

# Kinematic Chain:

Relationship for a kinematic chain having

Lower pairs:  $SS - 4$

$$N = 2P - 4 \quad \dots \rightarrow (1)$$

$$J = \frac{3N}{2} - 2 \quad \dots \rightarrow (2)$$

where

$N =$  no. of links

$P =$  no. of pairs

$J =$  no. of joints

Conditions:

- (i)  $LHS > RHS$  - Chain is locked
- (ii)  $LHS = RHS$  - Chain is constrained
- (iii)  $LHS < RHS$  - Chain is unconstrained

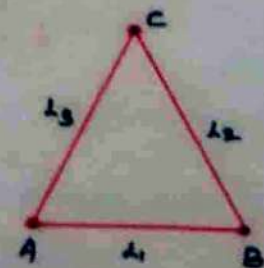
Problems:

- (1) Find the type of kinematic chain for the given arrangement of three links: R.S.K - 99

No. of links ( $l$ ) = 3

No. of joints ( $J$ ) = 3

No. of pairs ( $P$ ) = 3



$$l = 2P - 4$$

$$3 = 2(3) - 4$$

$$3 = 2$$

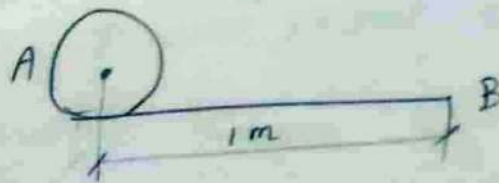
$$\Rightarrow LHS > RHS$$

## VELOCITY AND ACCELERATION ANALYSIS

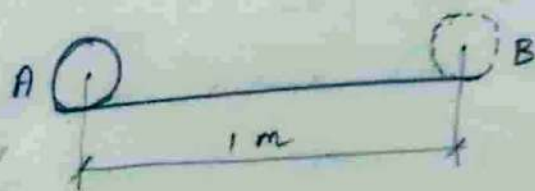
Analysis of mechanisms is the study of motions and forces concerning their different parts. The study of velocity analysis involves the linear velocities of various points on different links of a mechanism as well as the angular velocities of the links.

Linear Displacement: (R.S.K. 09)

It can be defined as the distance travelled or moved by a body with respect to a fixed point. The displacement may be along a straight line or a curved path.



Object at rest A



Object moved to point B.

Linear Velocity:

It can be defined as the rate of change of linear displacement of a body with respect to the time. Therefore velocity is always expressed in a particular direction. It is a vector quantity.

$$v = \frac{ds}{dt}$$

s - Displacement

t - Time

Linear Acceleration: (R.S.K. 09)

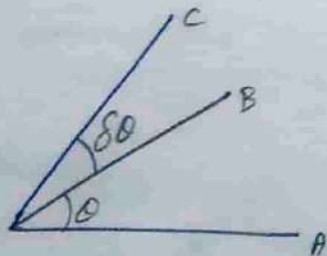
It may be defined as, the rate of change of linear velocity of a body with respect to time. It is also a vector quantity.

$$a = \frac{dv}{dt} = \frac{d}{dt} \left( \frac{ds}{dt} \right) = \frac{d^2s}{dt^2} \quad \left| \quad v = \frac{ds}{dt} \right.$$

The negative acceleration is known as deceleration or retardation.

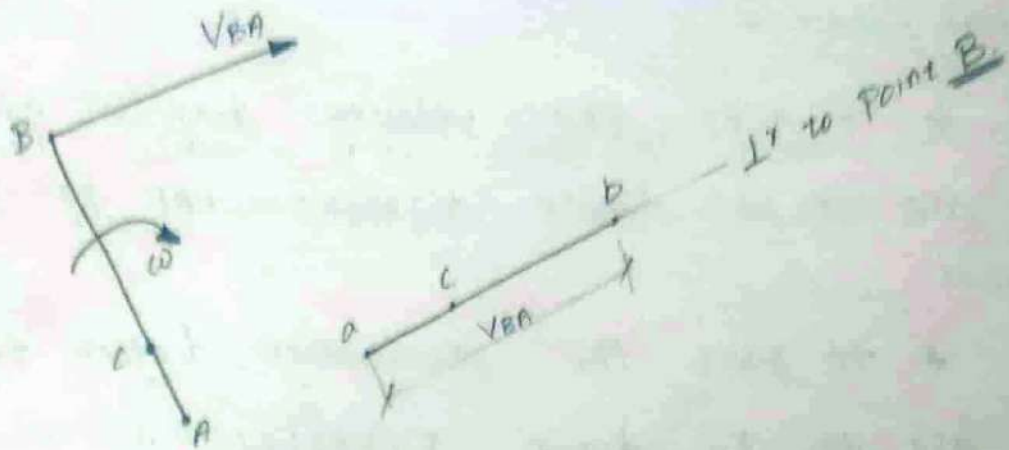
Angular Displacement: (R.S.K. 17)

It may be defined as the angle described or traced by a particle from one point to another point (or) from one position to another position.



$\delta\theta$  - Angular displacement

# Motion of a link (R.S.K-145) (for velocity)



Consider two points A and B on a rigid link AB. as  
 Let one of the point (B) of the link move relative  
 to A in a C.W direction.

The distance from A to B remains same, therefore  
 there can be no relative motion between A and B  
 along the line AB. It is obvious that, the relative  
 motion of B with respect to point A must be perpendicular  
 to AB.

Velocity of any point on a link w.r to another  
 point on the same link is always perpendicular to  
 the line joining these points on the Configuration  
 (or) Space diagram.

