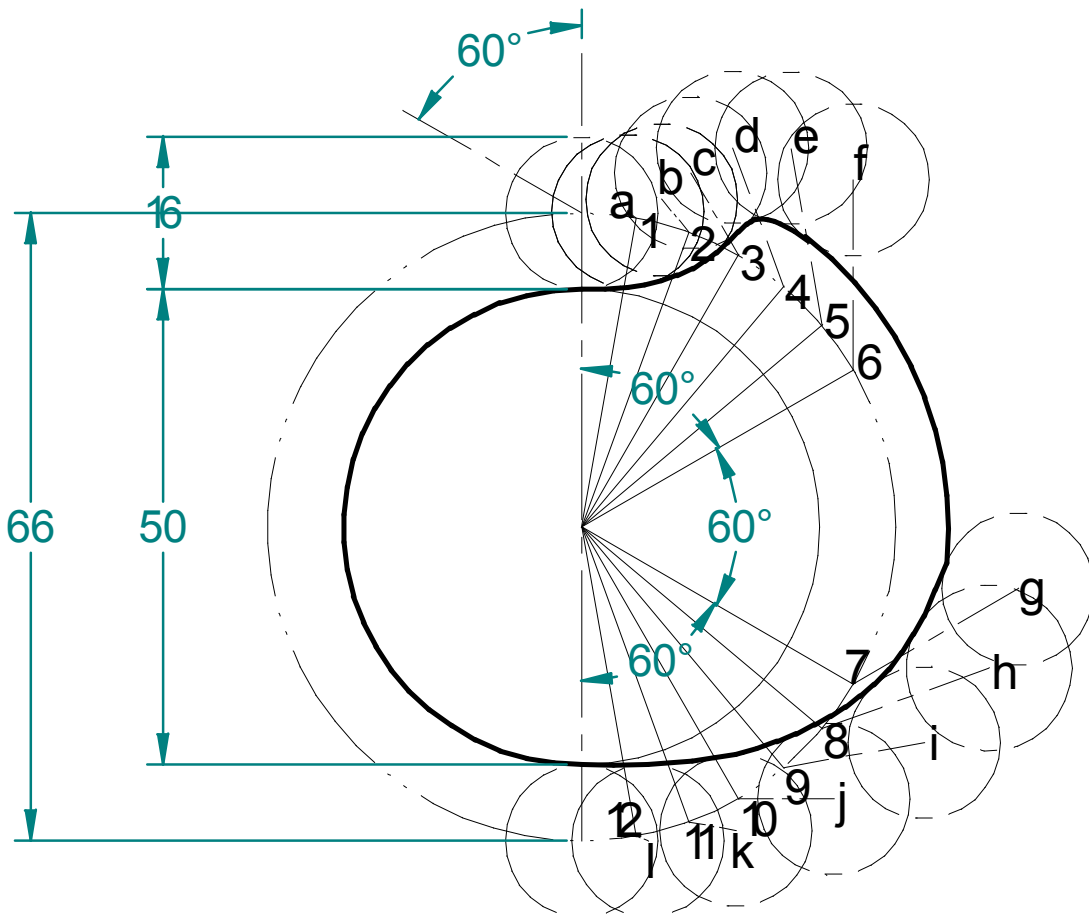


(12) A push rod of valve of an IC engine ascends with UARM, along a path inclined to the vertical at 60° . The same descends with SHM. The base circle diameter of the cam is 50mm and the push rod has a roller of 60mm diameter, fitted to its end. The axis of the roller and the cam fall on the same vertical line. The stroke of the follower is 20mm. The angle of action for the outstroke and the return stroke is 60° each, interposed by a dwell period of 60° . Draw the profile of the cam.

