SNS COLLEGE OF TECHNOLOGY

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DEPARTMENT OF FOOD TECHNOLOGY

19FTT202 – HEAT AND MASS TRANSFER

II YEAR IV SEM

UNIT 1 – CONDUCTION

Topic 9 Conduction with Internal Heat Generation

Conduction with Internal Heat Generation

- A medium through which heat is conducted may involve the conversion of mechanical, electrical, nuclear, or chemical energy into heat (or thermal energy).
- In heat conduction analysis, such conversion processes are characterized as heat generation.
- For example, the temperature of a resistance wire rises rapidly when electric current passes through it as a result of the electrical energy being converted to heat at high rate

Following are some of the cases where heat generation and heat conduction are encountered :

(*i*) Fuel rods – nuclear reactor;

(*ii*) Electrical conductors;

(*iii*) Chemical and combustion processes;

(*iv*) Drying and setting of concrete.

It is of paramount importance that the heat generation rate be controlled otherwise the equipment may fail (*e.g.*, some nuclear accidents, electrical fuses blowing out). Thus, in the design of the thermal systems temperature distribution within the medium and the rate of heat dissipation to the surroundings assumes ample importance / significance.

Conduction with Internal Heat Generation

- A large amount of heat is generated in the fuel elements of nuclear reactors as a result of nuclear fission that serves as the *heat source* for the nuclear power plants

- The natural disintegration of radioactive elements in nuclear waste or other radioactive material also results in the generation of heat throughout the body

- The heat generated in the sun as a result of the fusion of hydrogen into helium makes the sun a large nuclear reactor that supplies heat to the earth

Conduction with Internal Heat Generation

Refer to Fig. 2.91. Consider a plane wall of thickness L (small in comparison with other dimension) of uniform thermal conductivity k and in which heat sources are uniformly distributed in the whole volume. Let the wall surfaces are maintained at temperatures t_1 and t_2.

Let us assume that heat flow is one-dimensional, under steady state conditions, and there is a *uniform volumetric heat generation* within the wall.

Consider an element of thickness at a distance x from the left hand face of the wall.

Heat conducted in at distance x,

$$Q_x = -kA\frac{dt}{dx}$$

Heat generated in the element,

$$Q_g = A \, . \, dx \, . \, q_g$$

(where q_g = heat generated per unit volume per unit time in the element)

Heat conducted out at distance $(x + dx)$, $Q_{(x+dx)} = Q_x + \frac{d}{dx}(Q_x)\,dx$

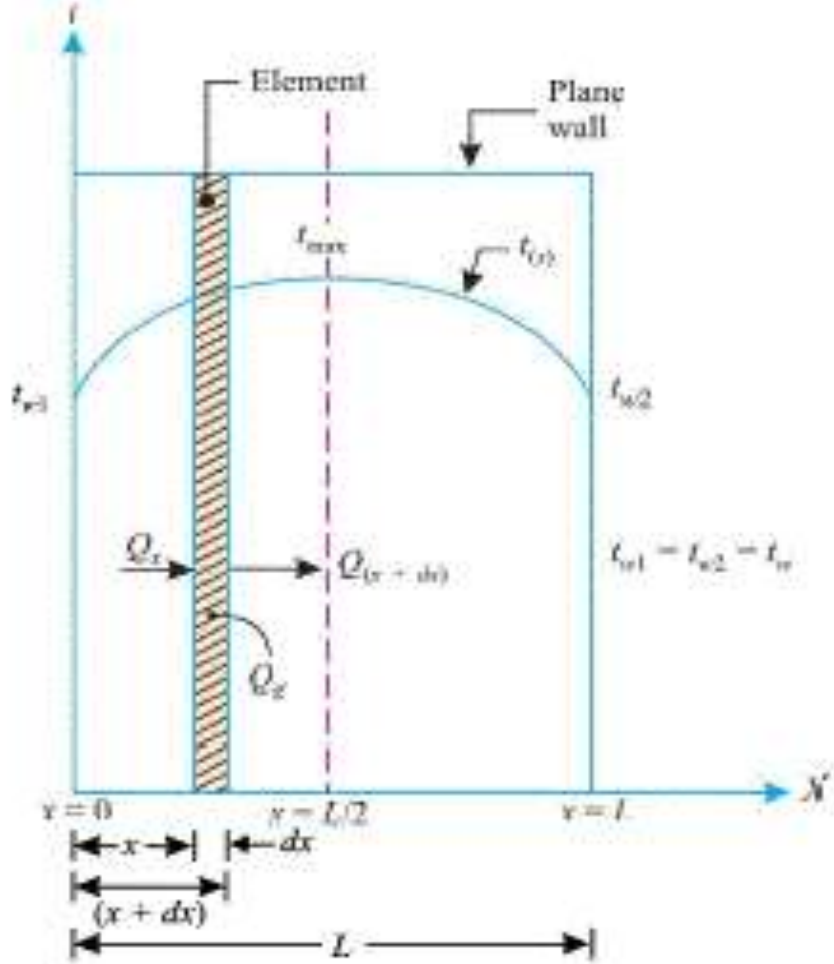

Fig. 2.91. Plane wall uniform heat generation. Both the surfaces maintained at a common temperature.

