SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB212 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 2 – IIR FILTER DESIGN

TOPIC – ANALOG FILTER
Impulse Invariant & Bilinear Transformation

IIR FILTERS

- **Infinite Impulse Response (IIR) Systems:** Length of Unit sample response (or) Impulse response h(n) is infinite
- **Infinite Impulse Response (IIR) Filters:** The filters designed by considering all the infinite samples of impulse responses
- In design of IIR filter, the specification of an IIR filter is transformed to specification of an analog filter and an analog filter with transfer function
- H(s) is designed to satisfy the specification. Then the analog filter is transformed to digital filter with transfer function H(z)

ADVANTAGES OF DIGITAL FILTERS

- The values of resistors, capacitors and inductors used in analog filters changes with temperature, since digital filters do not have these components, they have high thermal stability
- The digital filters are programmable. Hence the filter coefficients can be changed at any time to implement adaptive filters
- A single filter can be used to process multiple signals by using the techniques of multiplexing.
- In digital filters the precision of the filter depends on the length (or size) of the registers used to store the filter coefficients.

DISADVANTAGES OF DIGITAL FILTERS & FEATURES OF IIR FILTERS

DISADVANTAGES:

- The bandwidth of the discrete signal is limited by the sampling frequency. The bandwidth of real discrete signal is half the sampling frequency
- The performance of the digital filter depends on the hardware (i.e., depends on the bit length of the registers in the hardware) used to implement the filter

FEATURES:

- The physically realizable IIR filters do not have linear phase
- The IIR filter specifications include the desired characteristics for the magnitude response only.

COMPARISON OF DIGITAL & ANALOG FILTERS

S.No.	Digital Filter	Analog Filter
1	Operates on digital samples of the signal	Operates on analog signals
2	It is governed by linear difference equation	It is governed by linear differential equation
3	It consists of adders, multipliers and delays implemented in digital logic	It consists of electrical components like resistors, capacitors and inductors
4	The filter coefficients are designed to satisfy the desired frequency response	The approximation problem is solved to satisfy the desired frequency response

FREQUENCY RESPONSE OF ANALOG AND DIGITAL IIR FILTERS

- The filters are frequency selective devices and so they are designed to pass the spectral content of the input signal in a specified band of frequencies
- Hence based on frequency response the filters are classified into four basic types. They are
- **Low pass**
- **High pass**
- **Band pass and**
- **Band stop**

FREQUENCY RESPONSE OF ANALOG AND DIGITAL IIR FILTERS

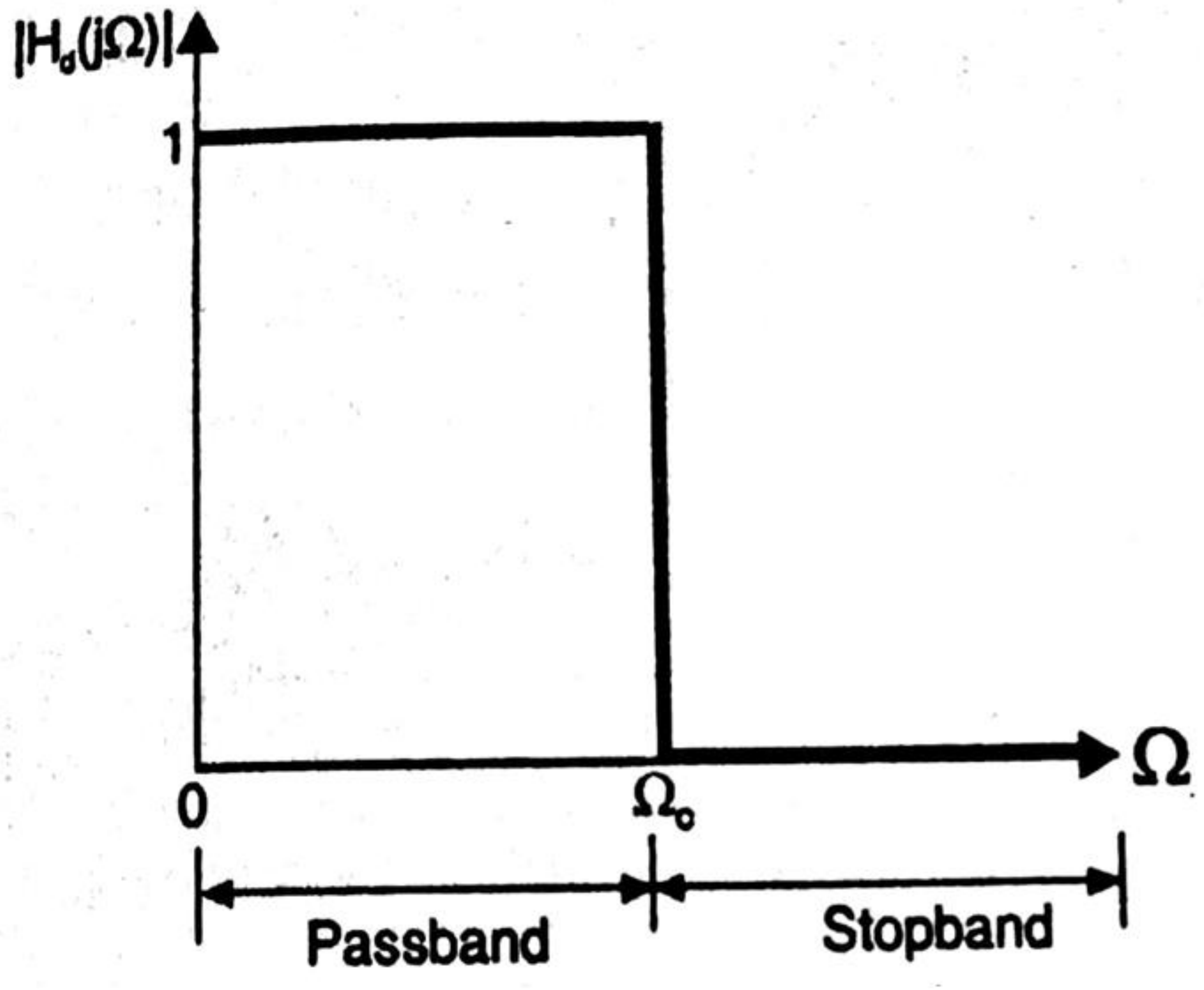
- The approximation problem is solved to meet a specified tolerance in the pass band and stop band
- In the pass band the magnitude is approximated to unity with in an error of δ_p
- In the stop band the magnitude is approximated to zero with in an error of δ_s
- Here δ_p , δ_s are the limits of the tolerance in the pass band and stop band
- The δ_p and δ_s are also called ripples

SPECIFICATION FOR PRACTICAL ANALOG FILTER & DIGITAL IIR FILTER

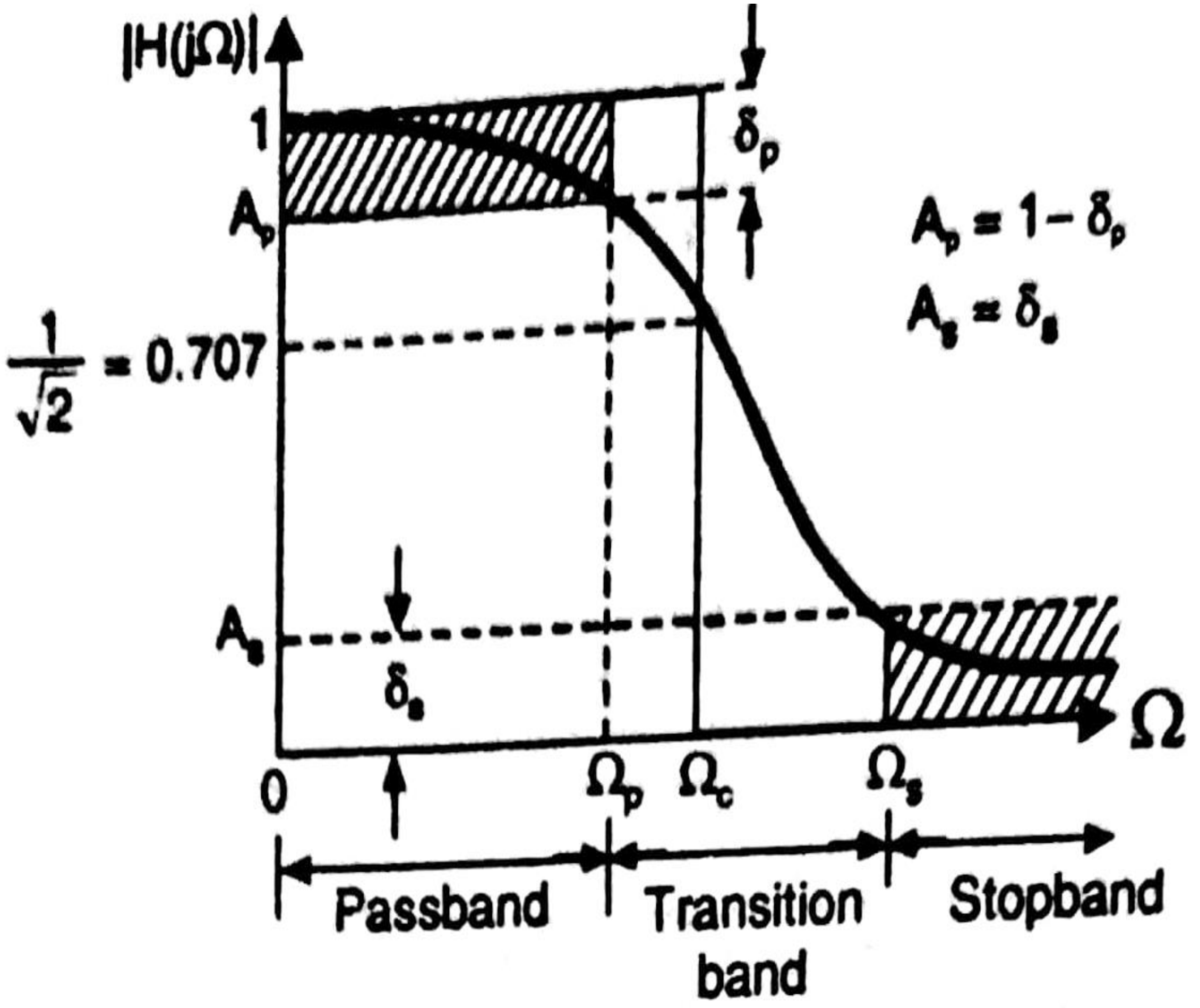
- $\mathbf{\Omega_p}$ - Pass band edge frequency in rad /second
- $\mathbf{\Omega_s}$ - Stop band edge frequency in rad /second
- $\mathbf{A_p}$ - Gain at pass band edge frequency
- $\mathbf{A_s}$ - Gain at Stop band edge frequency
- $\boldsymbol{\omega_p}$ - Pass band edge frequency in rad /sample
- $\boldsymbol{\omega_s}$ - Stop band edge frequency in rad/sample
- $\mathbf{A_p}$ - Gain at pass band edge frequency
- $\mathbf{A_s}$ - Gain at Stop band edge frequency