Displacement diagram:



Cam profile:



6.1 Introduction

6.1.1 A Simple Experiment: What is a Cam?

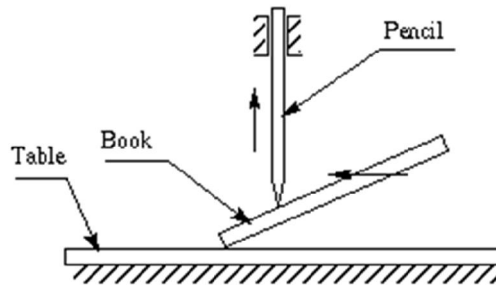


Figure 6-1 Simple Cam experiment

Take a pencil and a book to do an experiment as shown above. Make the book an inclined plane and use the pencil as a slider (use your hand as a guide). When you move the book smoothly upward, what happens to the pencil? It will be pushed up along the guide. By this method, you have transformed one motion into another motion by a very simple device. This is the basic idea of a cam. By rotating the cams in the figure below, the bars will have either translational or oscillatory motion.

6.1.2 Cam Mechanisms

The transformation of one of the simple motions, such as rotation, into any other motions is often conveniently accomplished by means of a **cam mechanism**. A cam mechanism usually consists of two moving elements, the cam and the follower, mounted on a fixed frame. Cam devices are versatile, and almost any arbitrarily-specified motion can be obtained. In some instances, they offer the simplest and most compact way to transform motions.

A **cam** may be defined as a machine element having a curved outline or a curved groove, which, by its oscillation or rotation motion, gives a predetermined specified motion to another element called the **follower**. The cam has a very important function in the operation of many classes of machines, especially those of the automatic type, such as printing presses, shoe machinery, textile machinery, gear-cutting machines, and screw machines. In any class of machinery in which automatic control and accurate timing are paramount, the cam is an indispensable part of mechanism. The possible applications of cams are unlimited, and their shapes occur in great variety. Some of the most common forms will be considered in this chapter.

6.2 Classification of Cam Mechanisms

We can classify cam mechanisms by the modes of input/output motion, the configuration and arrangement of the follower, and the shape of the cam. We can also classify cams by the different types of motion events of the follower and by means of a great variety of the motion characteristics of the cam profile. ([Chen 82](#))

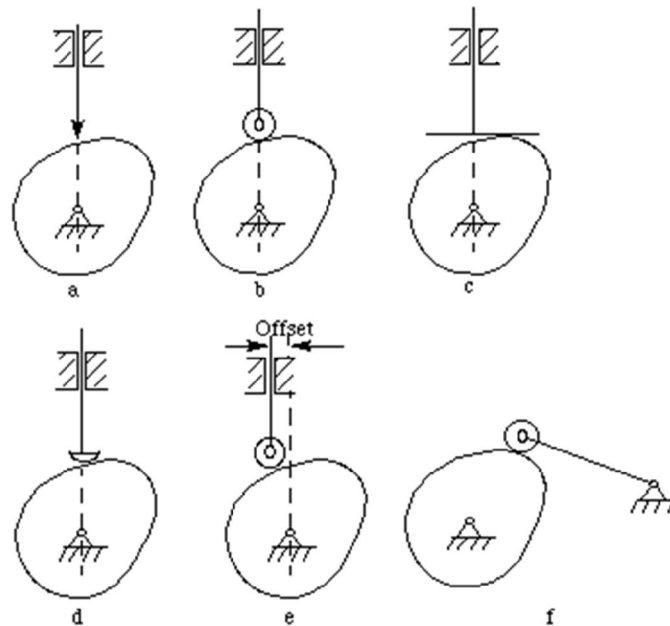


Figure 6-2 Classification of cam mechanisms

4.2.1 Modes of Input/Output Motion

1. Rotating cam-translating follower. ([Figure 6-2a,b,c,d,e](#))
2. Rotating follower ([Figure 6-2f](#)):
The follower arm swings or oscillates in a circular arc with respect to the follower pivot.
3. Translating cam-translating follower ([Figure 6-3](#)).
4. Stationary cam-rotating follower:
The follower system revolves with respect to the center line of the vertical shaft.

