SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF FOOD TECHNOLOGY

UNIT 1 - CONDUCTION

Topic 3: 1D SS Conduction equation for wall

Conduction Heat transfer

→ Objectives

1) i° objective is to determine the temp field (T_x) in a body (how temp varies with positⁿ within the body)

2) $T(x)$ depends on boundary conditions, initial, material ppts (ρ, k, C_p and geometry)

3) Why we need $T(x)$

   a) Compute heat flow at any point

   b) Compute thermal stress, expansion, deflection due to temp

   c) Design pdts in applications such as insulation thickness, Heat treatment of metals.

Boundary & Initial Conditions.

1) Heat equation is second order on spatial coordinates, hence 2 boundary condⁿ needed for each coordinate

The initial conditions describe the temperature distribution in a medium at the initial moment of time and these are needed only for time dependent pblms.

$t \geq 0, \; T = T(x, y, z)$

One dimensional steady state heat conduction without heat generation

- Consider a plane wall of a material of uniform thermal conductivity, k, which is assumed to be extending to infinity in y and z directions.

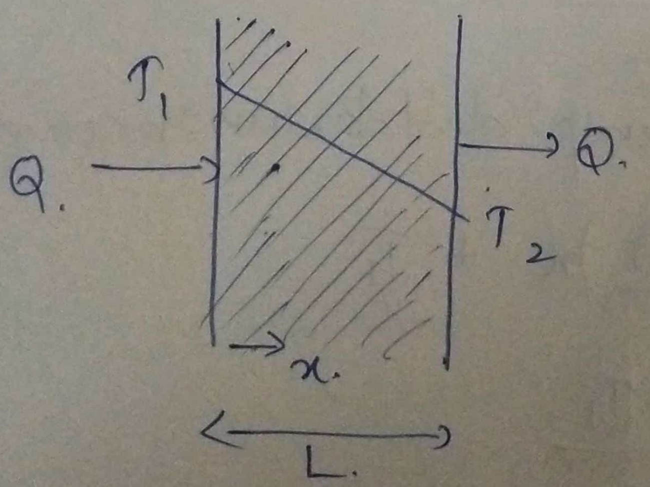
Walls of room may be considered as a plane, if the energy loss through the edges is negligible.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

$$\frac{\partial T}{\partial t} = 0 \text{ (steady state)}$$

$$\frac{\partial^2 T}{\partial x^2} \pm \frac{\partial^2 T}{\partial z^2} = 0 \text{ (one dimensional)}$$

$$\frac{q}{k} = 0 \text{ (no heat generation)}$$



Conduction equation simplifies to

$$\frac{\partial^2 T}{\partial x^2} = 0 \quad \text{or} \quad \frac{d^2 T}{dx^2} = 0 \quad \text{--- (1)}$$

This is a 2^{nd} order differential eq^n requiring 2 BC

$$T = T_1 \text{ at } x = 0$$

$$T = T_2 \text{ at } x = L$$

Integrating (1) twice we get

$$T = C_1 x + C_2$$

where C_1 and C_2 can be determined from bounda[ry] $cond^n$.

$x = 0,\ T = T_1$ so that $C_2 = T_1$

$x = L,\ T = T_2$ so that $T_2 = C_1 L + T_1$

$$C_1 = \frac{T_2 - T_1}{L}$$

So the eq^n for temp distribution becomes

$$T = \left(\frac{T_2 - T_1}{L}\right) x + T_1$$

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

$$Q = -KA\frac{dT}{dx} = \frac{-KA(T_2 - T_1)}{L} = \frac{KA(T_1 - T_2)}{L}$$

The thermal qty of heat supplied to the left face of wall to maintain a temperature difference $T_1 - T_2$ accross it.