NORMALIZED MAGNITUDE RESPONSE

IDEAL ANALOG LPF

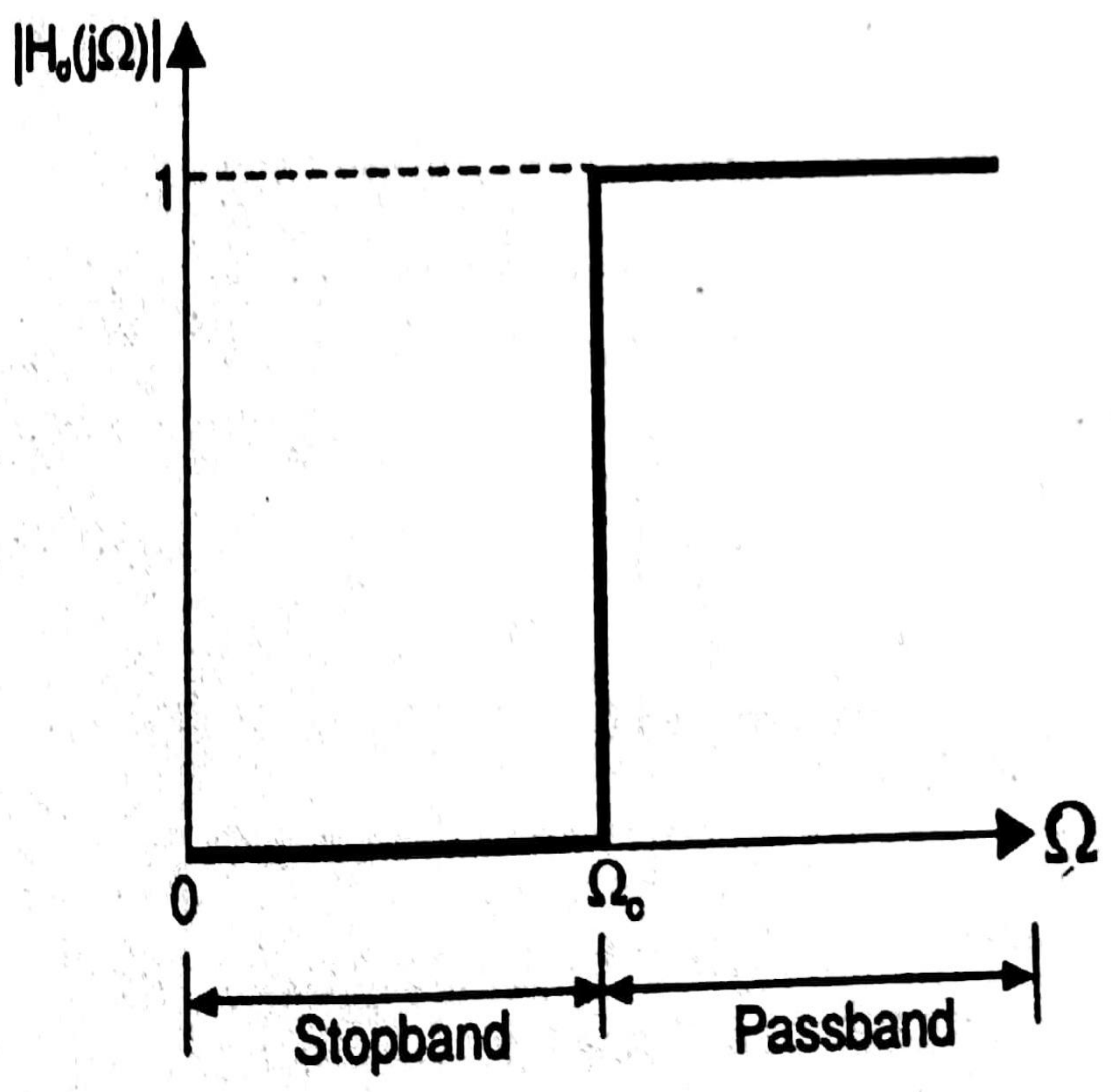


PRACTICAL ANALOG LPF

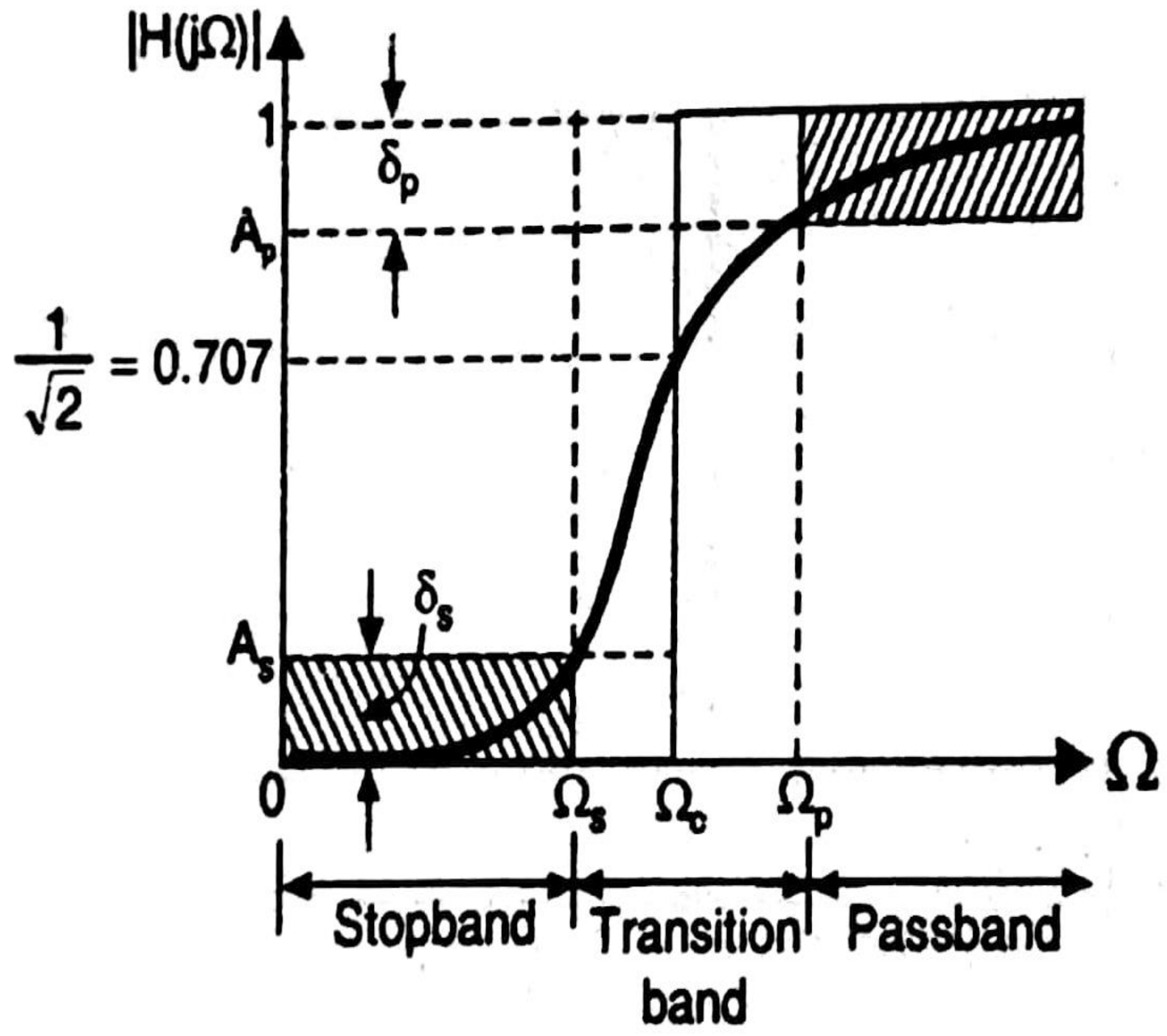


NORMALIZED MAGNITUDE RESPONSE

IDEAL ANALOG HPF

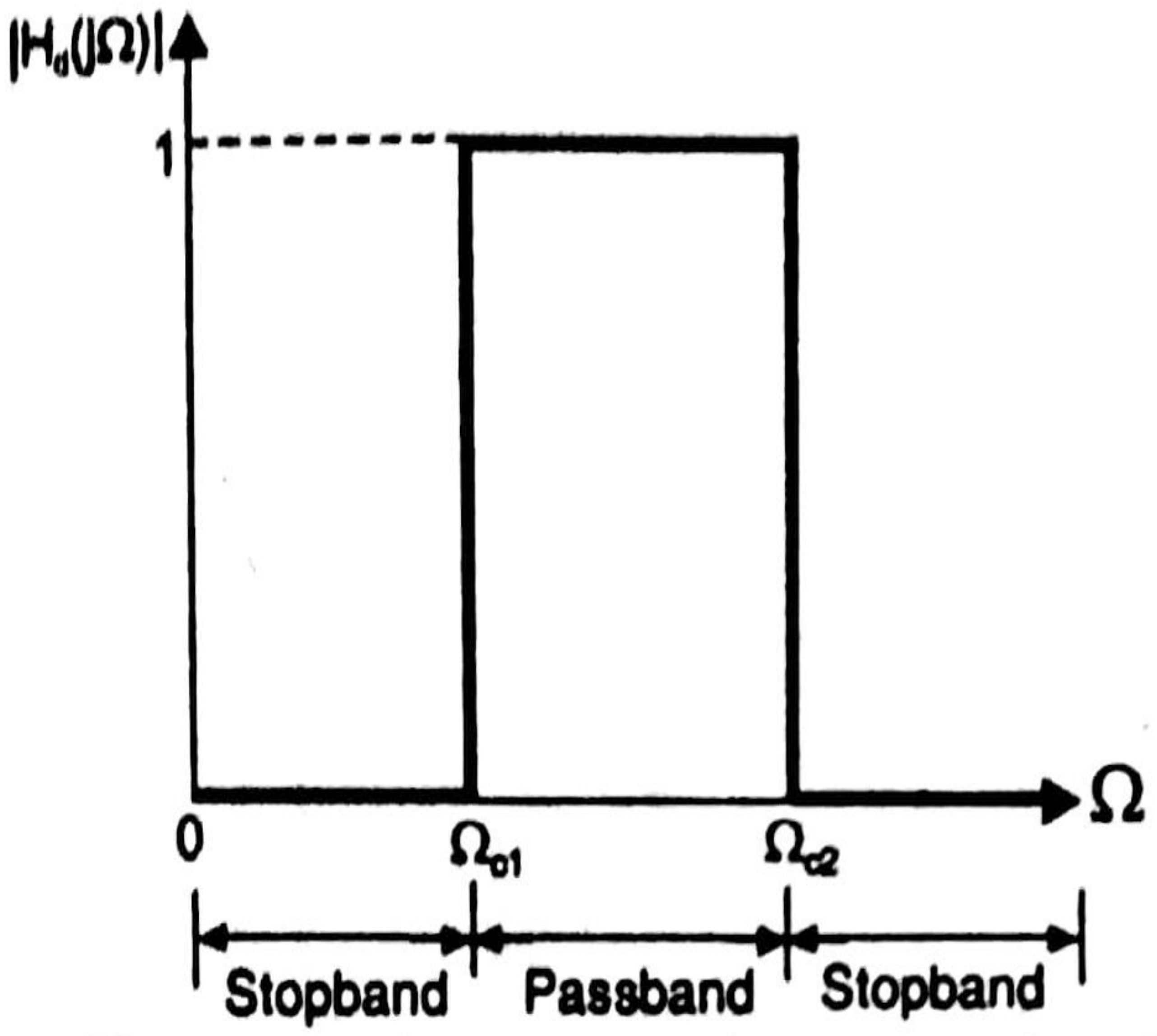


PRACTICAL ANALOG HPF

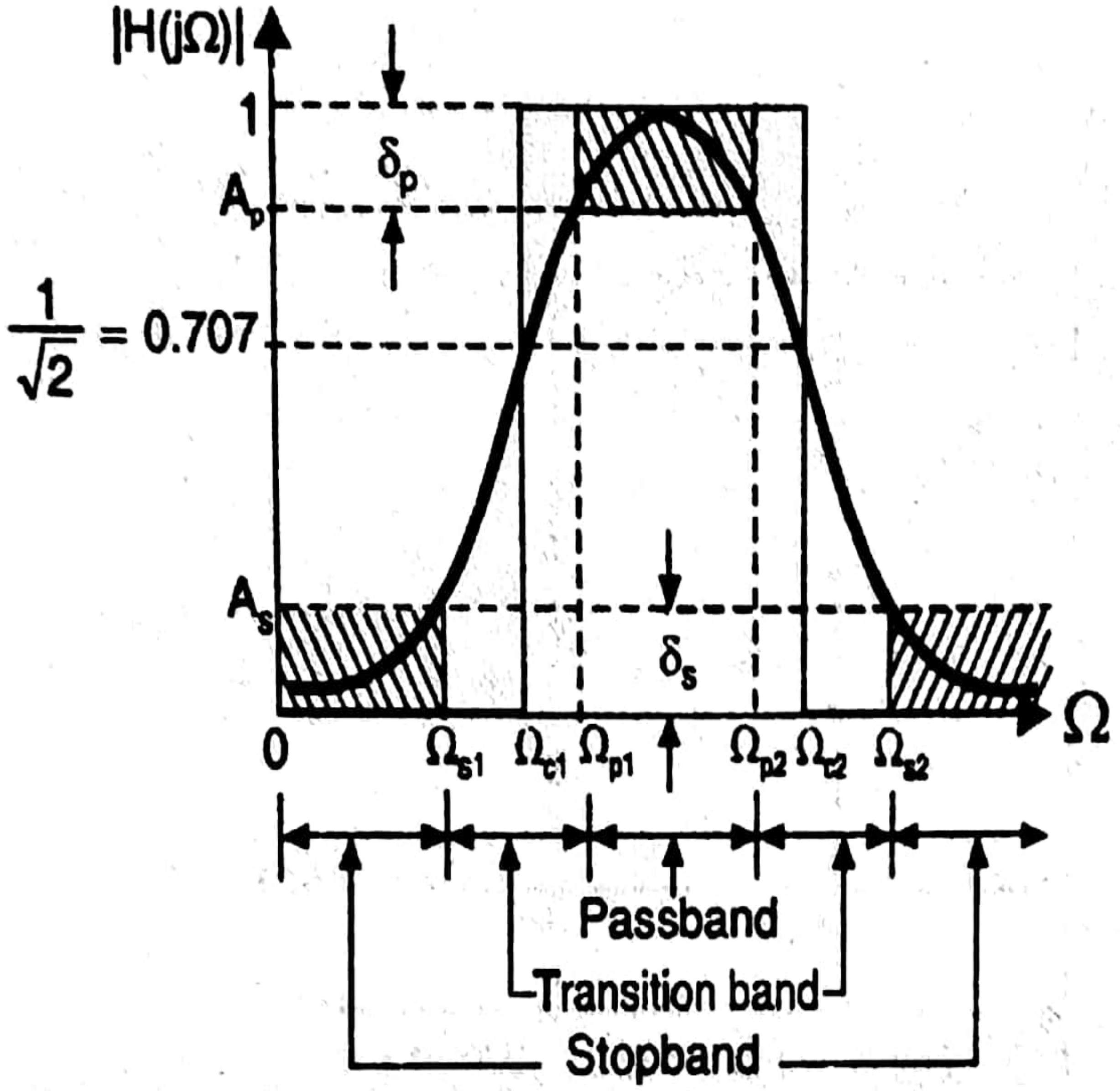


NORMALIZED MAGNITUDE RESPONSE

