Static Equilibrium

A body is in static equilibrium if it remains in its state of rest or uniform motion.

If the body is at rest, it tends to remain at rest and if in motion, it tends to keep the motion.

Conditions:

1) The vector sum of all the forces acting on the body is zero.

2) The vector sum of all the moments about any arbitrary point is zero.

Mathematically,

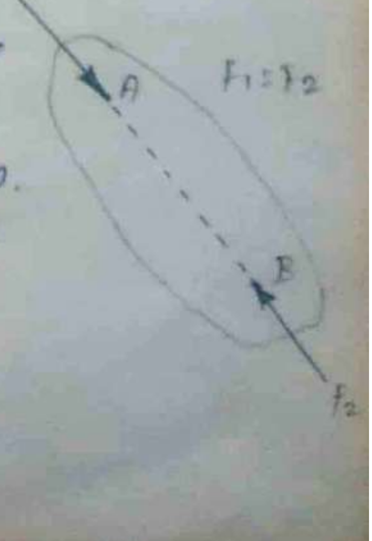
$$\sum F = 0, \quad \sum T = 0.$$

Equilibrium of two, three and four force members

Two force members:

A member under the action of two forces will be in equilibrium, if,

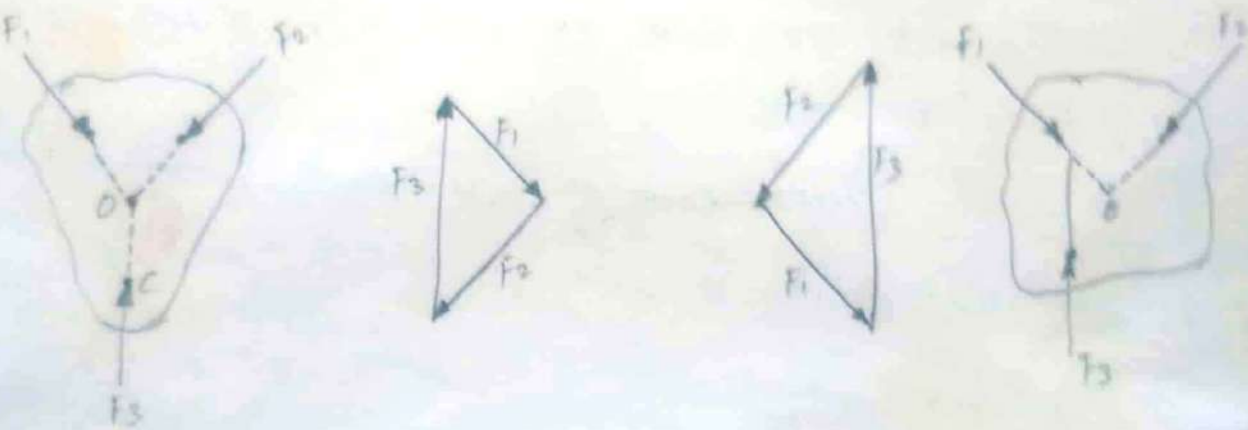
- The forces along the same line
- The forces are in opposite direction.
- The magnitude of the forces should be same



Three force members:

A member under the action of three forces will be in equilibrium if,

- The resultant of the forces is zero,
- The lines of action of the forces intersect at a point. (Known as Point of Concurrency)

