PART B

1. A wall is constructed of several layers. The first layer consists of masonry brick 20 cm. thick of thermal conductivity 0.66 W/mK, the second layer consists of 3 cm thick mortar of thermal conductivity 0.6 W/mK, the third layer consists of 8 cm thick lime stone of thermal conductivity 0.58 W/mK and the outer layer consists of 1.2 cm thick plaster of thermal conductivity 0.6 W/mK. The heat transfer coefficient on the interior and exterior of the wall are 5.6 W/m^2K and 11 W/m^2K respectively. Interior room temperature is 22°C and outside air temperature is -5°C. Calculate

i)Overall heat transfer coefficient
ii)Overall thermal resistance
iii)The rate of heat transfer
iv)The temperature at the junction between the mortar and the limestone.

Given Data

Thickness of masonry L_1 = 20cm = 0.20 m

Thermal conductivity K_1 = 0.66 W/mK

Thickness of mortar L_2 = 3cm = 0.03 m

Thermal conductivity of mortar K_2 = 0.6 W/mK

Thickness of limestone L_3 = 8 cm = 0.08 m

Thermal conductivity K_3 = 0.58 W/mK

Thickness of Plaster L_4 = 1.2 cm = 0.012 m

Thermal conductivity K_4 = 0.6 W/mK

Interior heat transfer coefficient h_a = 5.6 W/m^2K

Exterior heat transfer co-efficient h_b = 11 W/m^2K

Interior room temperature T_a = 22°C + 273 = 295 K

Outside air temperature T_b = -5°C + 273 = 268 K.

Solution:

Heat flow through composite wall is given by

$$Q = \frac{\Delta T_{overall}}{R}$$ [From equation (13)] (or) [HMT Data book page No. 34]

Where, $\Delta T = T_a - T_b$

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{L_4}{K_4 A} + \frac{1}{h_b A}$$

$$\Rightarrow Q = \frac{T_c - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{L_4}{K_4 A} + \frac{1}{h_b A}}$$

$$\Rightarrow Q / A = \frac{295 - 268}{\frac{1}{5.6} + \frac{0.20}{0.66} + \frac{0.03}{0.6} + \frac{0.08}{0.58} + \frac{0.012}{0.6} + \frac{1}{11}}$$

Heat transfer per unit area Q/A = 34.56 W/m^2

We know, Heat transfer Q = UA ($T_a - T_b$) [From equation (14)]

Where U – overall heat transfer co-efficient

$$\Rightarrow U = \frac{Q}{A \times (T_a - T_b)}$$

$$\Rightarrow U = \frac{34.56}{295 - 268}$$

Overall heat transfer co - efficient U = 1.28 W/m^2K

We know

Overall Thermal resistance (R)

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{L_4}{K_4 A} + \frac{1}{h_b A}$$

For unit Area

$$R = \frac{1}{h_a} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{L_4}{K_4} + \frac{1}{h_b}$$

$$= \frac{1}{56} + \frac{0.20}{0.66} + \frac{0.03}{0.6} + \frac{0.08}{0.58} + \frac{0.012}{0.6} + \frac{1}{11}$$

$R = 0.78\ K/W$

Interface temperature between mortar and the limestone T_3

Interface temperatures relation

$$\Rightarrow Q = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} = \frac{T_4 - T_5}{R_4} = \frac{T_5 - T_b}{R_b}$$

$$\Rightarrow Q = \frac{T_a - T_1}{R_a}$$

$$Q = \frac{295 - T_1}{1/h_a A} \qquad \left[\because R_a = \frac{1}{h_a A}\right]$$

$$\Rightarrow Q/A = \frac{295 - T_1}{1/h_a}$$

$$\Rightarrow 34.56 = \frac{295 - T_1}{1/5.6}$$

$$\Rightarrow \boxed{T_1 = 288.8\ K}$$

$$\Rightarrow Q = \frac{T_1 - T_2}{R_1}$$

$$Q = \frac{288.8 - T_2}{\frac{L_1}{K_1 A}} \qquad \left[\because R_1 = \frac{L_1}{k_1 A}\right]$$

