1) calculate Order and cutoff Frequency for the given IIR filter using bilinear transformation, Take $T = 0.5$ sec

$$\begin{cases} 0.707 \le |H e^{j\omega}| < 1.0 & \text{for } 0 \le \omega \le 0.45\pi \\ |H e^{j\omega}| \le 0.2 & \text{for } 0.65\pi \le \omega \le \pi \end{cases}$$

Soln :-

$A_p = 0.707$ $\qquad$ $\omega_p = 0.45\pi$

$A_s = 0.2$ $\qquad$ $\omega_s = 0.65\pi$

Bilinear Method :-

$$\Omega_p = \frac{2}{T} \tan\left(\frac{\omega_p}{2}\right) \qquad \Omega_s = \frac{2}{T} \tan\left(\frac{\omega_s}{2}\right)$$

$$= \frac{2}{0.5} \tan\left(\frac{0.45\pi}{2}\right) \qquad = \frac{2}{0.5} \tan\left(\frac{0.65\pi}{2}\right)$$

$$= 4 \tan(0.7065) \qquad = 4 \tan(1.020)$$

$$\Omega_p = 3.413 \text{ rad/sec} \qquad \Omega_s = 6.527 \text{ rad/sec}$$

Order of the filter :-

$$N = \frac{1}{2} \left\{ \frac{\log\left[\dfrac{\left(\frac{1}{A_s^2} - 1\right)}{\left(\frac{1}{A_p^2} - 1\right)}\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} \right\}$$

$$= \frac{1}{2} \; \frac{\log\left[\dfrac{\left(\frac{1}{(0.2)^2} - 1\right)}{\left(\frac{1}{(0.7)^2} - 1\right)}\right]}{\log\left(\frac{6.527}{3.413}\right)}$$

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$$= \frac{1}{2} \frac{\log\left[\dfrac{\left(\dfrac{1}{0.04}-1\right)}{\left(\dfrac{1}{0.49}-1\right)}\right]}{\log(1.9123)}$$

$$= \frac{1}{2} \frac{\log\left[\dfrac{25-1}{2.0-1}\right]}{\log(1.9123)} \Rightarrow \frac{1}{2} \frac{\log\left(\dfrac{24}{1.0}\right)}{0.28155}$$

$$= \frac{1}{2}\left[\frac{1.380}{0.2815}\right] = 2.45$$

$$N = 3$$

cutoff Frequency :-

$$\Omega_c = \frac{\Omega_s}{\left(\dfrac{1}{A_s^2}-1\right)^{1/2N}} = \frac{6.527}{\left(\dfrac{1}{(0.2)^2}-1\right)^{1/6}}$$

$$= \frac{6.527}{\left(\dfrac{1}{0.04}-1\right)^{1/6}} \Rightarrow \frac{6.527}{(24)^{1/6}}$$

$$= \frac{6.527}{1.698} = 3.843$$

$$\Omega_c = 3.843 \text{ rad/sec}$$

N	Denominator of H(s)
1	$s+1$
2	$s^2 + \sqrt{2}\,s + 1$
3	$(s+1)(s^2+s+1)$
4	$(s^2 + 0.765s + 1)(s^2 + 1.847s + 1)$
5	$(s+1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$

Frequency transformation in Analog Domain :-

1) Normalised LPF to LPF

$$s \to \frac{s}{\Omega_c}$$

2) Normalised LPF to HPF

$$s \to \frac{\Omega_c}{s}$$

3) Normalised LPF to BSF

$$s \to \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_l + \Omega_u}$$

4) Normalised LPF to BPF

$$s \to \frac{s^2 + \Omega_l \Omega_u}{s(\Omega_u - \Omega_l)}$$

(2) Use bilinear transformation to design the first order butterworth low pass filter with 3dB and cut off frequency of 0.2π.

soln :-

$$w_c = 0.2\pi$$

$$\Omega_c = \frac{2}{T}\tan\left(\frac{w_c}{2}\right) = \frac{2}{1}\tan\left(\frac{0.2\pi}{2}\right)$$

$$= 2\tan(0.314)$$

$$\Omega_c = 0.649 \text{ rad/sec}$$

$$\therefore N = 1$$

$$H(S) = \frac{1}{S+1}$$

Replace $S \rightarrow \frac{S}{\Omega_c}$

$$H\left(\frac{S}{\Omega_c}\right) = \frac{1}{\frac{S}{\Omega_c}+1} = \frac{\Omega_c}{S+\Omega_c} = \frac{0.649}{S+0.649}$$