$$R_{th} = \frac{L}{KA}.$$

$$Q = -KA\frac{\partial T}{\partial x}$$

$$\int_0^L Q\,dx = -KA\int_{T_1}^{T_2} dT$$

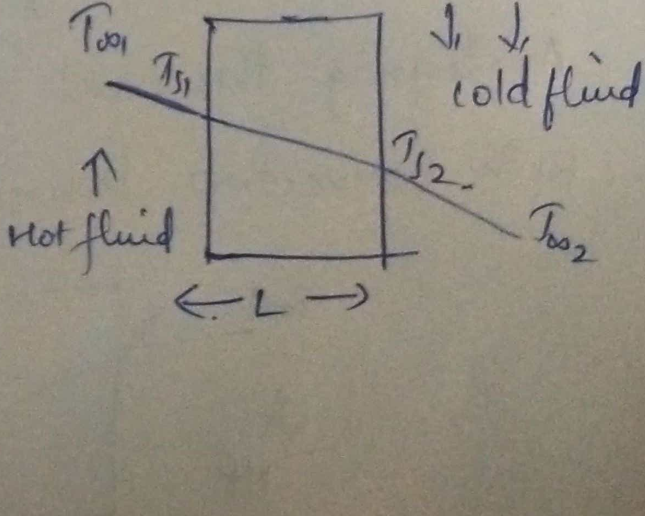
$$QL = -KA(T_2 - T_1)$$

$$Q = \frac{-KA(T_2 - T_1)}{L} = \frac{KA}{L}(T_1 - T_2) \quad \text{—— (2)}.$$

Plane wall with convective boundary conditions

The differential eq^n is

$$\frac{d}{dx}\left(k.\frac{dT}{dx}\right) = 0 \quad \text{—— (1)}$$



Integrating twice,

$$T(x) = C_1 x + C_2$$

Boundary conditions are,

a) $x=0,\ T=T_{s_1} \qquad T(0)=T_{s_1}$ — (2)

b) $x=L,\ T=T_{s_2} \qquad T(L)=C_1L+T_{s_1}$

$$\therefore C_1 = \frac{T(L)-T_{s1}}{L} \quad \text{— (3)}$$

$$\therefore T(x) = (T(L))-T_s)\frac{x}{L}+T_{s1} \quad \text{— (4)}$$

Heat flux across the wall is given by

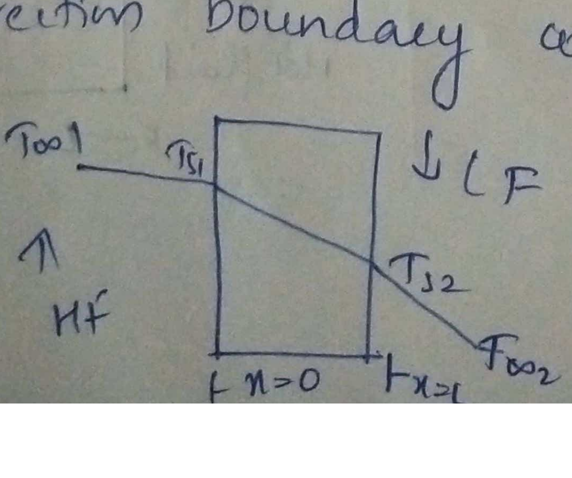
$$Q=-KA\frac{dT}{dx} = \frac{KA}{L}(T_{s_1}-T_{s_2}) = \frac{T_{s_1}-T_{s2}}{(L/KA)} \quad \text{— (3)}$$

<u>Thermal resistance concept for convection</u>

$$q=hA(T_{\infty}-T_{s_1}) = \ ,\ q=\frac{T_{\infty_1}-T_{s1}}{1/hA}$$

$$R_{th\,convec} = \frac{T_{\infty_1}-T_{s1}}{q} = 1/hA$$

Applying thermal resistance concept for plane wa[ll] with convection boundary conditions.



$$T_{\infty 1} \xrightarrow{q} T_{s1} \xrightarrow{q} T_{s2} \xrightarrow{q} T_{\infty 2}$$

$$q \quad 1/h_1A \quad q \quad L/KA \quad q \quad 1/h_2A$$

Heat transfer rate may be determined by considering each element of the resistance network as.

$$q = \frac{T_{\infty 1} - T_{s1}}{1/h_1A} = \frac{T_{s1} - T_{s2}}{L/KA} = \frac{T_{s2} - T_{\infty 2}}{1/h_2A} \quad \text{—} (1)$$

Since resistances are in series.

$$R_{total} = \Sigma R_{th} = 1/h_1A + L/KA + 1/h_2A \quad \text{—} (2)$$

$$= \frac{T_{\infty 1} - T_{s1}}{q} + \frac{T_{s1} - T_{s2}}{q} + \frac{T_{s2} - T_{\infty 2}}{q}.$$

1) Heat transfer deals with the rate of

a. work transfer
b. temperature transfer
c. energy transfer
d. none of the above

Answer :
c. energy transfer

MCQ

2. The amount of heat required to raise the temperature of a substance by 1°C is called:

A. work capacity

B. heat capacity

C. Energy capacity

D. none of the above

Ans: B

3 Heat bring.........................change

A. Physical

B. chemical

C. reversible

D. periodic

Ans: B   Heat bring chemical change

4. The process of transfer of heat in liquids & gases is called:

A. Conduction

B. Radiation

C. Convection

D. Absorption

Ans: C It is the process of transfer of heat in liquids & gases

MCQ

5) Solids are not heated by convection because:

solid are not free to move from one place to another

A. molecules only vibrate about a fixed position

B. both A and B

C. none of the above

Ans: C Solids are not heated by convection because the molecules of a solid are not free to move from one place to another; they can only vibrate about a fixed position