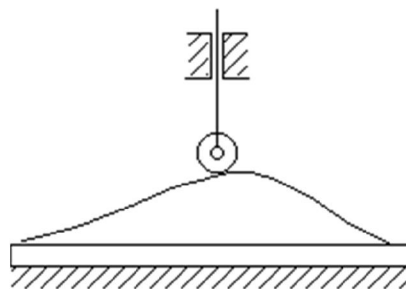


Figure 6-3 Translating cam - translating follower

6.2.1 Follower Configuration

1. **Knife-edge follower** ([Figure 6-2a](#))
2. **Roller follower** ([Figure 6-2b,e,f](#))
3. **Flat-faced follower** ([Figure 6-2c](#))
4. **Oblique flat-faced follower**

5. **Spherical-faced follower** ([Figure 6-2d](#))

6.2.2 Follower Arrangement

1. **In-line follower:**
The center line of the follower passes through the center line of the camshaft.
2. **Offset follower:**
The center line of the follower does not pass through the center line of the cam shaft. The amount of **offset** is the distance between these two center lines. The offset causes a reduction of the side thrust present in the roller follower.

6.2.3 Cam Shape

1. **Plate cam or disk cam:**
The follower moves in a plane perpendicular to the axis of rotation of the camshaft. A translating or a swing arm follower must be constrained to maintain contact with the cam profile.
2. **Grooved cam** or closed cam ([Figure 6-4](#)):
This is a plate cam with the follower riding in a groove in the face of the cam.

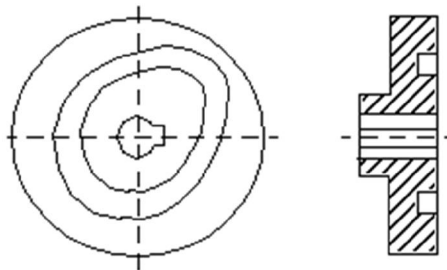


Figure 6-4 Grooved cam

3. **Cylindrical cam** or barrel cam ([Figure 6-5a](#)):
The roller follower operates in a groove cut on the periphery of a cylinder. The follower may translate or oscillate. If the cylindrical surface is replaced by a conical one, a conical cam results.
4. **End cam** ([Figure 6-5b](#)):
This cam has a rotating portion of a cylinder. The follower translates or oscillates, whereas the cam usually rotates. The end cam is rarely used because of the cost and the difficulty in cutting its contour.

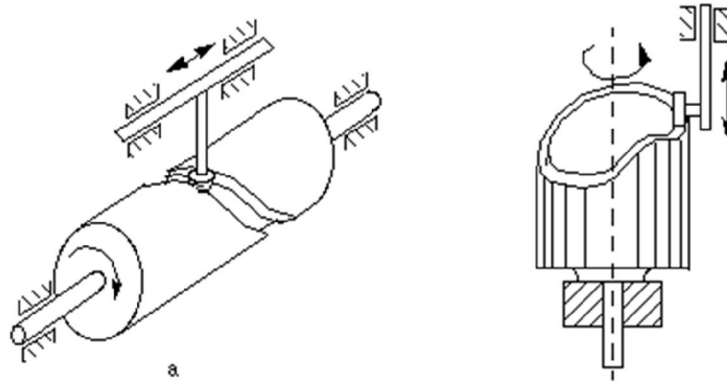


Figure 6-5 Cylindrical cam and end cam

6.2.4 Constraints on the Follower

1. Gravity constraint:
The weight of the follower system is sufficient to maintain contact.
2. Spring constraint:
The spring must be properly designed to maintain contact.
3. Positive mechanical constraint:
A groove maintains positive action. (Figure 6-4 and Figure 6-5a) For the cam in Figure 6-6, the follower has two rollers, separated by a fixed distance, which act as the constraint; the mating cam in such an arrangement is often called a *constant-diameter cam*.

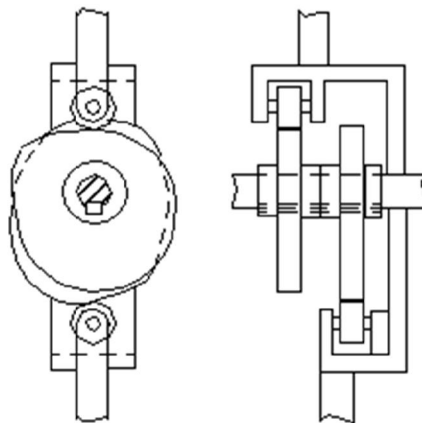


Figure 6-6 Constant diameter cam

A mechanical constraint cam also be introduced by employing a dual or conjugate cam in arrangement similar to what shown in Figure 6-7. Each cam has its own roller, but the rollers are mounted on the same reciprocating or oscillating follower.