$$\Rightarrow Q/A = \frac{288.8 - T_2}{\frac{L_1}{K_1}}$$

$$\Rightarrow 34.56 = \frac{288.8 - T_2}{\frac{0.20}{0.66}}$$

$$\Rightarrow \boxed{T_2 = 278.3\ K}$$

$$\Rightarrow Q = \frac{T_2 - T_3}{R_2}$$

$$Q = \frac{278.3 - T_3}{\frac{L_2}{K_2 A}} \qquad \left[\because R_2 = \frac{L_2}{K_2 A}\right]$$

$$\Rightarrow Q/A = \frac{278.3 - T_3}{\frac{L_2}{K_2}}$$

$$\Rightarrow 34.56 = \frac{278.3 - T_3}{\frac{0.03}{0.6}}$$

$$\Rightarrow \boxed{T_3 = 276.5\ K}$$

Temperature between Mortar and limestone (T_3 is 276.5 K)

2. A furnace wall made up of 7.5 cm of fire plate and 0.65 cm of mild steel plate. Inside surface exposed to hot gas at 650°C and outside air temperature 27°C. The convective heat transfer co-efficient for inner side is 60 W/m^{2}K. The convective heat transfer co-efficient for outer side is 8W/m^{2}K.

Calculate the heat lost per square meter area of the furnace wall and also find outside surface temperature.

Given Data

Thickness of fire plate L_1 = 7.5 cm = 0.075 m

Thickness of mild steel L_2 = 0.65 cm = 0.0065 m

Inside hot gas temperature T_a = 650°C + 273 = 923 K

Outside air temperature T_b = 27°C + 273 = 300°K

Convective heat transfer co-efficient for

Inner side $h_a = 60 W/m^2K$

Convective heat transfer co-efficient for

Outer side $h_b = 8 W/m^2K$.

Solution:

(i) Heat lost per square meter area (Q/A)

Thermal conductivity for fire plate

$K_1 = 1035 \times 10^{-3}$ W/mK [From HMT data book page No.11]

Thermal conductivity for mild steel plate

K_2 = 53.6W/mK [From HMT data book page No.1]

Heat flow $Q = \frac{\Delta T_{overall}}{R}$, Where

$\Delta T = T_a - T_b$

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}$$

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}}$$

[The term L_3 is not given so neglect that term]

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}}$$

The term L_3 is not given so neglect that term]

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{1}{h_b A}}$$

$$Q/A = \frac{923 - 300}{\frac{1}{60} + \frac{0.075}{1.035} + \frac{0.0065}{53.6} + \frac{1}{8}}$$

$$\boxed{Q/A = 2907.79\ W/m^2}$$

(ii) **Outside surface temperature T_3**

We know that, Interface temperatures relation

$$Q = \frac{T_a - T_b}{R} = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_b}{R_b} \text{......}(A)$$

$$(A) \Rightarrow Q = \frac{T_3 - T_b}{R_b}$$

where

$$R_b = \frac{1}{h_b A}$$

$$\Rightarrow Q = \frac{T_3 - T_b}{\frac{1}{h_b A}}$$

$$\Rightarrow Q/A = \frac{T_3 - T_b}{\frac{1}{h_b}}$$

$$\Rightarrow 2907.79 = \frac{T_3 - 300}{\frac{1}{8}}$$

$$\boxed{T_3 = 663.473\ K}$$

3. A steel tube (K = 43.26 W/mK) of 5.08 cm inner diameter and 7.62 cm outer diameter is covered with 2.5 cm layer of insulation (K = 0.208 W/mK) the inside surface of the tube receivers heat from a hot gas at the temperature of 316°C with heat transfer co-efficient of 28 W/m^{2}K. While the outer surface exposed to the ambient air at 30°C with heat transfer co-efficient of 17 W/m^{2}K. Calculate heat loss for 3 m length of the tube.