IDEAL ANALOG BPF

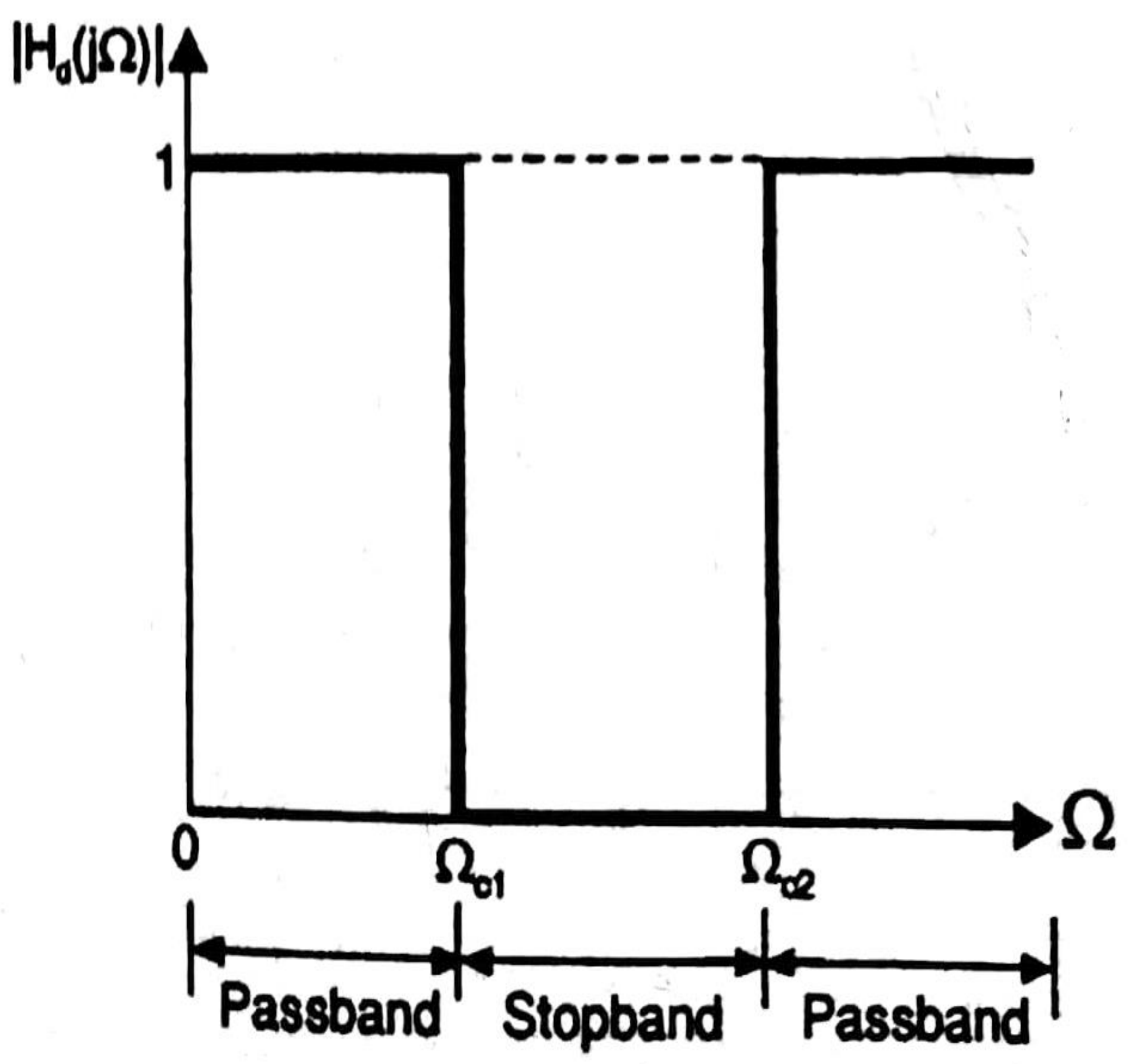


PRACTICAL ANALOG BPF

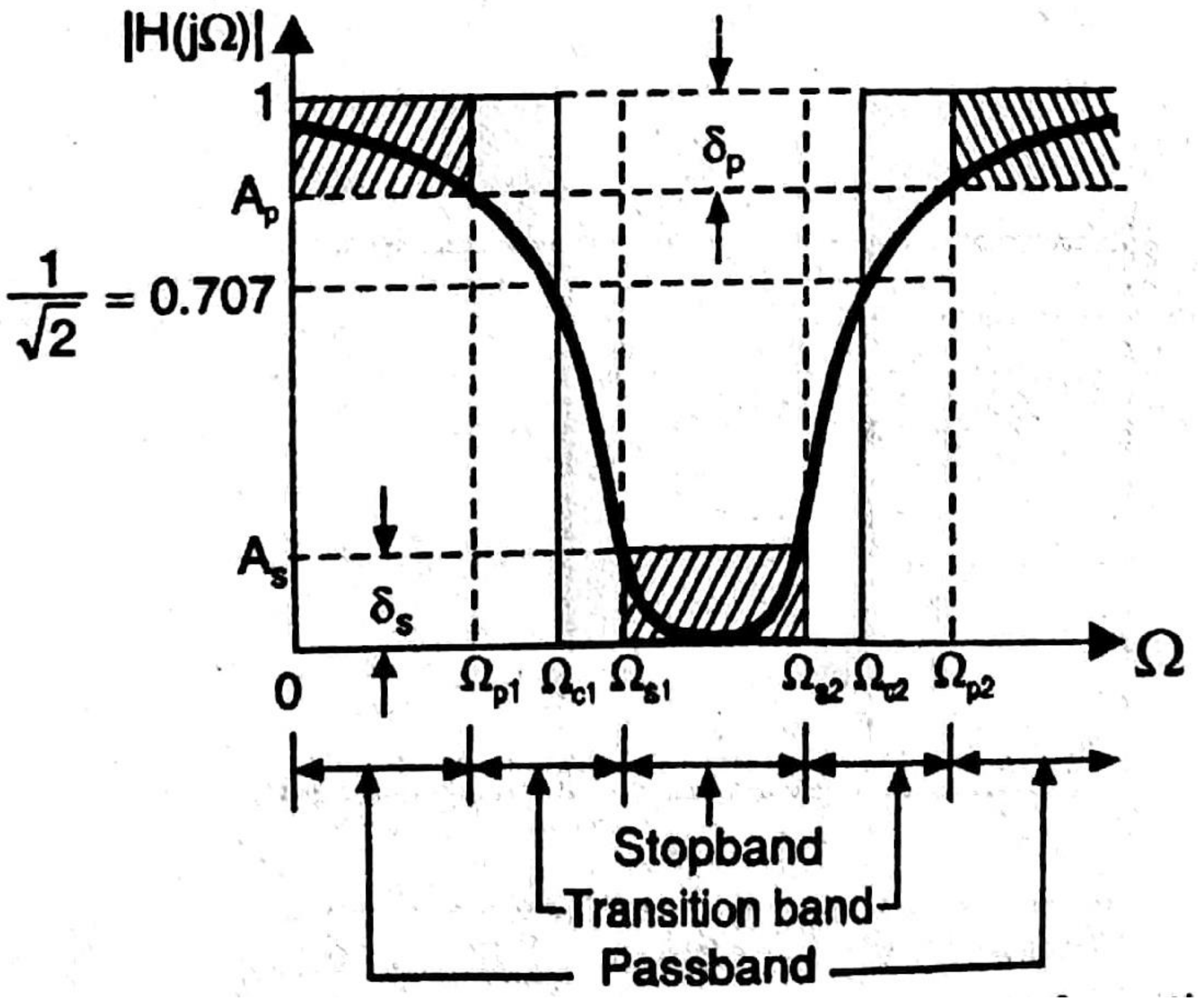


NORMALIZED MAGNITUDE RESPONSE

IDEAL ANALOG BSF

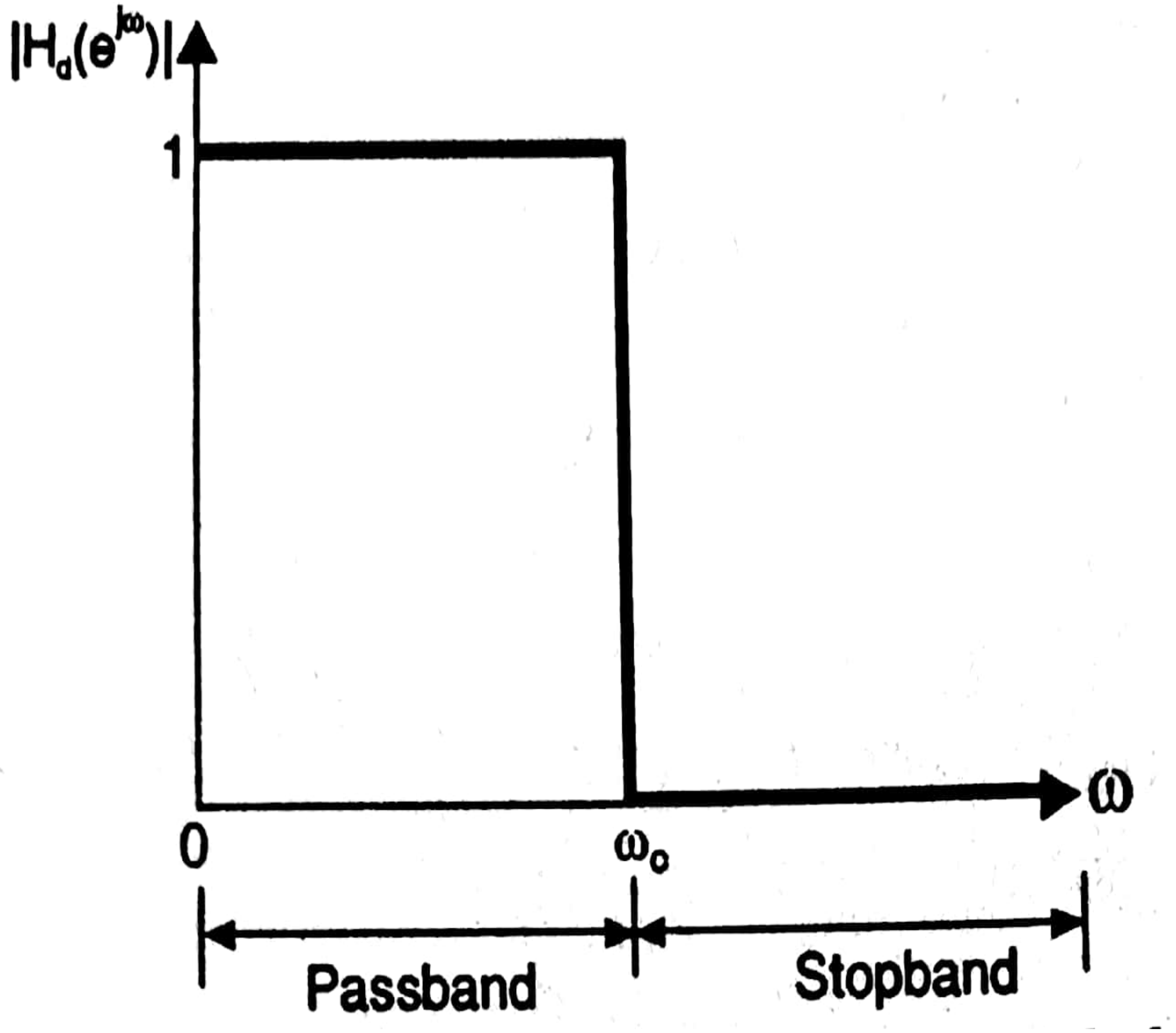


PRACTICAL ANALOG BSF

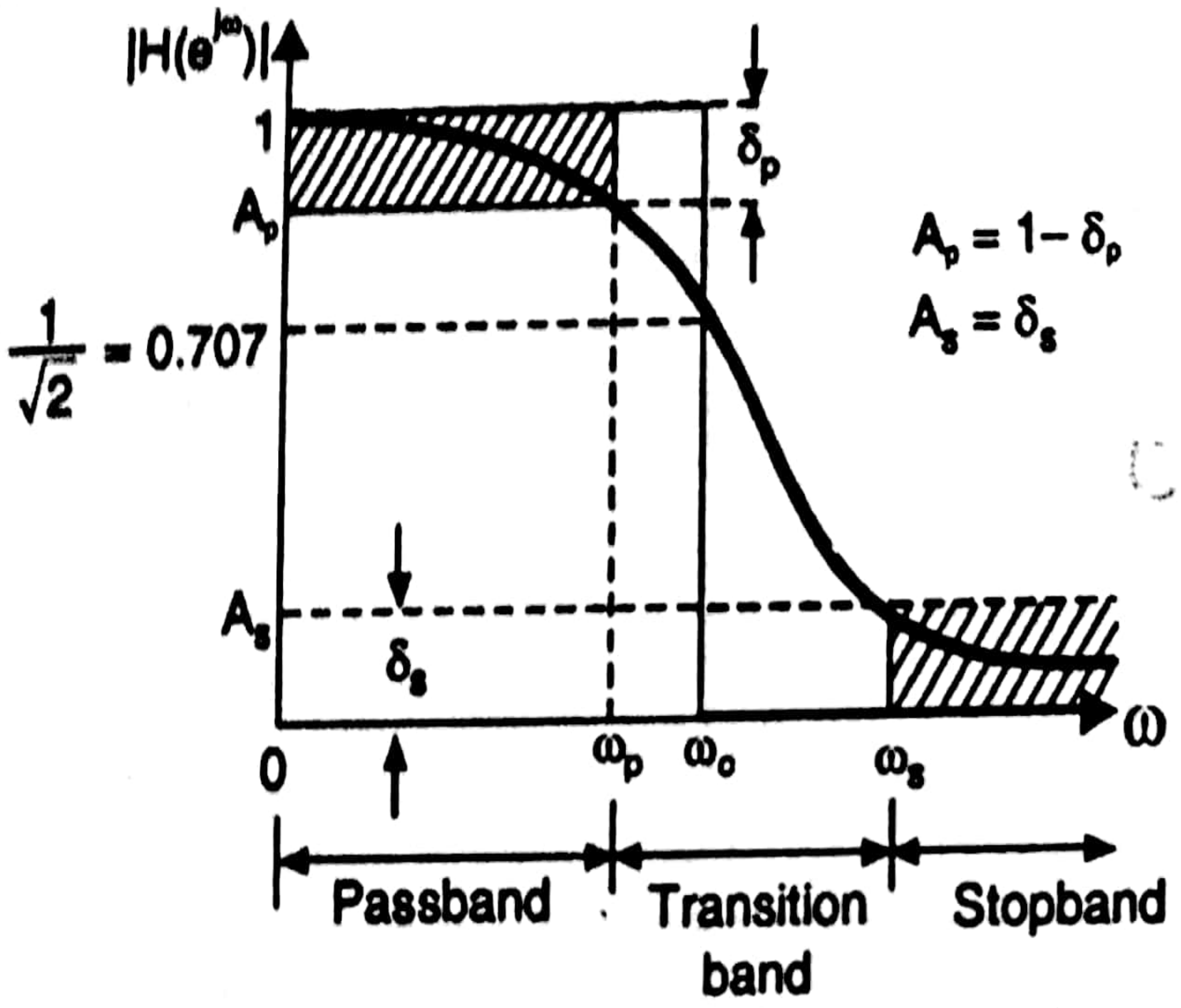


NORMALIZED MAGNITUDE RESPONSE

IDEALDIGITAL IIR LPF

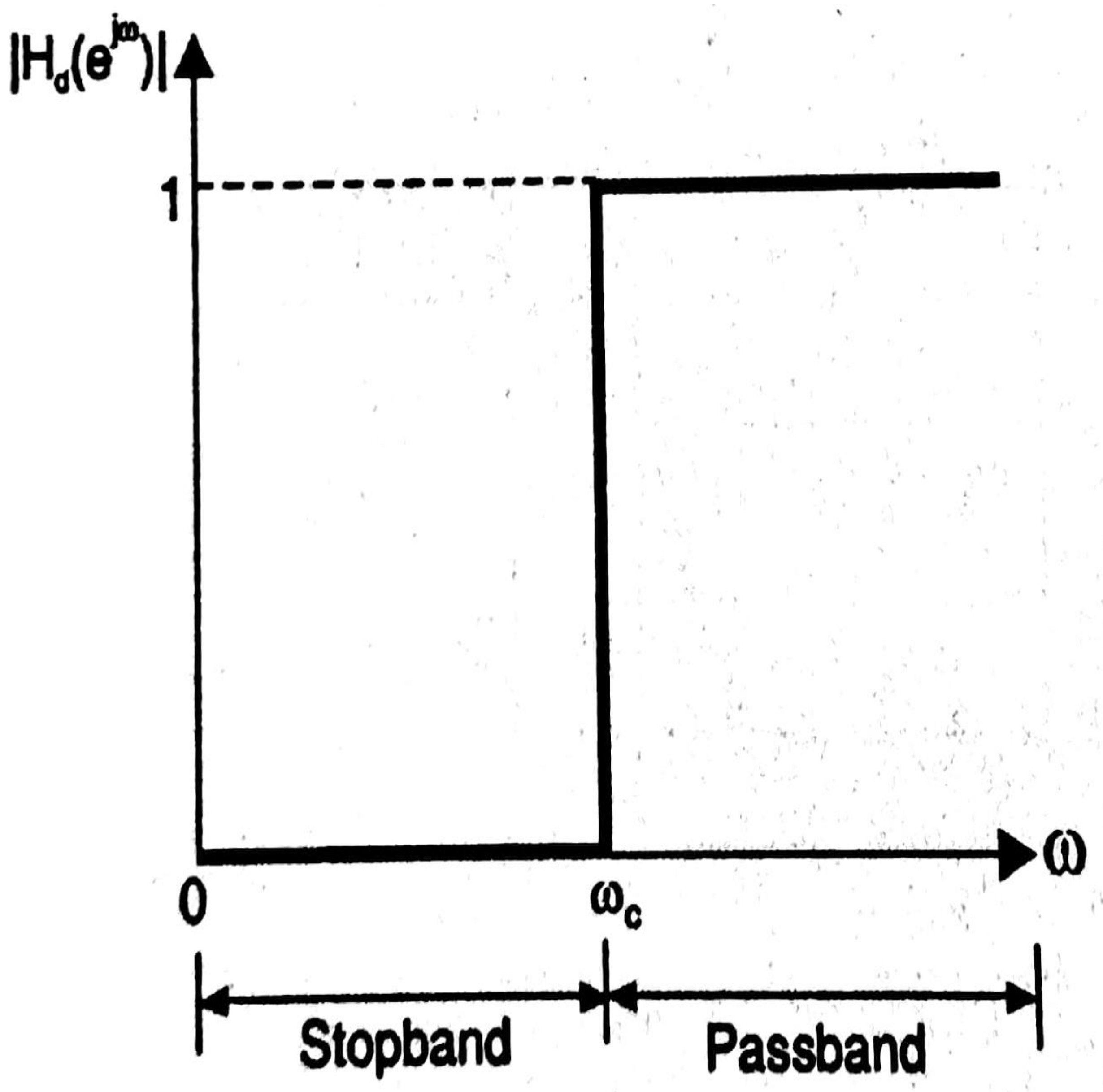