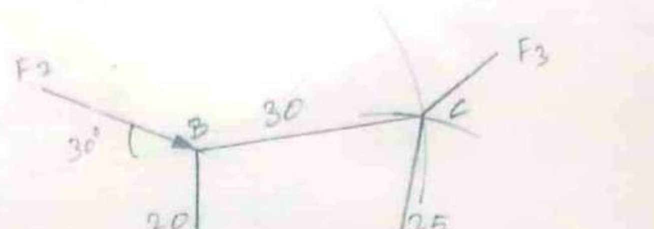


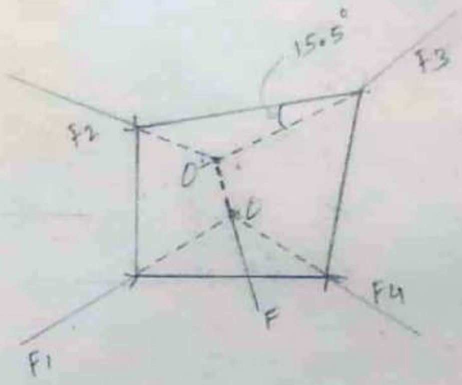
Four force members: 4.13

In some problems, it may found that the number of forces on a member is four or even more than that. In such cases, first look for the forces completely known and combined them into a single force representing the sum of the known forces.

This may reduce the number of forces acting on a body to two or three.

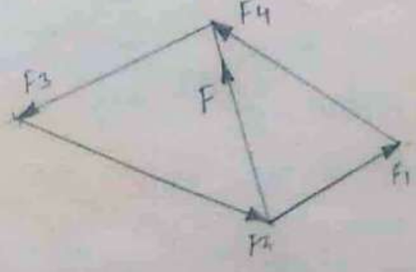
The following figure shows a quaternary link ABCD under the action of forces F_1, F_2, F_3 and F_4 acting at A B C and D respectively. The link is in static equilibrium. Determine the magnitude of the forces F_2 and F_3 and the direction of the force F_3. S.S.R. 412





RESULT

Force Triangle Scale: 1cm = 100N



Book Ans

F_2 = 370 N (380 N)

F_3 = 285 N (284 N)

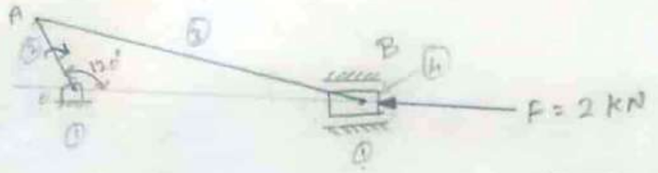
L.O.A of F_3 is 15.5° with CB

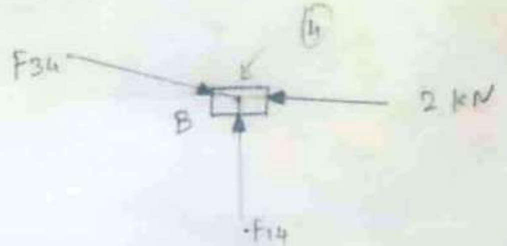
Static force analysis in simple machine members

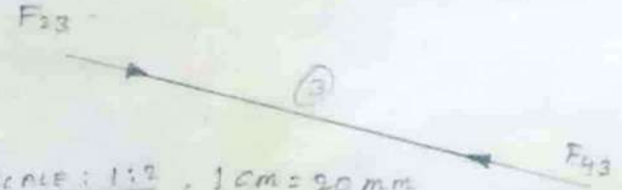
A slider crank mechanism withe the following dimensions is acted upon by a force F = 2 KN at B as shown in figure. OA = 100 mm, AB = 450 mm. Determine the input torque T on the link OA for the static equilibrium of the mechanism for the given configurations. S.S.R. 414