Steel tube thermal conductivity K_1 = 43.26 W/mK
Inner diameter of steel d_1 = 5.08 cm = 0.0508 m
Inner radius r_1 = 0.0254 m
Outer diameter of steel d_2 = 7.62 cm = 0.0762 m
Outer radius r_2 = 0.0381 m
Radius $r_3 = r_2$ + thickness of insulation
Radius r_3 = 0.0381 + 0.025 m r_3 = 0.0631 m
Thermal conductivity of insulation K_2 = 0.208 W/mK
Hot gas temperature T_a = 316°C + 273 = 589 K
Ambient air temperature T_b = 30°C + 273 = 303 K
Heat transfer co-efficient at inner side h_a = 28 W/m^{2}K
Heat transfer co-efficient at outer side h_b = 17 W/m^{2}K
Length L = 3 m

Solution :

Heat flow $Q = \frac{\Delta T_{overall}}{R}$ [From equation No.(19) or HMT data book Page No.35]

Where $\Delta T = T_a - T_b$

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{K_1} In\left[\frac{r_2}{r_1}\right] + \frac{1}{K_2} In\left[\frac{r_3}{r_2}\right] + \frac{1}{K_3} In\left[\frac{r_4}{r_3}\right] + \frac{1}{h_b r_4}\right]$$

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{K_1} In\left[\frac{r_2}{r_1}\right] + \frac{1}{K_2} In\left[\frac{r_3}{r_2}\right] + \frac{1}{K_3} In\left[\frac{r_4}{r_3}\right] + \frac{1}{h_b r_4}\right]}$$

[The terms K_3 and r_4 are not given, so neglect that terms]

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{K_1} In\left[\frac{r_2}{r_1}\right] + \frac{1}{K_2} In\left[\frac{r_3}{r_2}\right] + \frac{1}{h_b r_3}\right]}$$

$$\Rightarrow Q = \frac{589 - 303}{\frac{1}{2\pi \times 3}\left[\frac{1}{28 \times 0.0254} + \frac{1}{43.26} In\left[\frac{0.0381}{0.0254}\right] + \frac{1}{0.208} In\left[\frac{0.0631}{0.0381}\right] + \frac{1}{17 \times 0.0631}\right]}$$

$$\boxed{Q = 1129.42\ W}$$

Heat loss Q = 1129.42 W.

4. An aluminium alloy fin of 7 mm thick and 50 mm long protrudes from a wall, which is maintained at 120°C. The ambient air temperature is 22°C. The heat transfer coefficient and conductivity of the fin material are 140 W/m^{2}K and 55 W/mK respectively. Determine
   i) Temperature at the end of the fin
   ii) Temperature at the middle of the fin.
   iii) Total heat dissipated by the fin.

Given

Thickness t = 7mm = 0.007 m
Length L= 50 mm = 0.050 m
Base temperature T_b = 120°C + 273 = 393 K
Ambient temperature T_∞ = 22° + 273 = 295 K
Heat transfer co-efficient h = 140 W/m^2K
Thermal conductivity K = 55 W/mK.

Solution :

Length of the fin is 50 mm. So, this is short fin type problem. Assume end is insulated.

We know

Temperature distribution [Short fin, end insulated]

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cos h\ m\ [L - x]}{\cos h\ (mL)} \quad ....(A)$$

[From HMT data book Page No.41]

(i) Temperature at the end of the fin, Put x = L

$$(A) \Rightarrow \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cos h\ m\ [L-L]}{\cos h\ (mL)}$$

$$\Rightarrow \frac{T - T_\infty}{T_b - T_\infty} = \frac{1}{\cos h\ (mL)} \quad ...(1)$$

where

$$m = \sqrt{\frac{hP}{KA}}$$

P = Perimeter = 2 × L (Approx)

= 2 × 0.050

$$\boxed{P = 0.1\ m}$$

A – Area = Length × thickness = 0.050 × 0.007

$$\boxed{A = 3.5 \times 10^{-4}\ m^2}$$

$$\Rightarrow m = \sqrt{\frac{hP}{KA}}$$

$$= \sqrt{\frac{140 \times 0.1}{55 \times 3.5 \times 10^{-4}}}$$

$$\boxed{m = 26.96}$$

(1) $\Rightarrow \frac{T - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh(26.9 \times 0.050)}$

$\Rightarrow \frac{T - T_\infty}{T_b - T_\infty} = \frac{1}{2.05}$

$\Rightarrow \frac{T - 295}{393 - 295} = \frac{1}{2.05}$

$\Rightarrow T - 295 = 47.8$

$\Rightarrow \boxed{T = 342.8\ K}$

$\boxed{\text{Temperature at the end of the fin } T_{x=L} = 342.8\ K}$

(ii) Temperature of the middle of the fin,

Put x = L/2 in Equation (A)