PRACTICAL DIGITAL IIR LPF

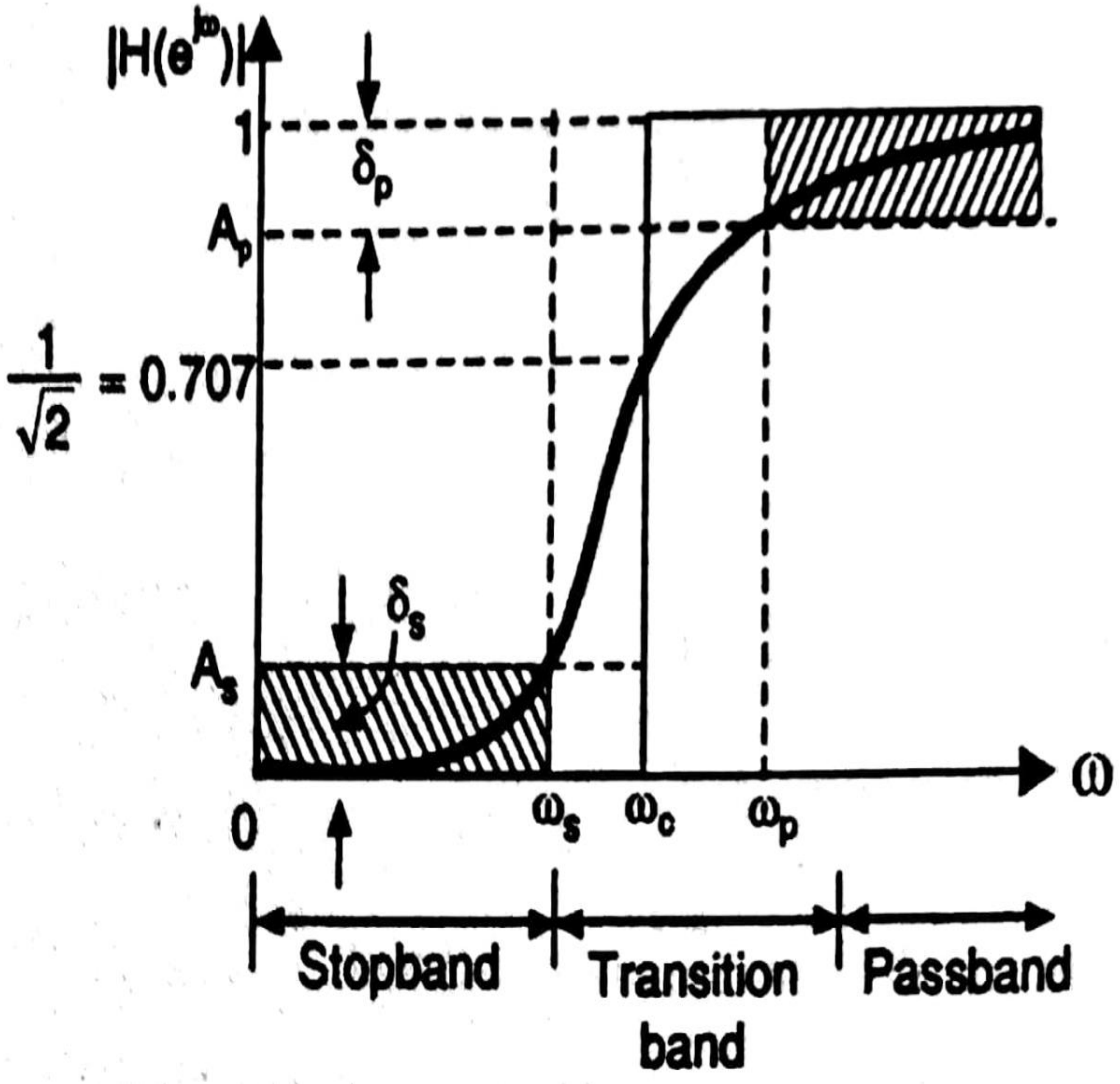


NORMALIZED MAGNITUDE RESPONSE

IDEALDIGITAL IIR HPF

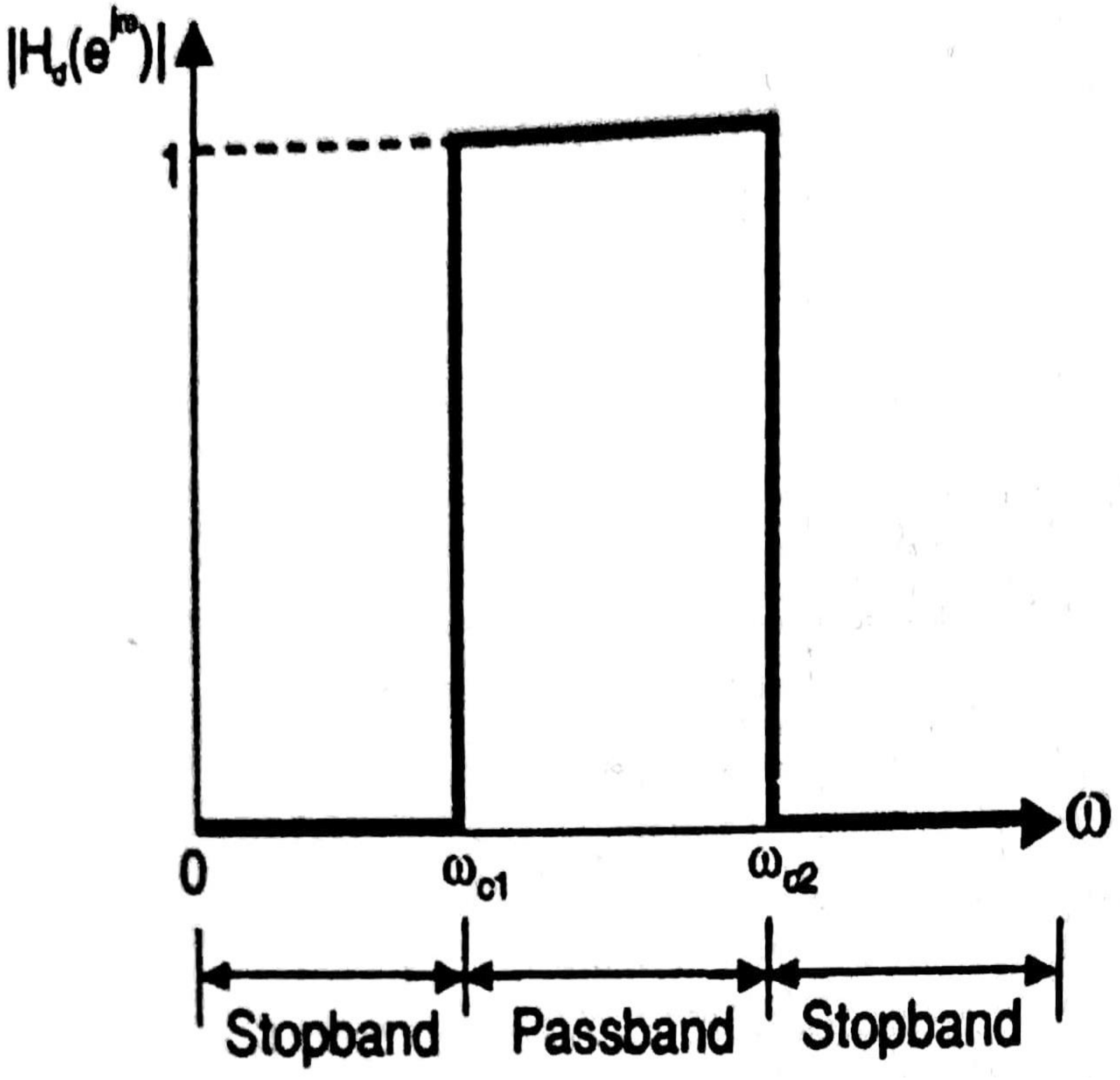


PRACTICAL DIGITAL IIR HPF

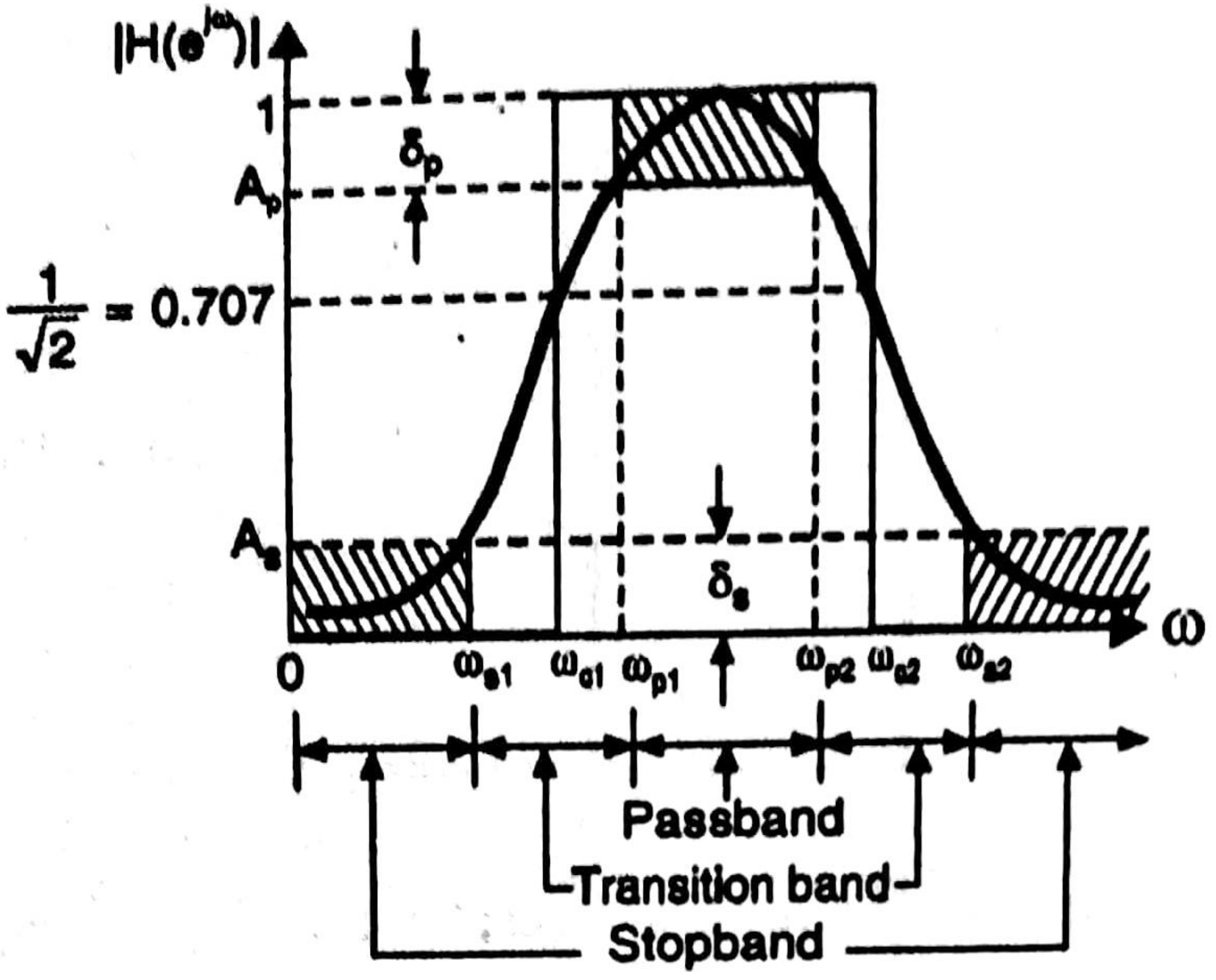


NORMALIZED MAGNITUDE RESPONSE

IDEALDIGITAL IIR BPF

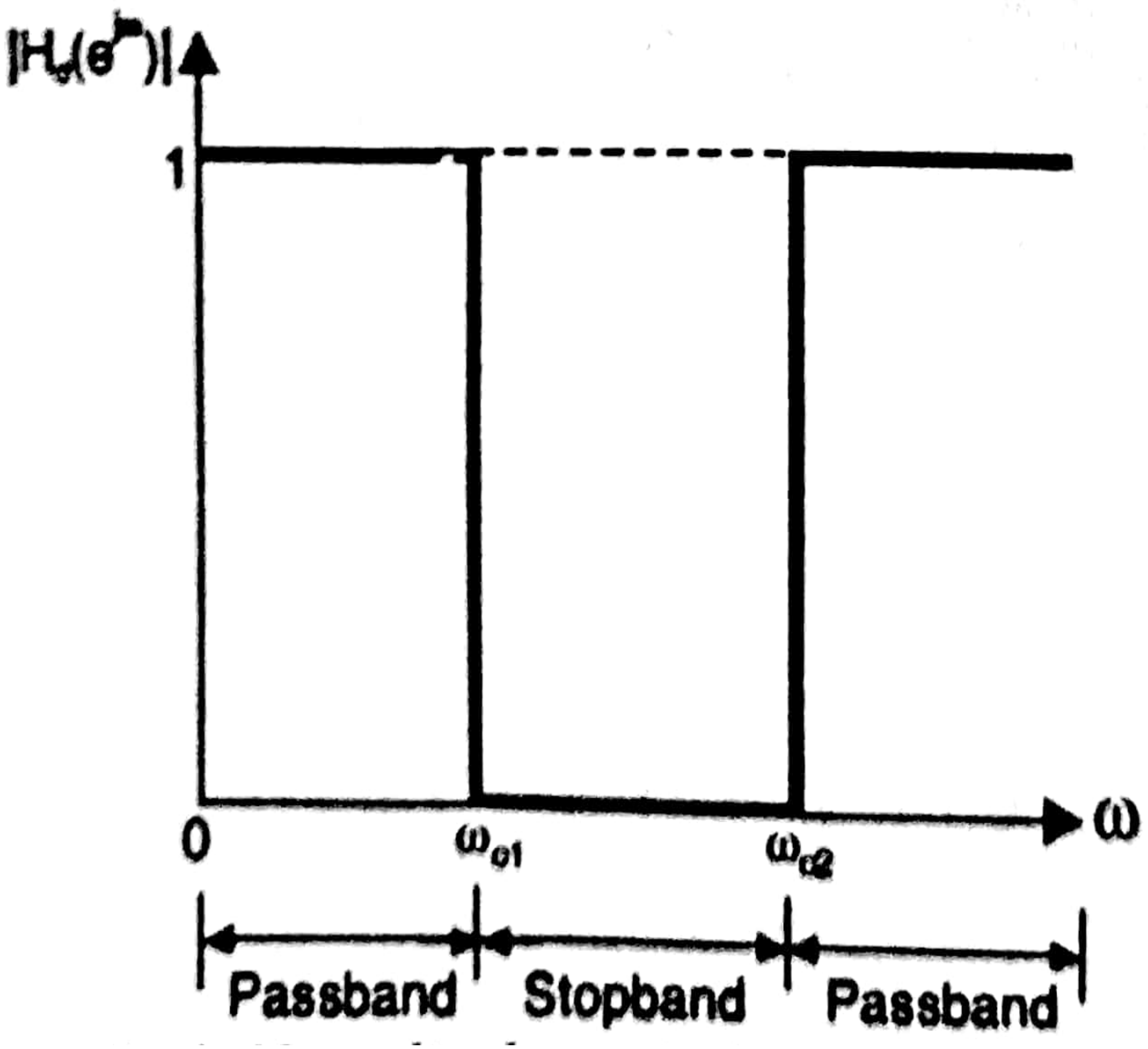