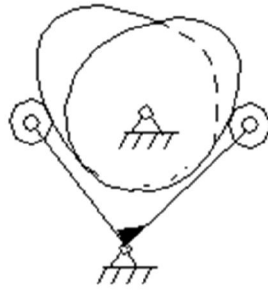


Figure 6-7 Dual cam

6.2.5 Examples in SimDesign

Rotating Cam, Translating Follower

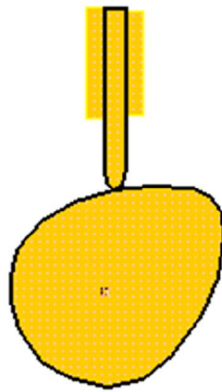


Figure 6-8 SimDesign translating cam

Load the SimDesign file `simdesign/cam.translating.sim`. If you turn the cam, the follower will move. The weight of the follower keeps them in contact. This is called a *gravity constraint* cam.

Rotating Cam/Rotating Follower

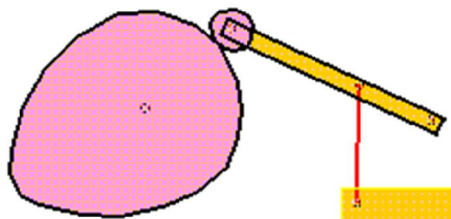


Figure 6-9 SimDesign oscillating cam

The SimDesign file is `simdesign/cam.oscillating.sim`. Notice that a roller is used at the end of the follower. In addition, a spring is used to maintain the contact of the cam and the roller.

If you try to calculate the [degrees of freedom \(DOF\)](#) of the mechanism, you must imagine that the roller is welded onto the follower because turning the roller does not influence the motion of the follower.

6.3 Cam Nomenclature

[Figure 6-10](#) illustrates some cam nomenclature:

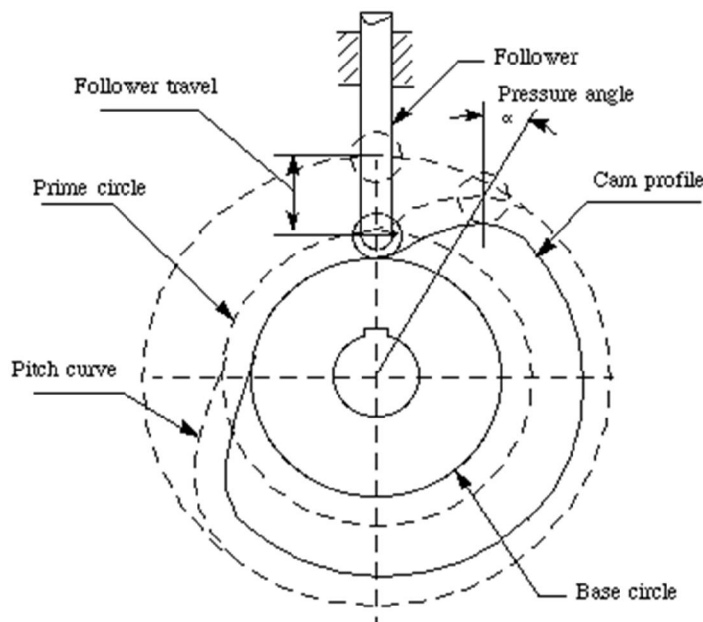


Figure 6-10 Cam nomenclature

- **Trace point:** A theoretical point on the follower, corresponding to the point of a fictitious *knife-edge follower*. It is used to generate the *pitch curve*. In the case of a *roller follower*, the trace point is at the center of the roller.
- **Pitch curve:** The path generated by the trace point at the follower is rotated about a stationary cam.
- **Working curve:** The working surface of a cam in contact with the follower. For the *knife-edge follower* of the plate cam, the *pitch curve* and the *working curves* coincide. In a *close or grooved cam* there is an *inner profile* and an *outer working curve*.
- **Pitch circle:** A circle from the cam center through the pitch point. The pitch circle radius is used to calculate a cam of minimum size for a given *pressure angle*.
- **Prime circle (reference circle):** The smallest circle from the cam center through the pitch curve.