(A) $\Rightarrow \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m[L - L/2]}{\cosh(mL)}$

$\Rightarrow \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh 26.9 \left(0.050 - \frac{0.050}{2} \right)}{\cosh [26.9 \times (0.050)]}$

$\Rightarrow \frac{T - 295}{393 - 295} = \frac{1.234}{2.049}$

$\Rightarrow \frac{T - 295}{393 - 295} = 0.6025$

$\boxed{T = 354.04\ K}$

Temperature at the middle of the fin

$\boxed{T_{x=L/2} = 354.04\ K}$

(iii) Total heat dissipated

[From HMT data book Page No.41]

$\Rightarrow Q = (hPKA)^{1/2} (T_b - T_\infty) \tanh(mL)$

$\Rightarrow [140 \times 0.1 \times 55 \times 3.5 \times 10^{-4}]^{1/2} \times (393 - 295) \times \tanh(26.9 \times 0.050)$

$\boxed{Q = 44.4\ W}$

5. a)A furnace wall consists of 200 mm layer of refractory bricks, 6mm layer of steel plate and a 100 mm layer of insulation bricks. The maximum temperature of the wall is 1150 °C on the furnace side and the minimum temperature is 40 °C on the outer side of the wall. An accurate energy balance over the furnace shows that the heat loss from wall is 400 W/m^{2}K. It is known that there is thin layer of air between the layers of refractory bricks and steel plate. Thermal conductivities for the three layers are 1.52, 45 and 0.138 W/m °C respectively. Find

1) To how may millimeters of insulation brick is the air layer equivalent?
2) What is the temperature of the outer surface of the steel plate?

$$R_c = \frac{1}{h_c}$$

$$R = \frac{L}{k}$$

Equivalent thickness is determined by

$$L = kR_c$$

b) Find out the amount of heat transferred through an iron fin of length 50 mm, width 100mm and thickness 5mm. Assume k= 210 kJ/mh°C and h = 42 kJ/m^{2}h°C for the material of the fin and the temperature at the base of the fin as 80 °C. Also determine and temperature at tip of the fin, if atmosphere temperature is 20°C **[NOV DEC 14]**

Refer problem No. 4 & 10

6. a) Derive general heat conduction equation in Cartesian coordinates.
b) Compute the heat loss per square meter surface area of 40 cm thick furnace wall having surface temperature of 300°C and 50 °C if the thermal conductivity k of the wall material is given by k = 0.005T - 5×10^{-6} T^2 where T= Temperature in °C **[NOV DEC 14]**

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction at}\\ x, y, \text{and } z\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x+\Delta x,\\ y+\Delta y, \text{and } z+\Delta z\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of}\\ \text{the element}\end{pmatrix}$$

or

$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho C\Delta x\Delta y\Delta z(T_{t+\Delta t} - T_t)$$
$$\dot{G}_{\text{element}} = \dot{g}V_{\text{element}} = \dot{g}\Delta x\Delta y\Delta z$$

$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{g}\Delta x\Delta y\Delta z = \rho C\Delta x\Delta y\Delta z\frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by $\Delta x\Delta y\Delta z$ gives

$$-\frac{1}{\Delta y\Delta z}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} - \frac{1}{\Delta x\Delta z}\frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} - \frac{1}{\Delta x\Delta y}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} + \dot{g} = \rho C\frac{T_{t+\Delta t} - T_t}{\Delta t}$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Sub values we have for cylinder

$A_1 = 2\pi(60\times10^{-3})60$

$A_3 = 2\pi(160\times10^{-3})60$

$$R_i = \frac{1}{h_1 A_1}$$

$$R_1 = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L K_1}$$

$$R_2 = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi L K_2}$$

$$R_0 = \frac{1}{h_2 A_2}$$

Rtotal = Ri+R1+R2+Ro

$$Q = \frac{T_{\infty1} - T_{\infty2}}{R_{Total}}$$

7. Alloy steel ball of 2 mm diameter heated to 800°C is quenched in a bath at 100°C. The material properties of the ball are K = 205 kJ/m hr K, ρ = 7860 kg/m^3, C_ρ = 0.45 kJ/kg K, h = 150 KJ/ hr m^2 K. Determine (i) Temperature of ball after 10 second and (ii) Time for ball to cool to 400°C.