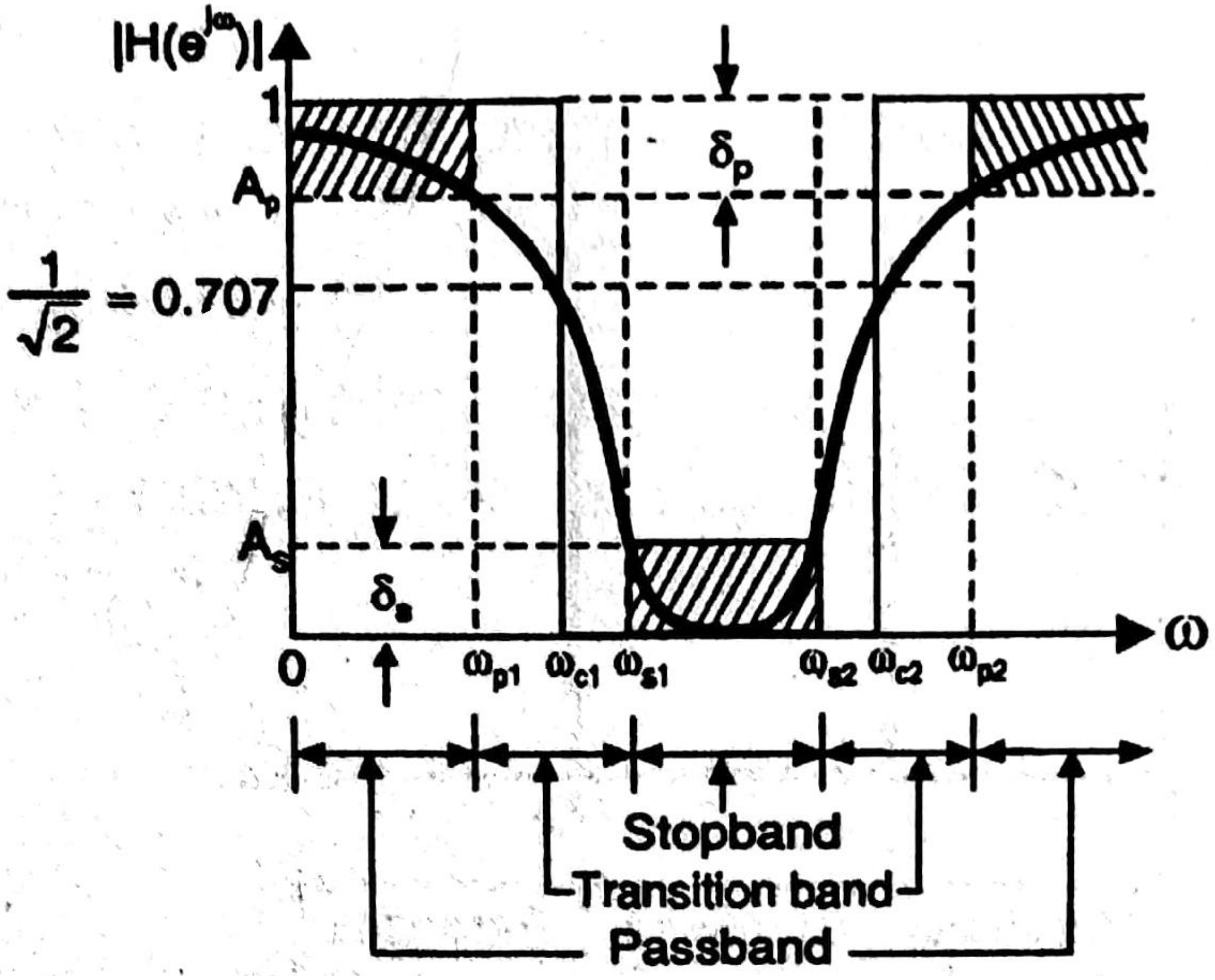
PRACTICAL DIGITAL IIR BPF



NORMALIZED MAGNITUDE RESPONSE

IDEALDIGITAL IIR BSF	PRACTICAL DIGITAL IIR BSF





IMPULSE INVARIANT TRANSFORMATION

- The objective of impulse invariant transformation is to develop an IIR filter transfer function whose impulse response is the sampled version of the impulse response of the analog filter
- The main idea is to preserve the frequency response characteristics of the analog filter.
- It can be stated that the frequency response of digital filter will be identical with the frequency response of the corresponding analog filter if the sampling time period T is selected sufficiently small to minimize the effects of aliasing