Bilinear Transformation :-

$$S = \frac{2}{T}\frac{(z-1)}{(z+1)}$$

$$H(z) = \frac{0.65}{\frac{2}{T}\frac{(z-1)}{(z+1)}+0.65} \Rightarrow \frac{0.65(z+1)}{2z-2+0.65z+0.65}$$

$$\therefore H(z) = \frac{0.65z+0.65}{2.65z-1.35}$$

③ Design a butterworth digital IIR Low pass filter using Bilinear Transformation $T = 0.1$ sec.

$$\begin{cases} 0.6 \le |H e^{j\omega}| \le 1.0 & \text{for } 0 \le \omega \le 0.35\pi \\ |H e^{j\omega}| \le 0.1 & \text{for } 0.7\pi \le \omega \le \pi \end{cases}$$

Soln :-

$A_p = 0.6$ $\qquad \omega_p = 0.35\pi$

$A_s = 0.1$ $\qquad \omega_s = 0.7\pi$

$$\Omega_p = \frac{2}{T} \tan\left(\frac{\omega_p}{2}\right)$$

$$= \frac{2}{0.1} \tan\left(\frac{0.35\pi}{2}\right) \Rightarrow 20 \tan(0.5495)$$

$$\Omega_p = 12.248 \text{ rad/sec}$$

$$\Omega_s = \frac{2}{T} \tan\left(\frac{\omega_s}{2}\right)$$

$$= \frac{2}{0.1} \tan\left(\frac{0.7\pi}{2}\right)$$

$$= 20 \tan(1.099) = 39.2 \text{ rad/sec}$$

$$\Omega_s = 39.2 \text{ rad/sec}$$

Order of the filter :-

$$N = \frac{1}{2} \frac{\log\left(\dfrac{\frac{1}{A_s^2} - 1}{\frac{1}{A_p^2} - 1}\right)}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

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$$= \frac{1}{2} \frac{\log\left[\dfrac{\left(\frac{1}{0.01}-1\right)}{\left(\frac{1}{0.36}-1\right)}\right]}{\log\left(\dfrac{39.25}{12.248}\right)} \Rightarrow \frac{1}{2} \frac{\log\left[\dfrac{100-1}{2.77-1}\right]}{\log(3.204)}$$

$$= \frac{1}{2} \frac{\log\left(\frac{99}{1.77}\right)}{\log(3.204)} \Rightarrow \frac{1}{2}\left(\frac{1.747}{0.505}\right) = 1.729$$

$\therefore N = 2$

cutoff Frequency :-

$$\Omega_c = \frac{\Omega_s}{\left[\frac{1}{A_s^2}-1\right]^{1/2N}} = \frac{39.25}{\left[\frac{1}{0.01}-1\right]^{1/4}}$$

$$= \frac{39.25}{(99)^{1/4}} = 12.44 \text{ rad/sec}$$

$$H(S) = \frac{1}{S^2+\sqrt{2}S+1}$$

$$S \to \frac{S}{\Omega_c} \Rightarrow \frac{1}{\left(\frac{S}{\Omega_c}\right)^2+\sqrt{2}\left(\frac{S}{\Omega_c}\right)+1} \Rightarrow \frac{1}{\frac{S^2}{\Omega_c^2}+\frac{\sqrt{2}S}{\Omega_c}+1}$$

$$= \frac{1}{\frac{s^2+\sqrt{2}\,s\,\Omega_c+\Omega_c^2}{\Omega_c^2}} \Rightarrow \frac{\Omega_c^2}{s^2+\sqrt{2}\,s\,\Omega_c+\Omega_c^2}$$

$\Omega_c = 12.44$ rad/sec

$$= \frac{(12.44)^2}{s^2+1.414\,(12.44)\,s+(12.44)^2}$$

$$H(z) = \frac{(12.44)^2}{\left[\frac{2}{T}\left(\frac{z-1}{z+1}\right)\right]^2 + 1.414\,(12.44)\left[\frac{2}{T}\left(\frac{z-1}{z+1}\right)\right] + (12.44)^2}$$

$$= \frac{154.75}{\frac{4}{T^2}\left[\frac{z-1}{z+1}\right]^2 + 17.59\left[\frac{2}{T}\left(\frac{z-1}{z+1}\right)\right] + 154.75}$$

$$H(z) = \frac{154.75}{\frac{4}{0.01}\left(\frac{z-1}{z+1}\right)^2 + 17.59\,\frac{2}{0.1}\left(\frac{z-1}{z+1}\right) + 154.75}$$