As Q_g represents an energy increase in the volume element, an energy balance on the element of thick dx is given by

$$Q_x + Q_g = Q_{(x+dx)}$$

$$= Q_x + \frac{d}{dx}(Q_x)\,dx$$

or, $$Q_g = \frac{d}{dx}(Q_x)\,dx$$

or, $$q_g \cdot A \cdot dx = \frac{d}{dx}\left[-k\,A\,\frac{dt}{dx}\right]dx$$

$$= -k\,A \cdot \frac{d^2t}{dx^2} \cdot dx$$

or, $$\frac{d^2t}{dx^2} + \frac{q_g}{k} = 0 \quad \text{...(2.88)}$$

Eqn. (2.88) may also be obtained from eqn. (2.8) by assuming one-dimensional steady state conditions.

The first and second integration of Eqn. (2.88) gives respectively

$$\frac{dt}{dx} = -\frac{q_g}{k}x + C_1 \quad \text{...(2.89)}$$

$$t = -\frac{q_g}{2k} \cdot x^2 + C_1\,x + C_2 \quad \text{... (2.90)}$$

Case I. Both the surfaces have the same temperature :

Refer to Fig. 2.92.

At $x = 0$ $\quad t = t_1 = t_w$, and

At $x = L$ $\quad t = t_2 = t_w$

(where t_w = temperature of the wall surface).

Using these boundary conditions in eqn. (2.90), we get

$$C_2 = t_w \text{ and } C_1 = \frac{q_g}{2k} \cdot L$$

Substituting these values of C_1 and C_2 in eqn. (2.90), we have

$$t = -\frac{q_g}{2k}x^2 + \frac{q_g}{2k} \cdot L \cdot x + t_w$$

or, $$t = \frac{q_g}{2k}(L - x)\,x + t_w \quad \text{...(2.91)}$$

In order to determine the location of the maximum temperature, differentiating the eqn. (2.91) w.r.t x and equating the derivative to *zero*, we have

$$\frac{dt}{dx} = \frac{q_g}{2k}(L - 2x) = 0$$

Since, $\frac{q_g}{2k} \neq 0$, therefore,

$$L - 2x = 0 \qquad \text{or} \qquad x = \frac{L}{2}$$

Thus the *distribution of temperature* given by eqn. (2.91) is the *parabolic* and *symmetrical* about the midplane. The maximum temperature occurs at $x = \frac{L}{2}$ and its value equals

$$t_{max} = \left[\frac{q_g}{2k}(L - x)\,x\right]_{x=\frac{L}{2}} + t_w$$

or, $$= \left[\frac{q_g}{2k}\left(L - \frac{L}{2}\right)\frac{L}{2}\right] + t_w$$

i.e. $$t_{max} = \frac{q_g}{8k} \cdot L^2 + t_w \quad ...(2.92)$$

Heat transfer then takes place towards both the surfaces, and for each surface it is given by

$$Q = -kA\left(\frac{dt}{dx}\right)_{x=0 \text{ or } x=L}$$

$$= -kA\left[\frac{q_g}{2k}(L - 2x)\right]_{x=0 \text{ or } x=L}$$

i.e., $$Q = \frac{AL}{2} \cdot q_g \quad ...(2.93)$$

When both the surfaces are considered,

$$Q = 2 \times \frac{AL}{2} q_g = A.L.q_g \quad ...[2.93\ (a)]$$

Also heat conducted to *each wall surface* is further dissipated to the surrounding atmosphere at temperature t_a.