- h(t) – Impulse response of analog filter
- The Laplace transform of the analog impulse response h(t) gives the transfer function of analog filter
- Transfer Function of analog filter **H(s) = L {h(t)}**

$$\frac{1}{s+p_k} \rightarrow \frac{1}{1-e^{-p_k T} z^{-1}}$$

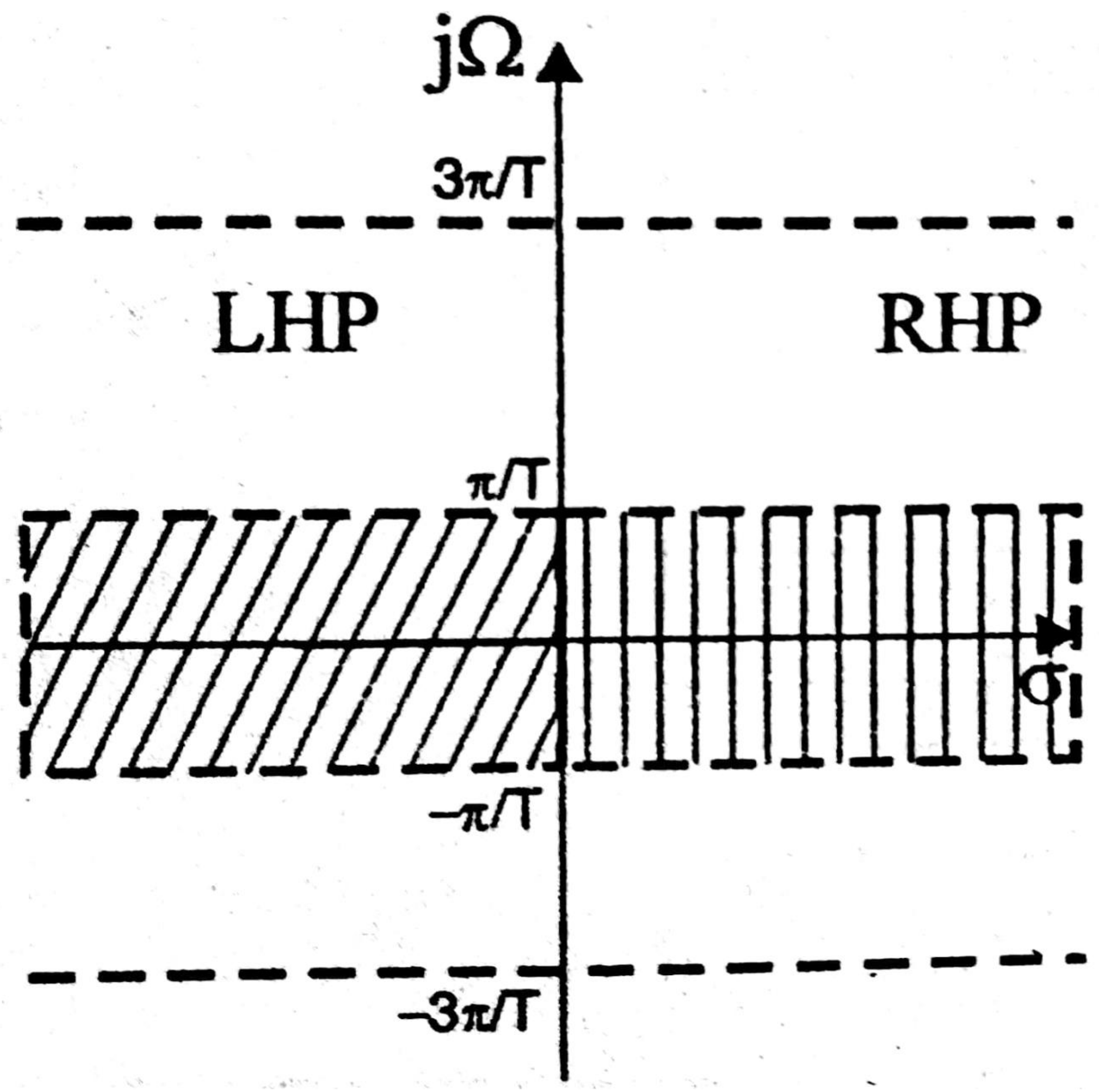
- **T** – Sampling time period

Relation between Analog and Digital Frequency:

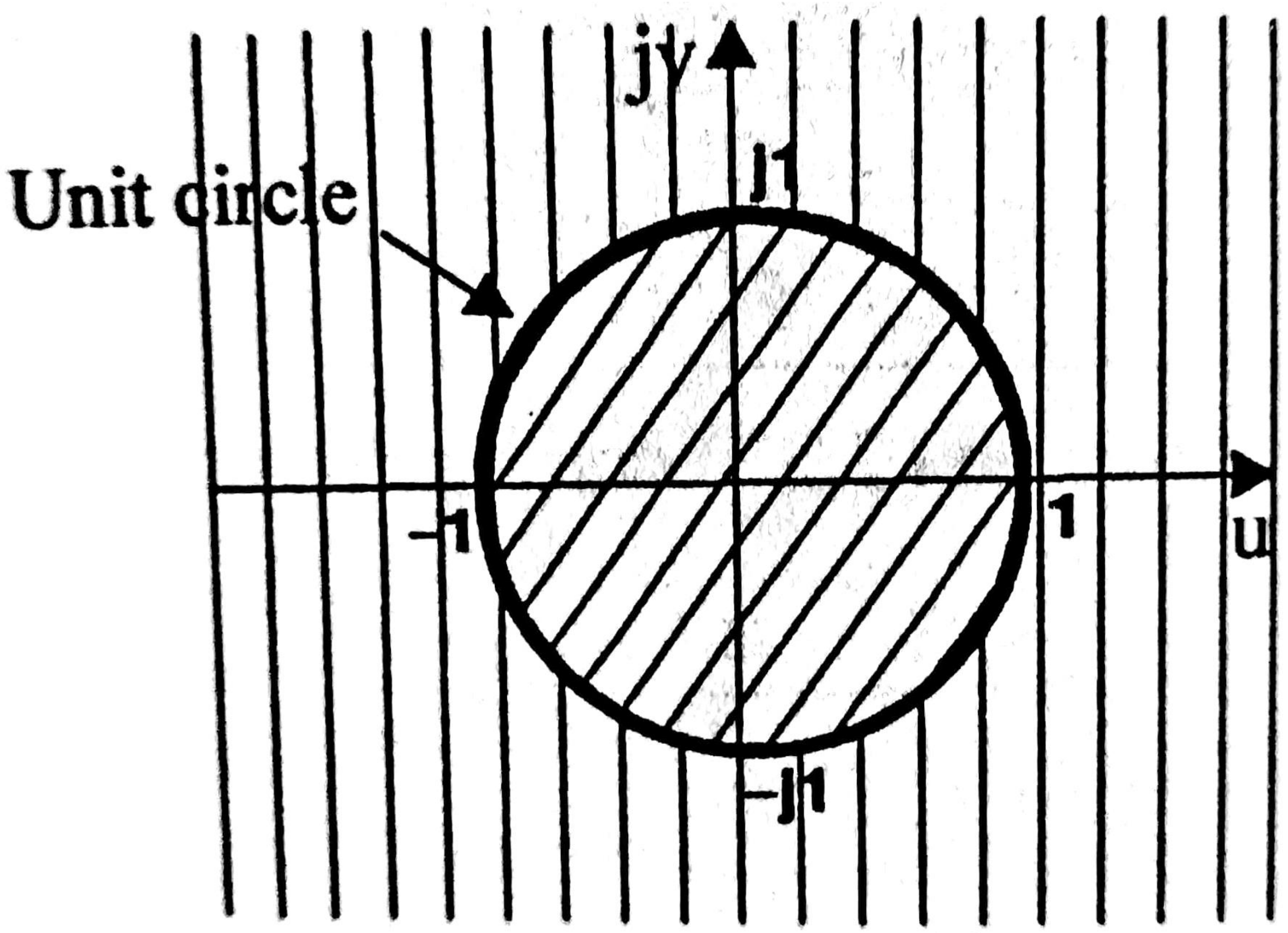
- Digital Frequency **ω = ΩT** (or) Analog Frequency **Ω = ω / T**