④ Design a butterworth digital IIR High pass filter using Bilinear Transformation:

$$0.6 \le |H e^{j\omega}| \le 1.0 \quad \text{for} \quad 0.7\pi \le \omega \le \pi$$

$$|H e^{j\omega}| \le 0.1 \quad \text{for} \quad 0 \le \omega \le 0.35\pi$$

Soln:

$A_p = 0.6$ $\qquad \omega_p = 0.7\pi$

$A_s = 0.1$ $\qquad \omega_s = 0.35\pi$

$$\Omega_p = \frac{2}{T} \tan\left(\frac{\omega_p}{2}\right)$$

$$= \frac{2}{0.1} \tan\left(\frac{0.7\pi}{2}\right) \Rightarrow 20 \tan(1.099)$$

$$\boxed{\Omega_p = 39.198 \text{ rad/sec}}$$

$$\Omega_s = \frac{2}{T} \tan\left(\frac{\omega_s}{2}\right)$$

$$= \frac{2}{0.1} \tan\left(\frac{0.35\pi}{2}\right) \rightarrow 20 \tan(0.549)$$

$$\boxed{\Omega_s = 12.248 \text{ rad/sec}}$$

order of the filter :-

$$N = \frac{1}{2} \frac{\log\left[\frac{\frac{1}{A_s^2} - 1}{\frac{1}{A_p^2} - 1}\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$= \frac{1}{2} \frac{\log\left[\frac{\frac{1}{(0.1)^2} - 1}{\frac{1}{(0.6)^2} - 1}\right]}{\log\left[\frac{12.24}{39.19}\right]} \Rightarrow \frac{1}{2} \frac{\log\left[\frac{100-1}{2.77-1}\right]}{\log(0.3123)}$$

$$= \frac{1}{2}\left[\frac{1.7459}{-0.5054}\right] \Rightarrow -1.727$$

$$\boxed{\therefore N = 2}$$

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cut off Frequency :-

$$\Omega_c = \frac{\Omega_s}{\left[\frac{1}{A_s^2} - 1\right]^{1/2N}} = \frac{12.248}{\left[\frac{1}{(0.1)^2} - 1\right]^{1/4}}$$

$$= \frac{12.248}{(99)^{1/4}} = \frac{12.248}{3.154} = 3.883 \text{ rad/sec}$$

$$\Omega_c = 3.883 \text{ rad/sec}$$

$$H(S) = \frac{1}{S^2 + \sqrt{2}S + 1}$$ Replace S by $\frac{\Omega_c}{S}$

$$= \frac{1}{\left(\frac{\Omega_c}{S}\right)^2 + \sqrt{2}\left(\frac{\Omega_c}{S}\right) + 1} \Rightarrow \frac{S^2}{\Omega_c^2 + \sqrt{2}\Omega_c S + S^2}$$

$$= \frac{S^2}{S^2 + \sqrt{2}(3.88)S + (3.88)^2} \Rightarrow \frac{S^2}{S^2 + 5.486S + 15.05}$$

Replace $S = \frac{2}{T}\left[\frac{Z-1}{Z+1}\right]$

$$H(Z) = \frac{\left[\frac{2}{T}\left(\frac{Z-1}{Z+1}\right)\right]^2}{\left[\frac{2}{T}\left(\frac{Z-1}{Z+1}\right)\right]^2 + 5.486\left[\frac{2}{T}\left(\frac{Z-1}{Z+1}\right)\right] + 15.05}$$

Sub T = 0.1 sec

$$H(z) = \frac{\left[\frac{2}{0.1}\left(\frac{z-1}{z+1}\right)\right]^2}{\left[\frac{2}{0.1}\left(\frac{z-1}{z+1}\right)\right]^2 + 5.486\left[\frac{2}{0.1}\left(\frac{z-1}{z+1}\right)\right] + 15.05}$$

$$= \frac{400\left(\frac{z-1}{z+1}\right)^2}{400\left(\frac{z-1}{z+1}\right)^2 + 5.486\,(20)\left(\frac{z-1}{z+1}\right) + 15.05}$$

$$H(z) = \frac{400\,(z-1)^2}{400\,(z-1)^2 + 109.72\,(z-1)(z+1) + 15.05(z+1)}$$