1) Design a linear phase FIR Lowpass filter using rectangular window by taking 7 samples of window sequence and with a cutoff freq $\omega_c = 0.2\pi$ rad/sample

Soln :- Symmetric Impulse response with symmetry condition

$$h(N-1-n) = h(n)$$

Desired Ideal frequency response $H_d(e^{j\omega})$ for FIR LPF is

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} , & -\omega_c \le \omega \le +\omega_c \\ 0 , & \text{otherwise} \end{cases}$$

The desired impulse response $h_d(n)$ is obtained by taking Inverse fourier transform of $H_d(e^{j\omega})$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega \Rightarrow \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\omega\alpha}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega(n-\alpha)}\, d\omega$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)} \right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega_c(n-\alpha)}}{j(n-\alpha)} - \frac{e^{-j\omega_c(n-\alpha)}}{j(n-\alpha)} \right]$$

$$\therefore h_d(n) = \frac{1}{\pi(n-\alpha)} \left[\frac{e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}}{2j} \right]$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\therefore h_d(n) = \frac{\sin\omega_c(n-\alpha)}{\pi(n-\alpha)} \; ; \text{ for all } n \text{ except } n = \alpha$$

$\therefore$ when $n=\alpha$; $h_d(n) = \underset{(n-\alpha)\to 0}{Lt} \dfrac{\sin \omega_c(n-\alpha)}{\pi(n-\alpha)}$

$$= \frac{1}{\pi} \underset{(n-\alpha)\to 0}{Lt} \frac{\sin \omega_c(n-\alpha)}{(n-\alpha)}$$

Using L' Hopital rule

$$\underset{\theta\to 0}{Lt} \frac{\sin A\theta}{\theta} = A$$

$$= \frac{1}{\pi} \times \omega_c \Rightarrow \frac{\omega_c}{\pi}$$

The impulse response $h(n)$ of FIR filter is obtained by multiplying $h_d(n)$ by window sequence.

Rectangular Window $W_R(n) : \begin{cases} 1 & \text{for } n=0 \text{ to } N-1 \\ 0 & \text{for otherwise} \end{cases}$

$\therefore$ Impulse response $h(n) = h_d(n) \times W_R(n)$

$$= h_d(n) \quad \text{for } n=0 \text{ to } N-1$$

$\therefore N=7$; $\omega_c = 0.2\pi$ rad/sample $\quad \alpha = \dfrac{N-1}{2} = \dfrac{7-1}{2} = 3$; $\{N-1=6\}$

Hence, calculate $h(n)$ for $n=0$ to 6

Impulse response $h(n)$ satisfies the symmetry condition

$$h(N-1-n) = h(n)$$

when $n=0$; $h(0) = \dfrac{\sin \omega_c(n-\alpha)}{\pi(n-\alpha)} \Rightarrow \dfrac{\sin(0.2\pi \times (0-3))}{\pi(0-3)} = 0.1009$

when $n=1$, $h(1) = \dfrac{\sin(0.2\pi \times (1-3))}{\pi(1-3)} = 0.1514$

when $n=2$, $h(2) = \dfrac{\sin(0.2\pi \times (2-3))}{\pi(2-3)} = 0.1871$

when $n=3$, $h(3) = \dfrac{0.2\pi}{\pi} = 0.2$

Using symmetry condition

$$h(N-1-n) = h(n) \Rightarrow h(6-n) = h(n)$$

when $n=4$, $h(4) = h(6-4) = h(2) = 0.1871$

when $n=5$, $h(5) = h(6-5) = h(1) = 0.1514$

when $n=6$, $h(6) = h(6-6) = h(0) = 0.1009$

The transfer function $H(z)$ of FIR Low pass filter is given by

$$H(z) = z\{h(n)\} = \sum_{n=0}^{N-1} h(n) z^{-n} \Rightarrow \sum_{n=0}^{6} h(n) z^{-n}$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4} + h(5) z^{-5} + h(6) z^{-6}$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(2) z^{-4} + h(1) z^{-5} + h(0) z^{-6}$$

$$= h(0)\left[1 + z^{-6}\right] + h(1)\left[z^{-1} + z^{-5}\right] + h(2)\left[z^{-2} + z^{-4}\right] + h(3) z^{-3}$$