IMPULSE INVARIANT TRANSFORMATION

S - PLANE



Z - PLANE



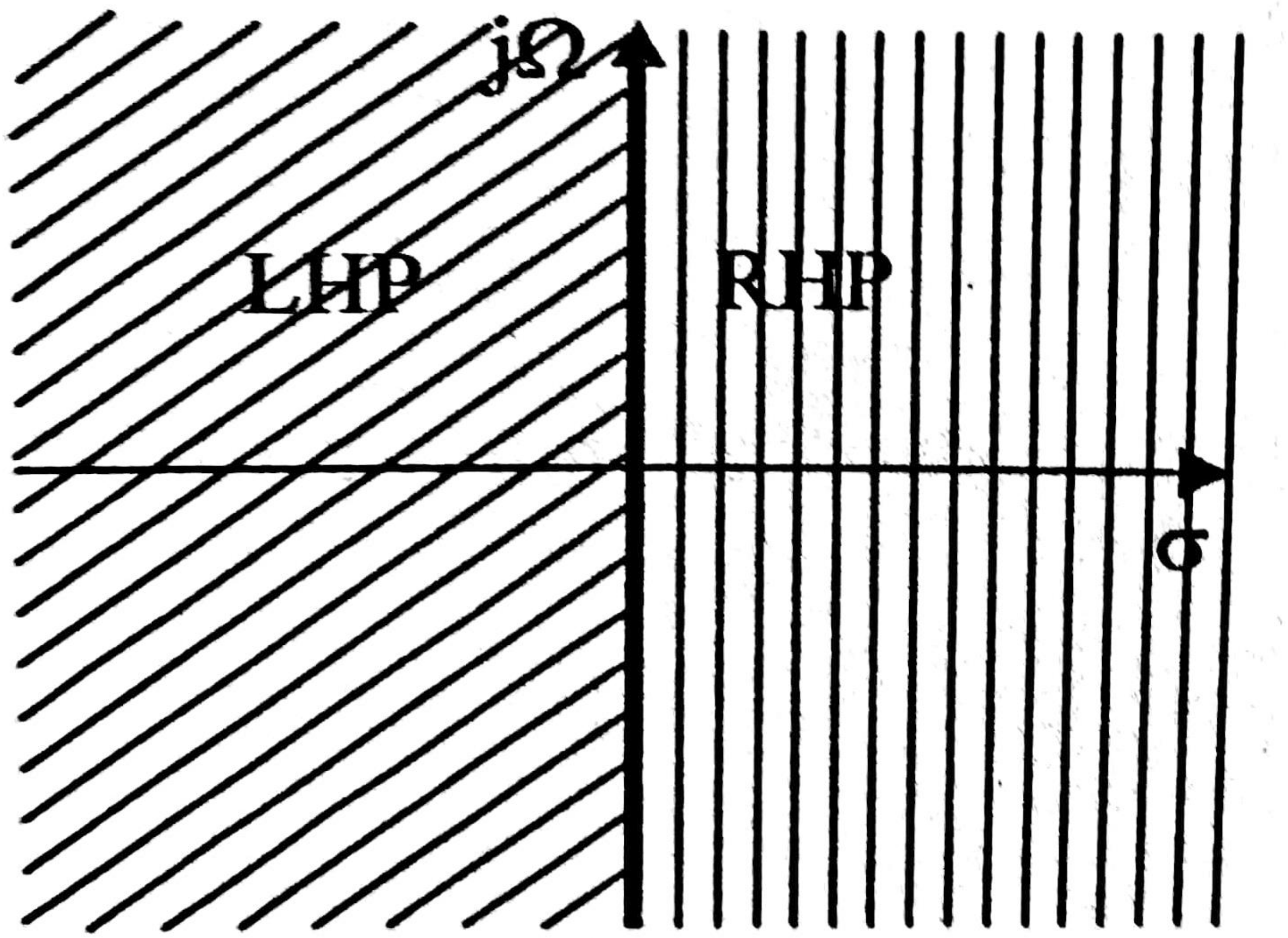
BILINEAR TRANSFORMATION

- The bilinear transformation is a conformal mapping that transforms the imaginary axis of s - plane into the unit circle in the z - plane only once, thus avoiding aliasing of frequency components
- In this mapping all points in the left half of s - plane are mapped inside the unit circle in the z - plane and all points in the right half of s - plane are mapped outside the unit circle in the z - plane. It is a one to one mapping
- T – Sampling time period

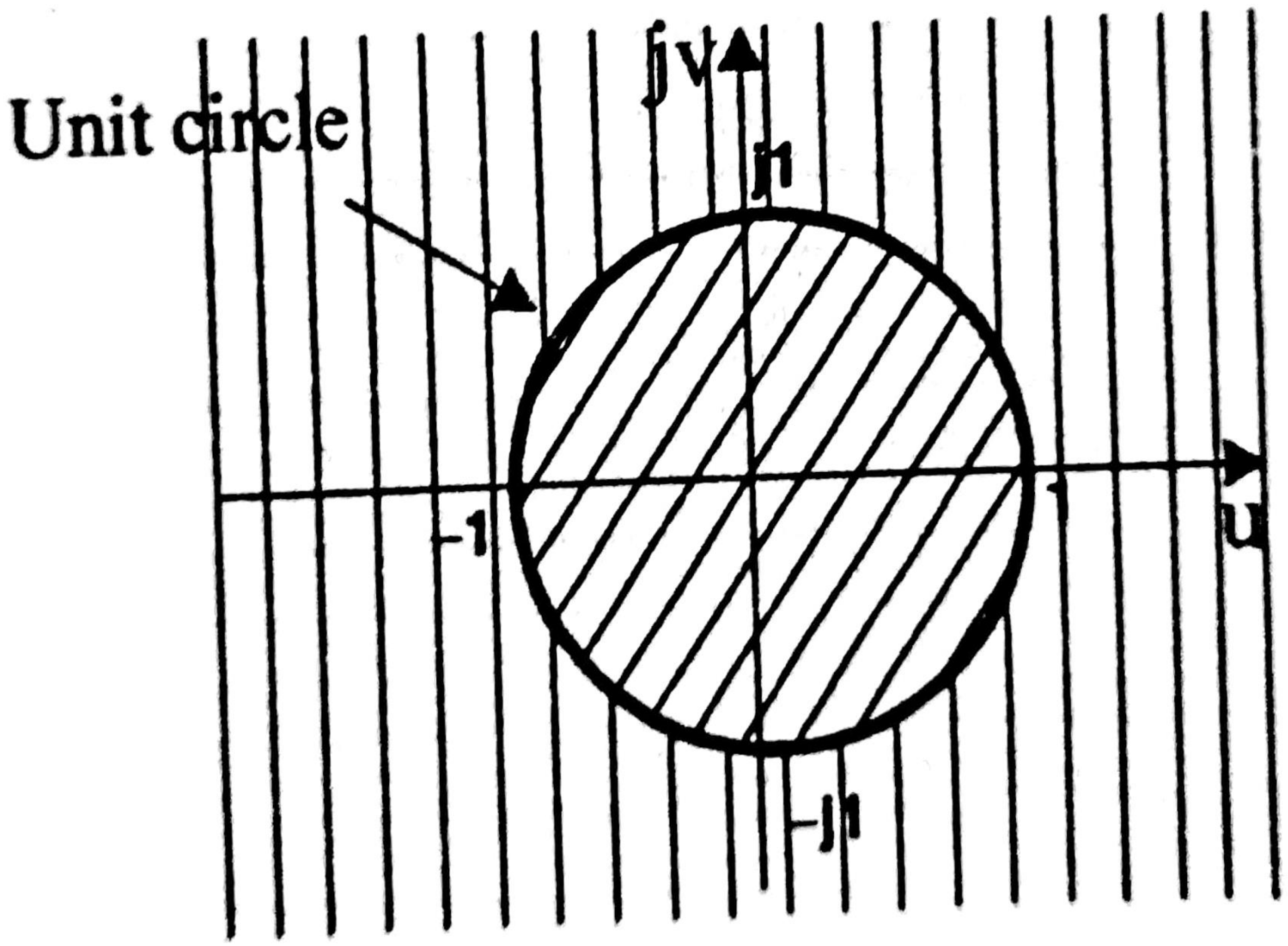
$$s = \frac{2}{T}\left[\frac{1 - z^{-1}}{1 + z^{-1}}\right]$$

BILINEAR TRANSFORMATION

S - PLANE



Z - PLANE

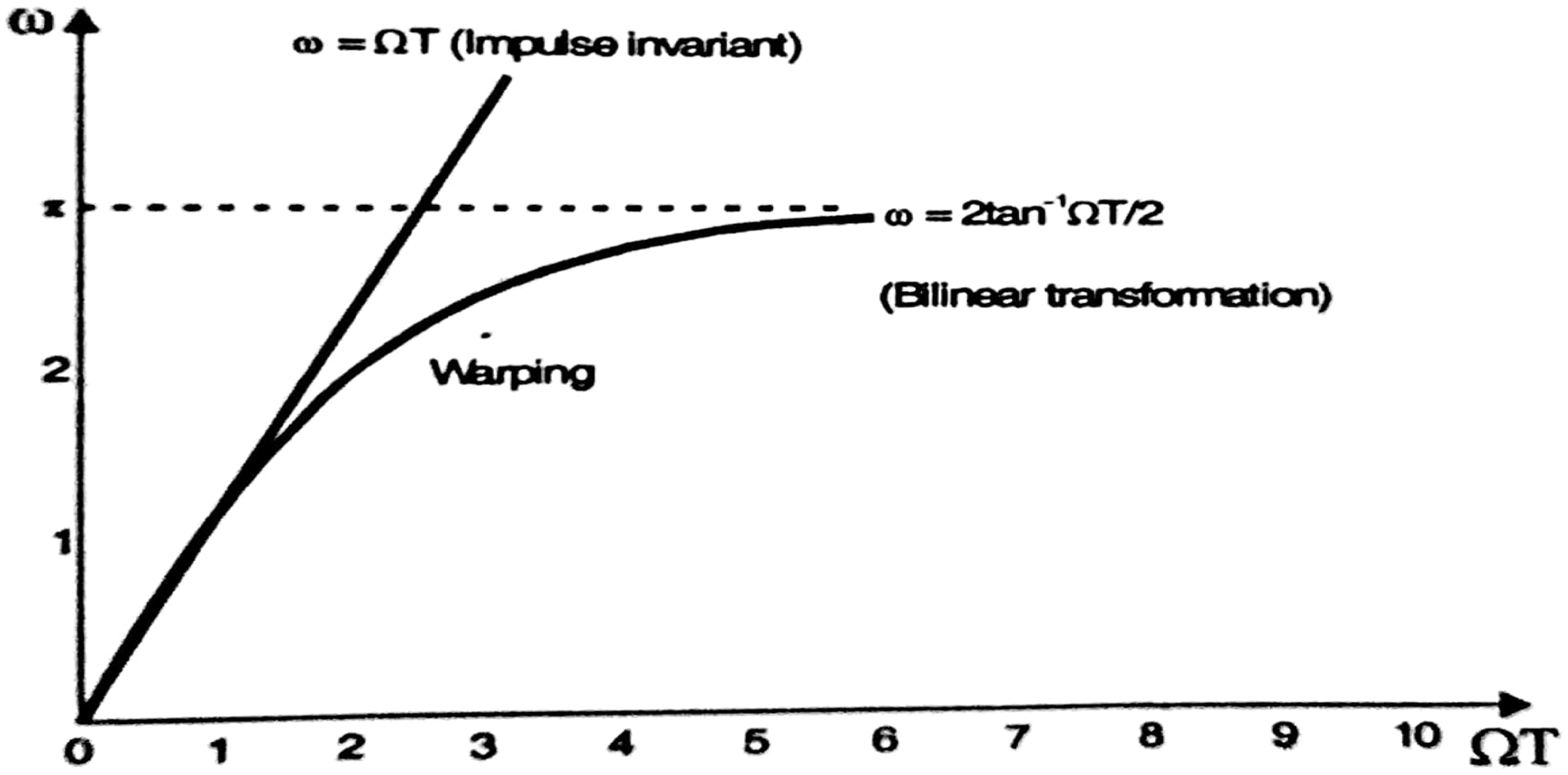


Relation between Analog and Digital Frequency:

- Analog Frequency $\boldsymbol{\Omega = 2 / T \tan \omega / 2}$
- Digital Frequency $\boldsymbol{\omega = 2 \tan^{-1} \Omega T/2}$
- In bilinear transformation the relation between analog and digital frequencies is nonlinear. When the s-plane is mapped into z-plane using bilinear transformation, this nonlinear relationship introduces distortion is frequency axis, which is called frequency warping
- In IIR Filter design using bilinear transformation, the conversion of the specified digital frequencies to analog frequencies is called prewarping

BILINEAR TRANSFORMATION

- The prewarping is to eliminate the effect of warping of amplitude response



ASSESSMENT

1. Define IIR Systems.
2. Mention the advantages and disadvantages of IIR Filters.
3. The δ_p and δ_s are also called -------------
4. Compare analog filter with digital filter.
5. Based on frequency response the filters are classified into four basic types. They are ------------, --------------, ------------------ and ----------------
6. Define bilinear transformation.
7. What is meant by prewarping?

THANK YOU