SCALE - 1:100

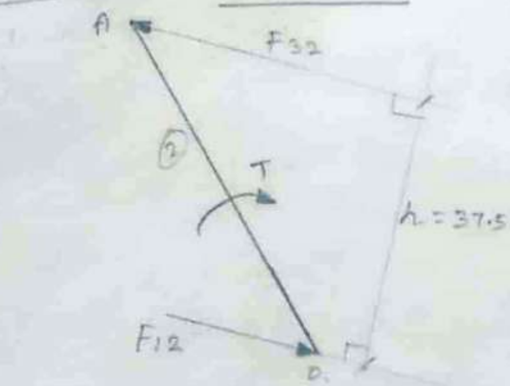
1 cm = 100 mm





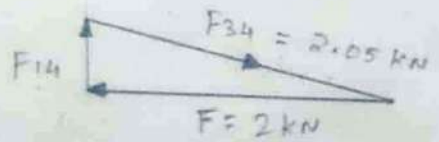


SCALE : 1:2 , 1 cm = 20 mm



FORCE TRIANGLE

SCALE - 2:1



$OA = 100$ mm

$h = 37.5 \times 2 = 75$ mm

$F_{34} = -F_{43} = F_{23} = -F_{32}$

$T = h \times -F_{32}$

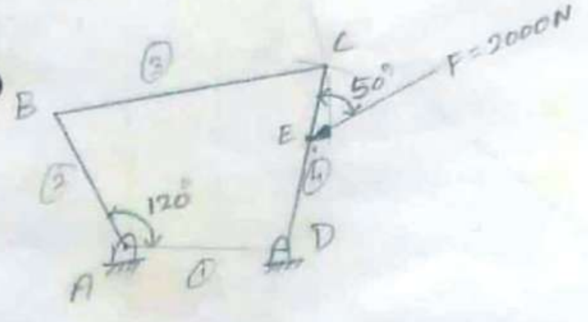
$= 75 \times (-2.05)$

$T = -153.75$ kN-mm

$\therefore \boxed{T = 153.75 \text{ kN}\cdot\text{m}}$ C.W

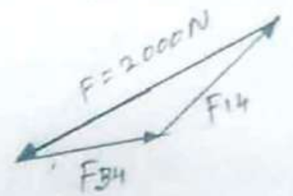
A four bar chain mechanism ABCD is shown in following figure. Calculate the required value of torque (T) and all the constraint forces on links for static equilibrium of the mechanism, if F = 2000N in the direction shown. The dimensions are as follows

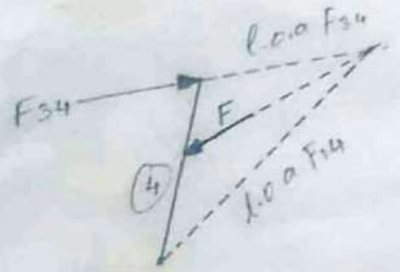
AB = 200 mm, BC = 370 mm, CD = 250 mm, AD = 215 mm and CE = 100 mm. Angle BAD = 120°



Force Triangle   Scale - 2:1

1 cm = 1000 N.

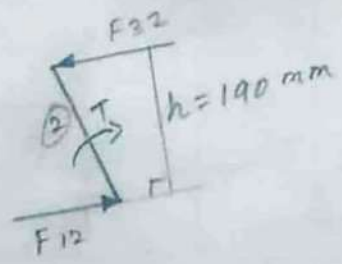




$F_{34} = 900\,N$

$F_{14} = 1150\,N.$

$F_{23} = -F_{32} = F_{34} = -F_{43}$



$T = -F_{32} \times h$

$= -900 \times 190$

$T = -171\ KN\text{-}mm$

$\boxed{T = 171\ N\text{-}m}$ Clockwise Direction

In a slider Crank mechanism, the length of the Crank and Connecting rod are 100 mm and 400 mm respectively. The Crank rotates uniformly at 600 rpm In clockwise. when the Crank has turned through 45° from inner dead centre. Find by analytical method

1) Velocity and acceleration of the slider

2) Angular velocity and angular acceleration of the Connecting rod.

Given:

$r = 100\ mm = 0.1\ m$

$l = 400\ mm = 0.4\ m$

$N = 600\ rpm.$

$\theta = 45°.$

Solution:

$$\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times600}{60}$$

$$\omega = 62.83\ rad/s.$$

$$\text{obliquity ratio},\ n = \frac{l}{r} = \frac{0.4}{0.1} = 4$$

1). Velocity and acceleration of the slider.