- **Base circle:** The smallest circle from the cam center through the cam profile curve.
- **Stroke or throw:** The greatest distance or angle through which the follower moves or rotates.
- **Follower displacement:** The position of the follower from a specific zero or rest position (usually its the position when the follower contacts with the *base circle* of the cam) in relation to time or the rotary angle of the cam.
- **Pressure angle:** The angle at any point between the normal to the pitch curve and the instantaneous direction of the follower motion. This angle is important in cam design because it represents the steepness of the cam profile.

6.4 Motion events

When the cam turns through one motion cycle, the follower executes a series of events consisting of rises, dwells and returns. **Rise** is the motion of the follower away from the cam center, **dwell** is the motion during which the follower is at rest; and **return** is the motion of the follower toward the cam center.

There are many follower motions that can be used for the rises and the returns. In this chapter, we describe a number of basic curves.

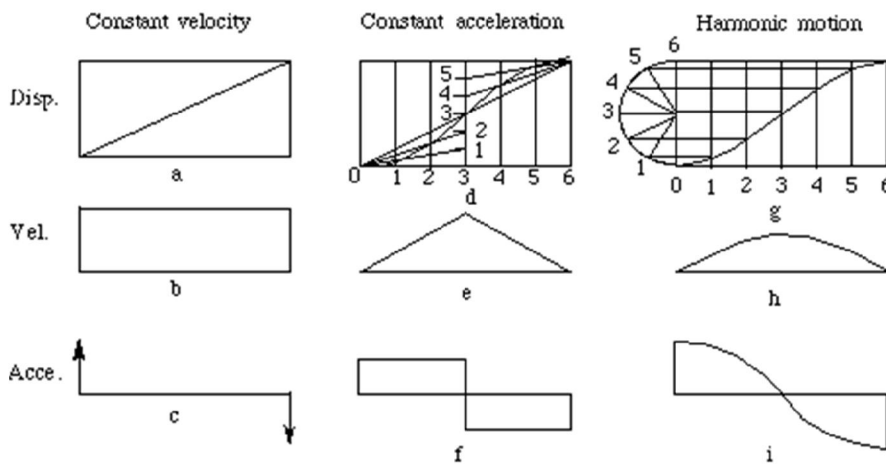


Figure 6-11 Motion events

Notation

- Φ : The rotary angle of the cam, measured from the beginning of the motion event;
- β : The range of the rotary angle corresponding to the motion event;
- h : The stroke of the motion event of the follower;
- S : Displacement of the follower;
- V : Velocity of the follower;
- A : Acceleration of the follower.

6.4.1 Constant Velocity Motion

If the motion of the follower were a straight line, [Figure 6-11a,b,c](#), it would have equal displacements in equal units of time, *i.e.*, uniform velocity from the beginning to the end of the stroke, as shown in b. The acceleration, except at the end of the stroke would be zero, as shown in c. The diagrams show abrupt changes of velocity, which result in large forces at the beginning and the end of the stroke. These forces are undesirable, especially when the cam rotates at high velocity. The *constant velocity motion* is therefore only of theoretical interest.

$$\begin{aligned} S(\phi) &= h \frac{\phi}{\beta} \\ V(\phi) &= \frac{h}{\beta} \\ A(\phi) &= 0 \end{aligned}$$

(6-1)

6.4.2 Constant Acceleration Motion

Constant acceleration motion is shown in [Figure 6-11d, e, f](#). As indicated in e, the velocity increases at a uniform rate during the first half of the motion and decreases at a uniform rate during the second half of the motion. The acceleration is constant and positive throughout the first half of the motion, as shown in f, and is constant and negative throughout the second half. This type of motion gives the follower the smallest value of maximum acceleration along the path of motion. In high-speed machinery this is particularly important because of the forces that are required to produce the accelerations.