Diameter of the ball D = 12 mm = 0.012 m
Radius of the ball R = 0.006m
Initial temperature T_0 = 800°C + 273 = 1073 K

Final temperature $T_\infty = 100°C + 273 = 373$ K
Thermal conductivity K = 205 kJ/m hr K

$$= \frac{205 \times 1000 J}{3600\ s\ mK}$$

$$= 56.94\ W/mK \qquad [\because J/s = W]$$

Density ρ = 7860 kg/m^3
Specific heat C_ρ = 0.45 kJ/kg K
= 450 J/kg K

Heat transfer co-efficient h = 150 kJ/hr m^2 K

$$= \frac{150 \times 1000 J}{3600\ s\ m^2 K}$$

$$= 41.66\ W/m^2 K$$

Solution

Case (i) Temperature of ball after 10 sec.

For sphere,

Characteristic Length $L_c = \frac{R}{3}$

$$= \frac{0.006}{3}$$

$$L_c = 0.002\ m$$

We know,

Biot number $B_i = \frac{hL_c}{K}$

$$= \frac{41.667 \times 0.002}{56.94}$$

$B_i = 1.46 \times 10^{-3} < 0.1$

Biot number value is less than 0.1. So this is lumped heat analysis type problem.

For lumped parameter system,

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-hA}{C_\rho \times V \times \rho} \times t\right]} \quad \text{……….(1)}$$

[From HMT data book Page No.48]

We know,

Characteristics length $L_c = \frac{V}{A}$

$$(1) \Rightarrow \frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-h}{C_\rho \times L_c \times \rho} \times t\right]} \quad \ldots\ldots\ldots(2)$$

$$\Rightarrow \frac{T - 373}{1073 - 373} = e^{\left[\frac{-41.667}{450 \times 0.002 \times 7860} \times 10\right]}$$

$$\Rightarrow \boxed{T = 1032.95\ K}$$

Case (ii) Time for ball to cool to 400°C

$\therefore T = 400°C + 273 = 673\ K$

$$(2) \Rightarrow \frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-h}{C_\rho \times L_c \times \rho} \times t\right]} \quad \ldots\ldots.(2)$$

$$\Rightarrow \frac{673 - 373}{1073 - 373} = e^{\left[\frac{-41.667}{450 \times 0.002 \times 7860} \times t\right]}$$

$$\Rightarrow \ln\left[\frac{673 - 373}{1073 - 373}\right] = \frac{-41.667}{450 \times 0.002 \times 7860} \times t$$

$$\Rightarrow \boxed{t = 143.849\ s}$$

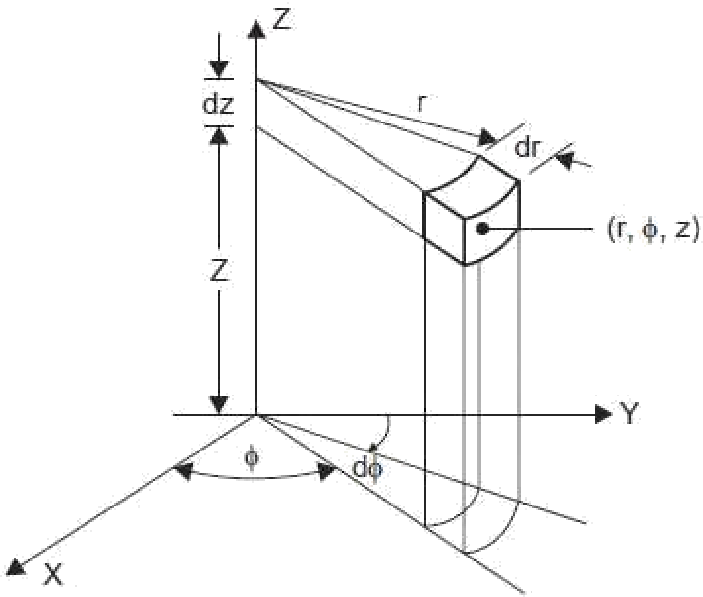
8. Derive the general heat conduction equation in cylindrical coordinate and solve the following. Hot air at a temperature of 65 °C is flowing through steel pipe of 120 mm diameter. The pipe is Covered with two layers of different insulating materials of thickness 60 mm and 40 mm and their Corresponding thermal conductivities are 0.24 and 0.4 W/ m K. The inside and outside heat transfer coefficients are 60 W/m^2 K and 12 W/m^2 K respectively. The atmosphere is at 20°C. Find the rate of heat loss from 60 m length of pipe. **[MAY-JUN 14]**