Velocity of the slider (piston).

$$V_P = \omega r \times \left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$$

$$= 62.83\times0.1\left[\sin 45 + \frac{\sin(2\times45)}{2\times4}\right]$$

$$V_P = 5.23\ m/s.$$

Acceleration of the Slider.

$$a_P = \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= (62.83)^2 \times 0.1 \left[\cos 45^\circ + \frac{\cos(2\times 45^\circ)}{4}\right]$$

$$a_P = \underline{279.14} \ m/s^2.$$

Angular velocity and angular acceleration of the Connecting rod.

Angular velocity of the connecting rod.

$$\omega_{PC} = \frac{\omega \cos\theta}{n} = \frac{62.83 \times \cos 45^\circ}{4} = \underline{11 \ rad/s}.$$

Angular acceleration of the connecting rod.

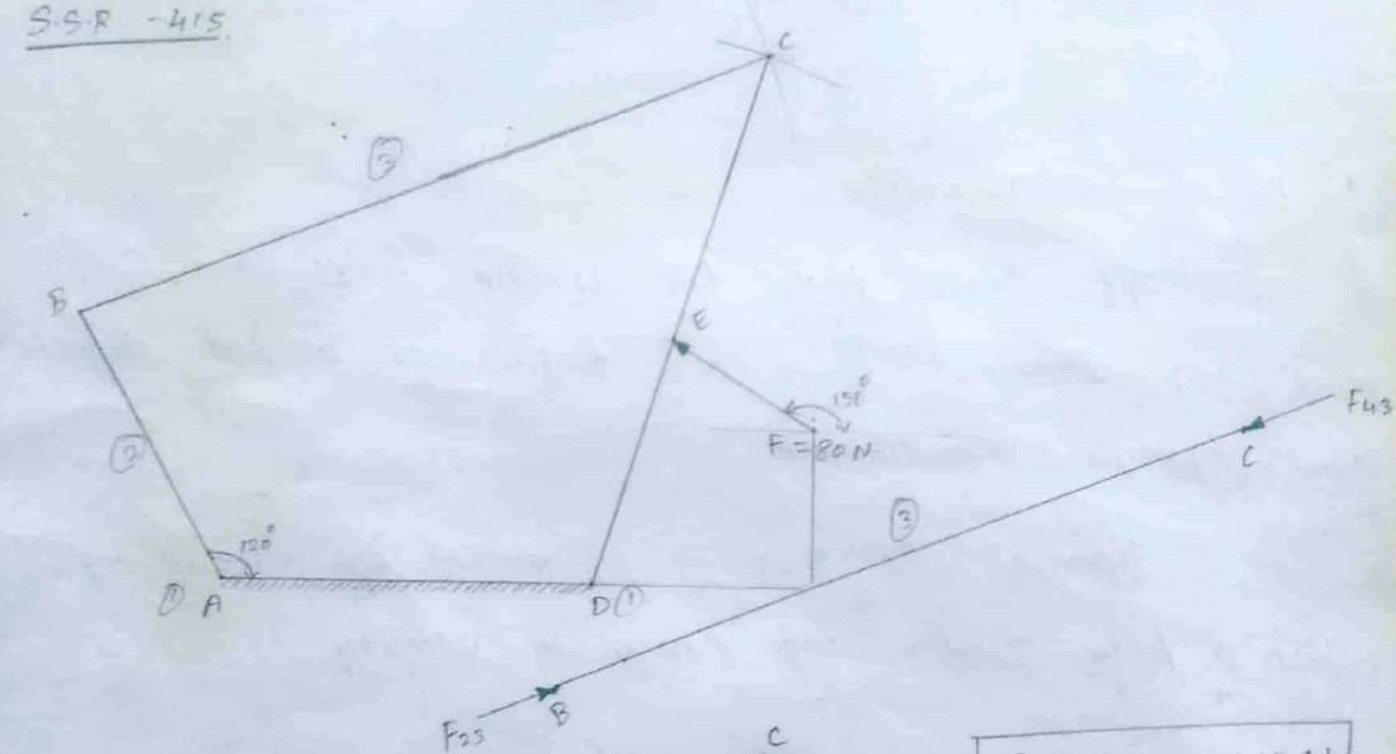
$$\alpha_{PC} = \frac{\omega^2 \sin\theta}{n} = \frac{(62.83)^2 \cdot \sin 45^\circ}{4} = \underline{697.8 \ rad/s^2}.$$