When

$$0 \leq \phi \leq \frac{\beta}{2},$$

$$\begin{aligned} S(\phi) &= 2h \frac{\phi^2}{\beta^2} \\ V(\phi) &= \frac{4h}{\beta^2} \phi \\ A(\phi) &= \frac{4h}{\beta^2} \end{aligned}$$

(6-2)

When

$$\frac{\beta}{2} \leq \phi \leq \beta,$$

$$\begin{aligned}
 S(\phi) &= h - \frac{2h}{\beta^2}(\beta - \phi)^2 \\
 V(\phi) &= \frac{4h}{\beta}\left(1 - \frac{\phi}{\beta}\right) \\
 A(\phi) &= \frac{4h}{\beta^2}
 \end{aligned}$$

(6-3)

6.4.3 Harmonic Motion

A cam mechanism with the basic curve like *g* in [Figure 6-7g](#) will impart *simple harmonic motion* to the follower. The velocity diagram at *h* indicates smooth action. The acceleration, as shown at *i*, is maximum at the initial position, zero at the mid-position, and negative maximum at the final position.

$$\begin{aligned}
 S(\phi) &= \frac{h}{2}\left(1 - \cos \frac{\pi\phi}{\beta}\right) \\
 V(\phi) &= \frac{h\pi}{2\beta} \sin \frac{\pi\phi}{\beta} \\
 A(\phi) &= \frac{h\pi^2}{2\beta^2} \cos \frac{\pi\phi}{\beta}
 \end{aligned}$$

(6-4)

6.5 Cam Design

The translational or rotational displacement of the follower is a function of the rotary angle of the cam. A designer can define the function according to the specific requirements in the design. The motion requirements, listed below, are commonly used in cam profile design.

6.5.1 Disk Cam with Knife-Edge Translating Follower

Figure 6-12 is a skeleton diagram of a disk cam with a knife-edge translating follower. We assume that the cam mechanism will be used to realize the displacement relationship between the rotation of the [cam](#) and the translation of the [follower](#).

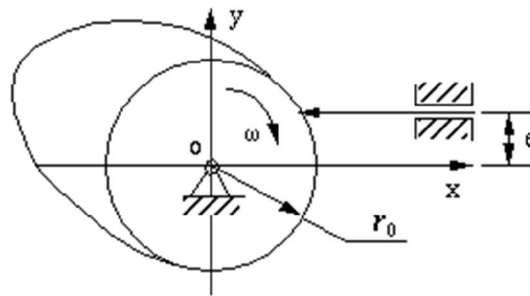


Figure 6-12 A Skeleton Diagram of disk cam with knife-edge translation

Below is a list of the essential parameters for the evaluation of these types of cam mechanisms. However, these parameters are adequate only to define a [knife-edge follower](#) and a [translating follower](#) cam mechanism.

Parameters:

r_o : The radius of the [base circle](#);

e : The [offset](#) of the follower from the rotary center of the cam. Notice: it could be negative.

s : The displacement of the follower which is a function of the rotary angle of the cam -- ϕ

IW : A parameter whose absolute value is 1. It represents the turning direction of the cam. When the cam turns clockwise: $IW=+1$, otherwise: $IW=-1$.

Cam profile design principle:

The method termed [inversion](#) is commonly used in cam profile design. For example, in a disk cam with [translating follower](#) mechanism, the follower translates when the cam turns. This means that the relative motion between them is a combination of a relative turning motion and a relative translating motion. Without changing this feature of their relative motion, imagine that the cam remains fixed. Now the follower performs both the relative turning and translating motions. We have inverted the mechanism.

Furthermore, imagine that the [knife-edge](#) of the follower moves along the fixed cam profile in the inverted mechanism. In other words, the [knife edge](#) of the follower draws the profile of the cam. Thus, the problem of designing the cam profile becomes a problem of calculating the trace of the knife edge of the follower whose motion is the combination of the relative turning and the relative translating.