$$H(z) = 0.1009\left[1 + z^{-6}\right] + 0.1514\left[z^{-1} + z^{-5}\right] + 0.1871\left[z^{-2} + z^{-4}\right] + 0.2 z^{-3}$$

structure :-

$$H(z) = \frac{Y(z)}{X(z)}$$

$$H(z) = \frac{Y(z)}{X(z)} = 0.1009\left[1 + z^{-6}\right] + 0.1514\left[z^{-1} + z^{-5}\right] + 0.1871\left[z^{-2} + z^{-4}\right] + 0.2 z^{-3}$$

$$\therefore Y(z) = 0.1009\left[X(z) + z^{-6} X(z)\right] + 0.1514\left[z^{-1} X(z) + z^{-5} X(z)\right] + 0.1871\left[z^{-2} X(z) + z^{-4} X(z)\right] + 0.2 z^{-3} X(z)$$

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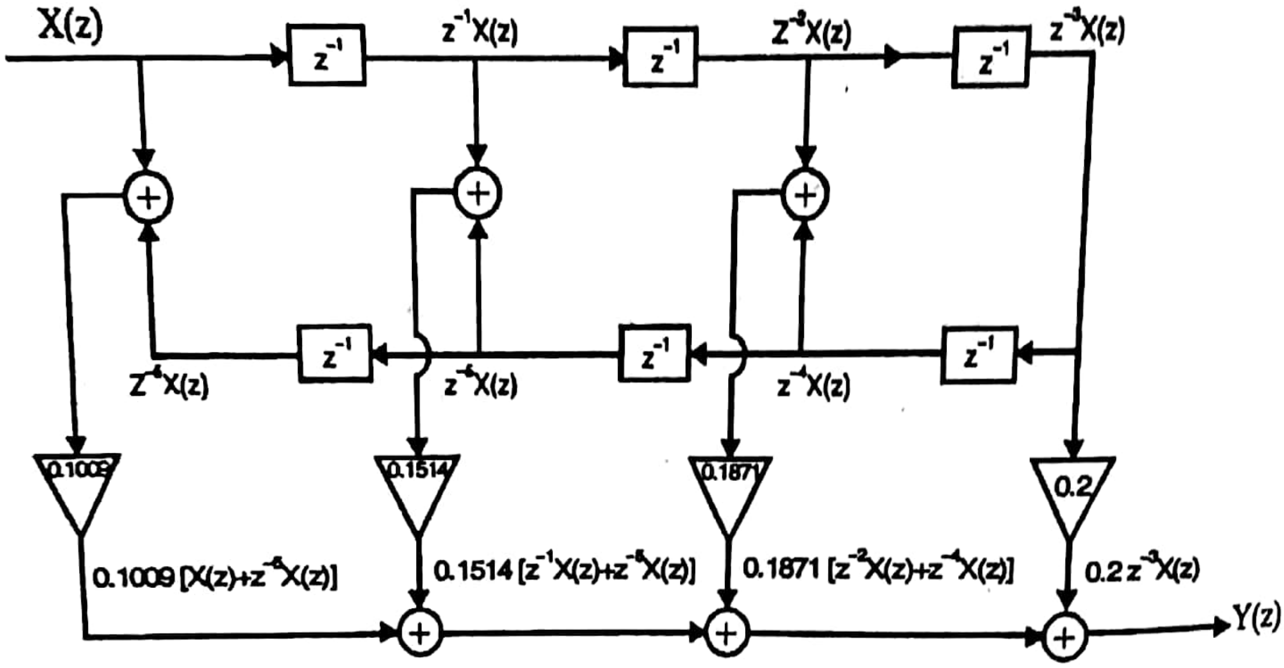
Linear phase structure of FIR Lowpass filter :-



Frequency Response :-

$$A(\omega) = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right) \cos\omega n$$

$$A(\omega) = h(3) + \sum_{n=1}^{3} 2h(3-n) \cos \omega n$$

$$= h(3) + 2h(2)\cos\omega + 2h(1)\cos 2\omega + 2h(0)\cos 3\omega$$

$$= 0.2 + 2\times 0.1871 \cos\omega + 2\times 0.1514 \cos 2\omega + 2\times 0.1009 \cos 3\omega$$

$$A(\omega) = 0.2 + 0.3742\cos\omega + 0.3028 \cos 2\omega + 0.2018 \cos 3\omega$$

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