A four link mechanism with the following dimensions is acted upon by a force of 80 N at ∠150° on the link DC as shown in the figure.

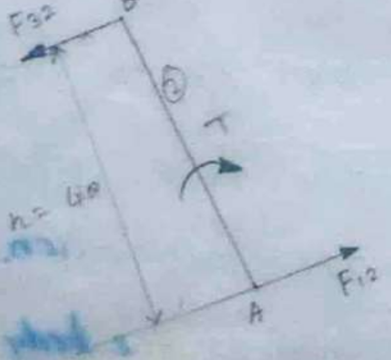
AD = 500 mm, AB = 400 mm, BC = 1000 mm, DC = 750 mm, DE = 350 mm.

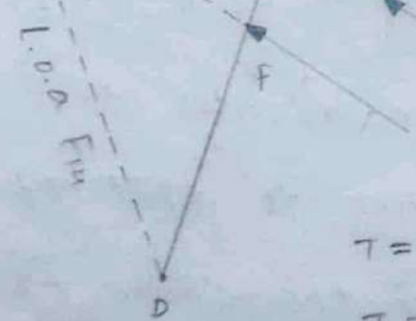
Determine the input torque T on the link AB for the static equilibrium of the mechanism for the given configuration.

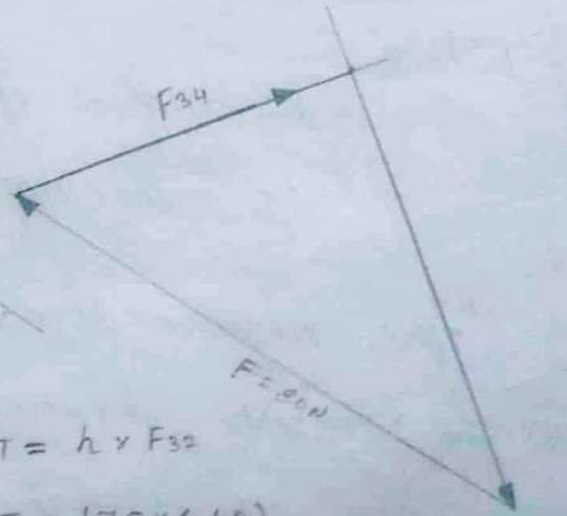
S.S.R - 415.



SCALE - 1 cm = 10 N







$F_{23} = -F_{32} = F_{34} = -F_{43}$

$T = h \times F_{32}$

$T = 470 \times (-40)$

$T = -18{,}800$ N-mm

T = 18.8 Nm C.W.

Book ans: $F_{34} = 47.8$, $h = 393$, $T = 18.7$ Nm

Dynamic force analysis. S.S.R - 434

Dynamic forces are associated with accelerating masses. As all machines have some accelerating parts, dynamic forces are always present when the machines operate. In situations where dynamic forces are dominant or comparable with magnitudes of external forces and operating speeds are high, dynamic analysis has to be carried out.

D'Alembert's Principle

It states that the inertia forces and couples, and the external forces and torques on a body together give statical equilibrium.

Inertia is a property of matter by virtue of which a body resists any change in velocity.

Inertia force $F_I = -m f_g$.

Where,

m = mass of the body

f_g = acceleration of centre of mass of the body.

The negative sign indicates that the force acts in the opposite direction to that of the acceleration. The forces acts through the centre of mass of the body.