Design equations:

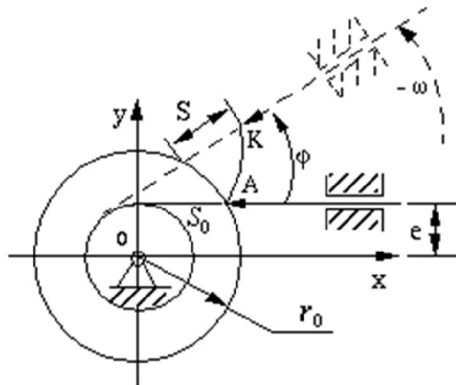


Figure 6-13 Profile design of translating cam follower

In Figure 6-13, only part of the cam profile AK is displayed. Assume the cam turns clockwise. At the beginning of motion, the knife edge of the follower contacts the point of intersection A of the [base circle](#) and the cam profile. The coordinates of A are (S_0, e) , and S_0 can be calculated from equation $S_0 = \sqrt{r_0^2 - e^2}$

Suppose the displacement of the follower is S when the angular displacement of the cam is ϕ . At this moment, the coordinates of the knife edge of the follower should be $(S_0 + S, e)$.

To get the corresponding position of the knife edge of the follower in the inverted mechanism, turn the follower around the center of the cam in the reverse direction through an angle of ϕ . The knife edge will be inverted to point K , which corresponds to the point on the cam profile in the inverted mechanism. Therefore, the coordinates of point K can be calculated with the following equation:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(IW \cdot \phi) & -\sin(IW \cdot \phi) \\ \sin(IW \cdot \phi) & \cos(IW \cdot \phi) \end{bmatrix} \begin{bmatrix} S_0 + S \\ e \end{bmatrix}$$

(6-5)

Note:

- The [offset](#) e is negative if the follower is located below the x axis.
- When the rotational direction of the cam is clockwise: $IW = +1$, otherwise: $IW = -1$.

6.5.2 Disk Cam with Oscillating Knife-Edge Follower

Suppose the cam mechanism will be used to make the knife edge oscillate. We need to compute the coordinates of the cam profile that results in the required motion of the follower.

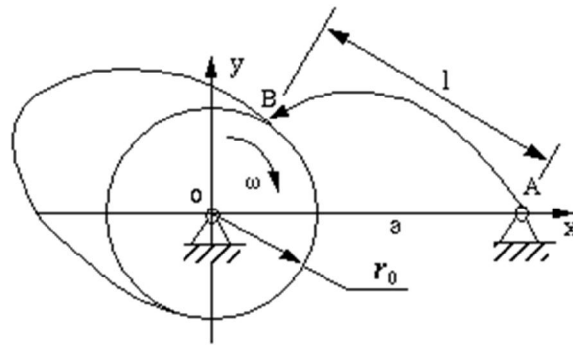


Figure 6-14 Disk cam with knife-edge oscillating follower

The essential parameters in this kind of cam mechanisms are given below.

r_o : The radius of the [base circle](#);

a : The distance between the pivot of the cam and the pivot of the follower.

l : The length of the follower which is a distance from its pivot to its knife edge.

Ψ : The angular displacement of the follower which is a function of the rotary angle of the cam -- ϕ .

IP : A parameter whose absolute value is 1. It represents the location of the follower.

When the follower is located above the x axis: $IP=+1$, otherwise: $IP=-1$.

IW : A parameter whose absolute value is 1. It represents the turning direction of the cam.

When the cam turns clockwise: $IW=+1$, otherwise: $IW=-1$.

Cam profile design principle

The fundamental principle in designing the cam profiles is still [inversion](#), similar to that for designing other cam mechanisms, (e.g., the [translating follower cam mechanism](#)). Normally, the follower oscillates when the cam turns. This means that the relative motion between them is a combination of a relative turning motion and a relative oscillating motion. Without changing this feature of their relative motion, let the cam remain fixed and the follower performs both the relative turning motion and oscillating motion. By imagining in this way, we have actually inverted the mechanism.

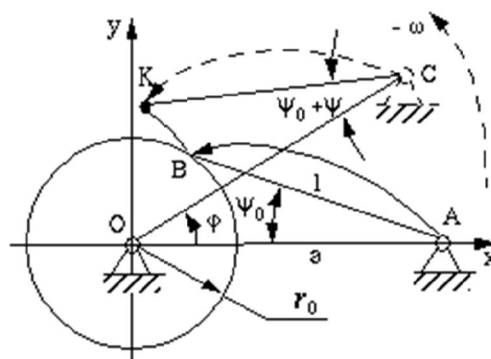


Figure 6-15 Cam profile design for a rotating follower

In Figure 6-15, only part of the cam profile BK is shown. We assume that the cam turns clockwise.