In cylindrical coordinate (r, Φ, z),

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \Phi}\left(k\frac{\partial T}{\partial \Phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+q=\rho c\frac{\partial T}{\partial \tau}$$

With k constant eqn. in 2.4 reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \Phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q}{k}=\frac{1}{\alpha}\cdot\frac{\partial T}{\partial \tau}$$



Elemental volume in cylindrical coordinates.

9. Steam in a heating system flows through tubes whose outer diameter is D_1= 3 cm and whose walls are maintained at a temperature of 120°C. Circular aluminum fins (k =180 W/m · °C) of outer diameter D_2 = 6 cm and constant thickness t = 2 mm are attached to the tube. The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at T_∞= 25°C, with a combined heat transfer coefficient of h = 60 W/m^2 °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.

$$A_{\text{no fin}} = \pi D_1 L = \pi(0.03\text{ m})(1\text{ m}) = 0.0942\text{ m}^2$$
$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty)$$
$$= (60\text{ W/m}^2\cdot{}^\circ\text{C})(0.0942\text{ m}^2)(120 - 25)^\circ\text{C}$$
$$= 537\text{ W}$$

$$A_{fin} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t$$
$$= 2\pi[(0.03\text{ m})^2 - (0.015\text{ m})^2] + 2\pi(0.03\text{ m})(0.002\text{ m})$$
$$= 0.00462\text{ m}^2$$

$$\dot{Q}_{fin} = \eta_{fin}\dot{Q}_{fin,\,max} = \eta_{fin} h A_{fin}(T_b - T_\infty)$$
$$= 0.95(60\text{ W/m}^2\cdot°\text{C})(0.00462\text{ m}^2)(120 - 25)°\text{C}$$
$$= 25.0\text{ W}$$

$$A_{unfin} = \pi D_1 S = \pi(0.03\text{ m})(0.003\text{ m}) = 0.000283\text{ m}^2$$
$$\dot{Q}_{unfin} = hA_{unfin}(T_b - T_\infty)$$
$$= (60\text{ W/m}^2\cdot°\text{C})(0.000283\text{ m}^2)(120 - 25)°\text{C}$$
$$= 1.60\text{ W}$$

$$\dot{Q}_{total,\,fin} = n(\dot{Q}_{fin} + \dot{Q}_{unfin}) = 200(25.0 + 1.6)\text{ W} = 5320\text{ W}$$

$$\dot{Q}_{increase} = \dot{Q}_{total,\,fin} - \dot{Q}_{no\,fin} = 5320 - 537 = \mathbf{4783\text{ W}} \quad \text{(per m tube length)}$$

10. Copper plate fins of rectangular cross section having thickness t = 1mm, height L = 10 mm and thermal conductivity k = 380 W/ mK are attached to a plane wall maintained at a temperature T_0 = 230°C. Fins dissipate heat by convection in to ambient air at T = 30 °C with a heat transfer coefficient h= 40 W/ m^2 K. Fins are spaced at 8mm (that is 125 fins per meter). Assume negligible heat loss from the tip.
    a) Determine the fin efficiency
    b) Determine the fin effectiveness.
    c) Determine the net rate of heat transfer per square meter of plane surface
    d) What would be heat transfer rate from the plane wall if there were no fins attached?

$$\eta_{insulated\ tip} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,\,max}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{hA_{fin}(T_b - T_\infty)} = \frac{\tanh aL}{aL}$$

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\,fin}} = \frac{\dot{Q}_{fin}}{hA_b(T_b - T_\infty)} = \frac{\eta_{fin} hA_{fin}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \frac{A_{fin}}{A_b}\eta_{fin}$$