Thus, $$\frac{AL}{2} \cdot q_g = hA(t_w - t_a)$$

or, $$t_w = t_a + \frac{q_g}{2h} \cdot L \quad ...(2.94)$$

Substituting this value of t_w in eqn. (2.91), we obtain

$$t = t_a + \frac{q_g}{2h} \cdot L + \frac{q_g}{2k}(L - x)x \quad ...(2.95)$$

At $x = L/2$ i.e., at the midplane :

$$t = t_{max} = t_a + \frac{q_g}{2h} \cdot L + \frac{q_g}{8k}L^2$$

or, $$t_{max} = t_a + q_g\left[\frac{L}{2h} + \frac{L^2}{8k}\right] \quad ...[2.95\ (a)]$$

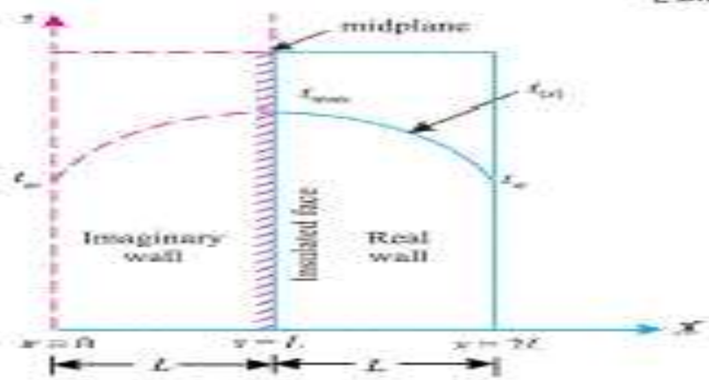

Fig. 2.92. Heat conduction in an insulated wall.

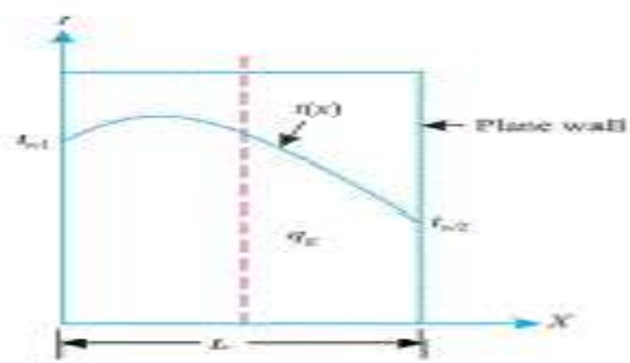

Fig. 2.93. Plane wall with uniform heat generation—Both the surfaces of the wall having different temperatures.

Conduction with Internal Heat Generation

The eqn. (2.95) also works well in case of conduction in an *insulated wall* Fig. (2.92).

The following boundary conditions apply in the full *hypothetical wall* of thickness $2L$:

$$\text{At } x = L \qquad \frac{dt}{dx} = 0$$

$$\text{At } x = 2L \qquad t = t_w$$

The location $x = L$ refers to the mid-plane of the hypothetical wall (or insulated face of given wall).

Eqns. (2.91) and (2.92) for temperature distribution and maximum temperature at the mid- plane (insulated end of the given wall) respectively can be written as

$$t = \frac{q_g}{2k}(2L - x)x + t_w \qquad ...(2.96)$$

$$t_{max} = \frac{q_g}{2k}L^2 + t_w \qquad ...(2.97)$$

[Substituting L = 2L in eqn. (2.91) and (2.92)]

Case II. Both the surfaces of the wall have different temperatures :

Refer to Fig. 2.93

The boundary conditions are :

$$\text{At } x = 0 \qquad t = t_{w1}$$

$$\text{At } x = L \qquad t = t_{w2}$$

Substituting these values in eqn. (2.90), we obtain the values of constant C_1 and C_2 as :

$$C_2 = t_{w1}; \qquad C_1 = \frac{t_{w2} - t_{w1}}{L} + \frac{q_g}{2k} \cdot L$$

Inserting these values in eqn. (2.90), we get

$$t = -\frac{q_g}{2k}x^2 + \frac{t_{w2} - t_{w1}}{L}x + \frac{q_g}{2k}L \cdot x + t_{w1}$$

$$= \frac{q_g}{2k}L \cdot x - \frac{q_g}{2k}x^2 + \frac{x}{L}(t_{w2} - t_{w1}) + t_1$$

or, $$t = \left[\frac{q_g}{2k}(L - x) + \frac{t_{w2} - t_{w1}}{L}\right]x + t_{w1} \qquad ...(2.98)$$

The temperature distribution, in dimensionless form can be obtained by making the following transformations :

$$t - t_{w2} = \frac{q_g}{2k}L^2\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] + \frac{x}{L}(t_{w2} - t_{w1}) + (t_{w1} - t_{w2})$$

or, $$\frac{t - t_{w2}}{t_{w1} - t_{w2}} = \frac{q_g}{2k} \cdot \frac{L^2}{(t_{w1} - t_{w2})}\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] - \frac{x}{L} + 1$$

or, $$\frac{t - t_{w2}}{t_{w1} - t_{w2}} = \frac{q_g}{2k} \cdot \frac{L^2}{(t_{w1} - t_{w2})} \cdot \frac{x}{L}\left[1 - \frac{x}{L}\right] + \left[1 - \frac{x}{L}\right]$$