(or) R.S.K 516 The resultant force acting on a body together with the reversed effective force (or inertia force) are in equilibrium.

Superposition Principle S.S.R - 4.14

In linear systems, if a number of loads act on a system of forces, the net effect is equal to the superposition of the effects of the individual loads taken at a time.

A linear system is one in which the output force is directly proportional to the input force. (i.e) in mechanisms where Coulomb or dry friction is neglected.

UNIT II

FORCE ANALYSIS

S.S.R-410

In all types of machinery, forces are transmitted from one Component to the other such as from a belt to a pulley, from a brake drum to a brake shoe, from a gear to shaft.

In the design of machine mechanisms, it is necessary to know the magnitudes as well as the directions of forces transmitted from input to the output.

Constraint and applied forces.

Constraint forces: A pair of action and reaction forces which Constrain two Connected bodies to behave in a particular manner depending upon the nature of Connection are known as Constraint forces.

Applied forces: Forces acting from outside on a system of bodies are called applied forces.

The forces like electric, magnetic and gravitational forces are considered as applied forces.

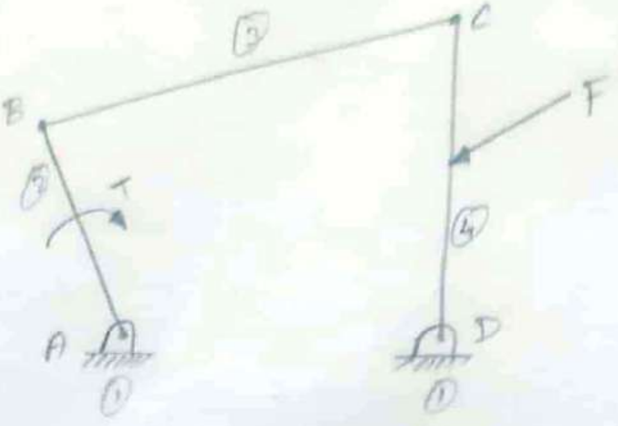
Static force analysis: Magnitude of the inertia forces are small when compared to externally applied loads.

Ex: Cranes.

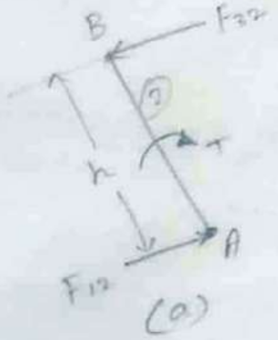
Dynamic force analysis: Inertia effect due to the mass of the components is also considered.

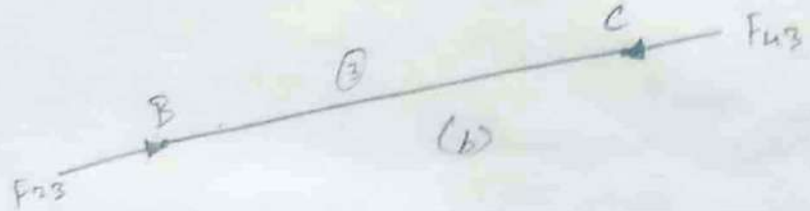
Free body diagrams: SSR-413

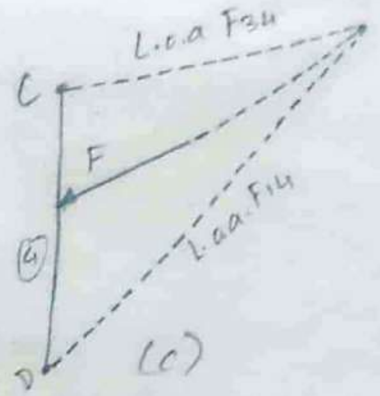
A free body diagram is a sketch or diagram of a part isolated from the mechanism in order to determine the nature of forces acting on it.



F.B.D







$F_{34} = F_{43} = F_{23} = F_{32}$

$T = F_{12} \times h = F_{32} \times h.$