$Q_{net} = Q_{fin} + Q_{unfin} = 26.87\ kW/m^2$

$Q_{nofin} = 8\ kW/m^2$

11. Derive the heat dissipation equation through pin fin with insulated end and solve the following. A temperature rise of 50 C in a circular shaft of 50 mm diameter is caused by the amount of heat generated due to friction in the bearing mounted on the crankshaft. The thermal conductivity of shaft material is 55 W/m K and heat transfer coefficient is 7 W/m^2 K. Determine the amount of heat transferred through shaft assume that the shaft is a rod of infinite length. **[MAY-JUN 14]**

$$\begin{pmatrix} \text{Rate of } \textit{heat} \\ \textit{conduction} \text{ into} \\ \text{the element at } x \end{pmatrix} = \begin{pmatrix} \text{Rate of } \textit{heat} \\ \textit{conduction} \text{ from the} \\ \text{element at } x + \Delta x \end{pmatrix} + \begin{pmatrix} \text{Rate of } \textit{heat} \\ \textit{convection} \text{ from} \\ \text{the element} \end{pmatrix}$$

$$\dot{Q}_{\text{cond}, x} = \dot{Q}_{\text{cond}, x + \Delta x} + \dot{Q}_{\text{conv}}$$

where

$$\dot{Q}_{\text{conv}} = h(p\,\Delta x)(T - T_\infty)$$

Substituting and dividing by Δx, we obtain

$$\frac{\dot{Q}_{\text{cond}, x + \Delta x} - \dot{Q}_{\text{cond}, x}}{\Delta x} + hp(T - T_\infty) = 0$$

Taking the limit as $\Delta x \to 0$ gives

$$\frac{d\dot{Q}_{\text{cond}}}{dx} + hp(T - T_\infty) = 0$$

From Fourier's law of heat conduction we have

$$\dot{Q}_{\text{cond}} = -kA_c \frac{dT}{dx}$$

$$\frac{d}{dx}\left(kA_c\frac{dT}{dx}\right) - hp(T - T_\infty) = 0$$

$$\frac{d^2\theta}{dx^2} - a^2\theta = 0$$

where

$$a^2 = \frac{hp}{kA_c}$$

and $\theta = T - T_\infty$ is the *temperature excess.* At the fin base we have $\theta_b = T_b - T_\infty$.

Boundary condition at fin tip: $\left.\frac{d\theta}{dx}\right|_{x=L} = 0$

Adiabatic fin tip: $\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh a(L - x)}{\cosh aL}$

Adiabatic fin tip:

$$\dot{Q}_{\text{insulated tip}} = -kA_c\left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL$$

Sub values we have

$Q_{\text{insulated tip}}$ = 17.23 W

12. (i) A turbine blade 6 cm long and having a cross sectional area 4.65 cm^2 and perimeter 12 cm is made of stainless steel (k = 23.3 W/m K). The temperature at the root is 500 °C. The blade is exposed to a hot gas at 870 °C. The heat transfer coefficient between the blade surface and gas is 442 W/m^2 K. Determine the temperature distribution and rate of heat flow at the root of the blade. Assume the tip of the blade to be insulated.

(ii) An ordinary egg can be approximated as 5 cm diameter sphere. The egg in initially at a uniform temperature of 5°C and is dropped in to boiling water at 95 °C. Taking the convection heat transfer coefficient to be h = 1200 W/m2 °C. determine how long it will take for the center of the egg to reach 70 °C. **[NOV-DEC 13]**

$$\text{Bi} = \frac{hr_0}{k} = \frac{(1200\ \text{W/m}^2\cdot{}^\circ\text{C})(0.025\ \text{m})}{0.627\ \text{W/m}\cdot{}^\circ\text{C}} = 47.8$$

$$\lambda_1 = 3.0753, \qquad A_1 = 1.9958$$

$$\frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70 - 95}{5 - 95} = 1.9958 e^{-(3.0753)^2 \tau} \longrightarrow \tau = 0.209$$

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.209)(0.025 \text{ m})^2}{0.151 \times 10^{-6} \text{ m}^2/\text{s}} = 865 \text{ s}$$