Replacing the parameter $\frac{q_g}{2k} \cdot \frac{L^2}{(t_{w1} - t_{w2})}$ (a constant) by a factor Z, we have

Vertical tank.

$$\frac{t - t_{w2}}{t_{w1} - t_{w2}} = Z \cdot \frac{x}{L}\left[1 - \frac{x}{L}\right] + \left[1 - \frac{x}{L}\right]$$

or, $$\frac{t - t_{w2}}{t_{w1} - t_{w2}} = \left[1 - \frac{x}{L}\right]\left[\frac{Z\,x}{L} + 1\right] \quad ...(2.99)$$

In order to get maximum temperature and its location, differentiating Eqn. (2.99) w.r.t x and equating the derivative to zero, we have

$$\frac{dt}{d(x/L)} = \left(1 - \frac{x}{L}\right)Z + \left(\frac{Zx}{L} + 1\right)(-1) = 0$$

or, $$Z - \frac{Zx}{L} - \frac{Zx}{L} - 1 = 0$$

or, $$\frac{2Zx}{L} = Z - 1$$

or, $$\frac{x}{L} = \frac{Z - 1}{2Z} \quad ...(2.100)$$

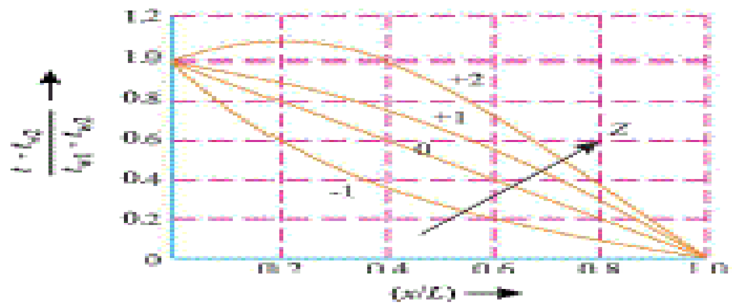

Fig. 2.94. Effect of factor Z on the temperature distribution in the plane wall.

Thus the maximum value of temperature occurs at $\frac{x}{L} = \frac{Z - 1}{2Z}$ and its value is given by:

$$\frac{t_{max} - t_{w2}}{t_{w1} - t_{w2}} = \left[1 - \frac{Z - 1}{2Z}\right]\left[Z \times \left(\frac{Z - 1}{2Z}\right) + 1\right]$$

or, $$\frac{t_{max} - t_{w2}}{t_{w1} - t_{w2}} = \left(\frac{Z + 1}{2Z}\right)\left(\frac{Z + 1}{2}\right) = \frac{(Z + 1)^2}{4Z} \quad ...(2.101)$$

Fig. 2.94 shows the effect of factor Z on the temperature distribution in the plane wall. The following points emerge :

- As the value of Z *increases* the slope of the curve changes; obviously the direction of heat flow can be reversed by an adequately large value of q_g.
- When $Z = 0$, the temperature distribution is *linear* (*i.e.*, no internal heat generation).
- When the value of Z is *negative*, q_g represents *absorption of heat* within the wall/body.

Case III. Current carrying electrical conductor :

When electrical current passes through a conductor, heat is generated (Q_g) in it and is given by

$$Q_g = I^2 R, \text{ where } R = \frac{\rho L}{A}$$

where,
I = Current flowing in the conductor,
R = Electrical resistance,
ρ = Specific resistance or resistivity,
L = Length of the conductor, and
A = Area of cross-section of the conductor.

Also, $$Q_g = q_g \times A \times L$$

$\therefore$ $$q_g \times A \times L = I^2 \times \frac{\rho L}{A} \quad \text{or,} \quad q_g = I^2 \times \frac{\rho L}{A} \times \frac{1}{AL} = \frac{I^2 \rho}{A^2}$$

1.The temperature drop in a plane wall with uniformly distributed heat generation can be decreased by reducing

a) Wall thickness

b) Heat generation rate

c) Thermal conductivity

d) Surface area

Answer: a

Explanation: On decreasing wall thickness, generally temperature drop decreases.

2.Notable example of uniform generation of heat within the conducting medium are

(i) Energy of a nuclear reactor

(ii) Liberation of energy due to some exothermic chemical reactions

(iii) Resistance heating in electrical appliances

Which of the statements made above are correct?

a) i, ii and iii

b) i and ii

c) i and iii

d) Only ii

Answer: a

Explanation: All the statements are correct with respect to plane wall heat conduction.

References

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