At the beginning of motion, the [knife edge](#) of the follower contacts the point of intersection (B) of the base circle and the cam profile. The initial angle between the follower (AB) and the line of two pivots (AO) is Ψ_0 . It can be calculated from the triangle OAB .

When the angular displacement of the cam is ϕ , the oscillating displacement of the follower is ψ which measures from its own initial position. At this moment, the angle between the follower and the line passes through two pivots should be $\psi + \Psi_0$.

The coordinates of the [knife edge](#) at this moment will be

$$(\alpha - l \cos [IP (\psi + \Psi_0)], l \sin [IP (\psi + \Psi_0)]) \quad (6-6)$$

To get the corresponding knife-edge of the follower in the inverted mechanism, simply turn the follower around the center of the cam in the reverse direction of the cam rotation through an angle of ϕ . The knife edge will be inverted to point K which corresponds to the point on the cam profile in the inverted mechanism. Therefore, the coordinates of point K can be calculated with the following equation:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos (IW \cdot \phi) & -\sin (IW \cdot \phi) \\ \sin (IW \cdot \phi) & \cos (IW \cdot \phi) \end{bmatrix} \begin{bmatrix} \alpha - l \cos [IP (\psi + \Psi_0)] \\ l \sin [IP (\psi + \Psi_0)] \end{bmatrix} \quad (6-7)$$

Note:

- When the initial position of the follower is above the x axis, $IP = +1$, otherwise: $IP = -1$.
- When the rotary direction of the cam is clockwise: $IW = +1$, otherwise: $IW = -1$.

6.5.3 Disk Cam with Roller Follower

Additional parameters:

- r : the radius of the roller.
- IM : a parameter whose absolute value is 1, indicating which envelope curve will be adopted.
- RM : inner or outer envelope curve. When it is an inner envelope curve: $RM = +1$, otherwise: $RM = -1$.

Design principle:

The basic principle of designing a cam profile with the [inversion](#) method is still used. However, the curve is not directly generated by inversion. This procedure has two steps:

1. Imagine the center of the roller as a knife edge. This concept is important in cam profile design and is called the [trace point](#) of follower. Calculate the [pitch curve](#) aa , that is, the trace of the pitch point in the inverted mechanism.
2. The cam profile bb is a product of the enveloping motion of a series of rollers.

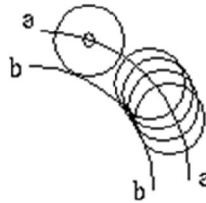


Figure 6-16 The trace point of the follower on a disk cam

Design equations:

The problem of calculating the coordinates of the cam profile is the problem of calculating the tangent points of a sequence of rollers in the inverted mechanism. At the moment shown Figure 6-17, the tangent point is P on the cam profile.

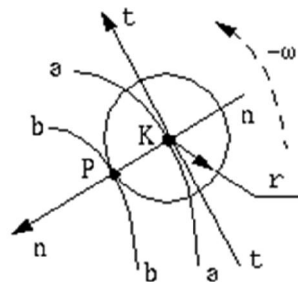


Figure 6-17 The tangent point, P, of a roller to the disk cam

The calculation of the coordinates of the point P has two steps:

1. Calculate the slope of the tangent tt of point K on [pitch curve](#), aa .
2. Calculate the slope of the normal nn of the curve aa at point K .

Since we have already have the coordinates of point K : (x, y) , we can express the coordinates of point P as

$$\begin{cases} x_p = x - IW \cdot RM \cdot r \cdot \frac{dy/d\phi}{\sqrt{(dx/d\phi)^2 + (dy/d\phi)^2}} \\ y_p = y + IW \cdot RM \cdot r \cdot \frac{dx/d\phi}{\sqrt{(dx/d\phi)^2 + (dy/d\phi)^2}} \end{cases}$$

(6-8)