1-42

# Acceleration analysis: (R.S.K 174)

## General Condition

\* To plot the velocity diagram the velocity links are to be drawn PERPENDICULAR to the space diagram.

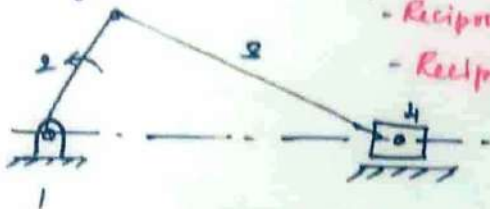
\* To plot the acceleration diagram the acceleration links are to be drawn PARALLEL to the space diagram.

Acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following components.

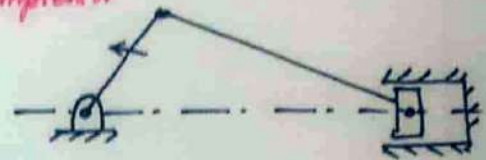
A slider crank chain has the following inversions:

(i) First Inversion:

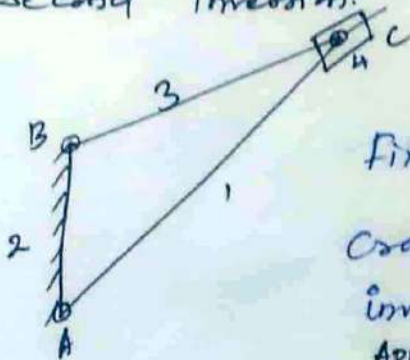
This inversion is obtained when link 1 is fixed & links 2 & 4 are made the crank & slider respectively.



APP:  
 - Reciprocating engine  
 - Reciprocating compressor



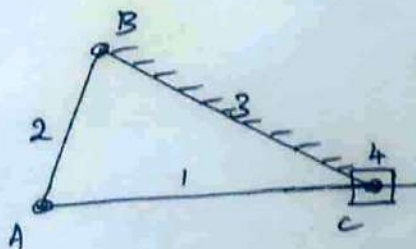
(ii) Second Inversion:



Fixing the link 2 of a slider crank chain gives the second inversion

APP:  
 - Whitworth quick return mechanism  
 - Rotary engine

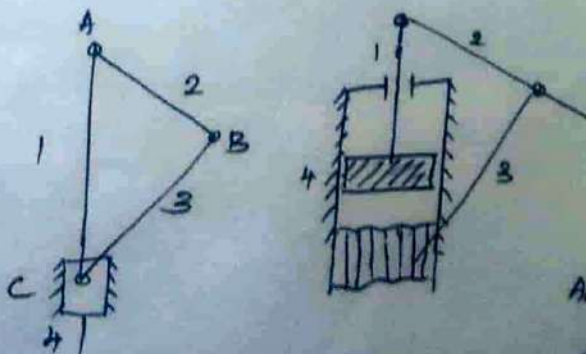
(iii) Third Inversion:



By fixing the link 3 of a slider crank mechanism, the third inversion is obtained.

APP:  
 - Oscillating cylinder engine  
 - Crank & slotted lever mechanism

(iv) Fourth Inversion:



If the link 4 of the slider crank mechanism is fixed, the fourth inversion is obtained.

APP:  
 - Hand pump

## Mechanism :

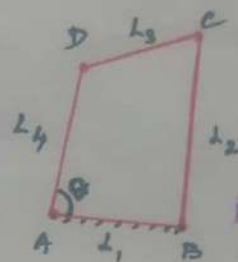
If a number of bodies are assembled in such a way that the motion of one body causes constrained & predictable motion to the other body, is known as mechanism.

The function of a mechanism is to transmit and modify a motion.

### (i) Simple Mechanism:

A mechanism with four links is known as

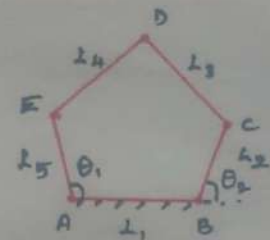
Simple mechanism



Four bar chain

### (ii) Compound Mechanism:

A mechanism with more than four links is known as Compound mechanism.

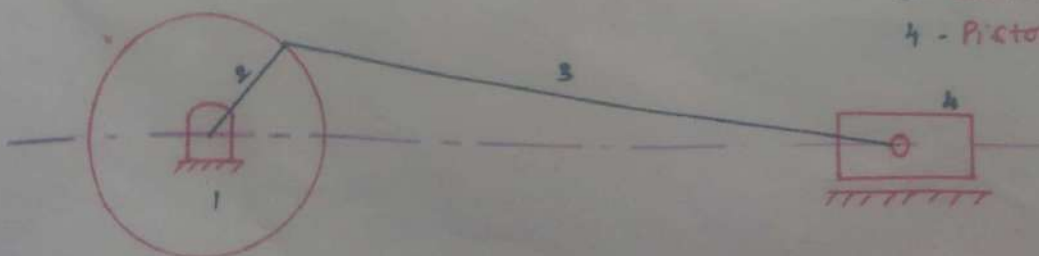


Five bar chain.

## Machine :

A machine is a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits & modifies the available energy into some kind of desired work.

- 1 - Fixed link
- 2 - Crank
- 3 - connecting rod
- 4 - Piston.



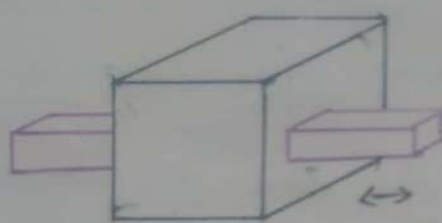
## Types of Constrained Motion:

There are three types of constrained motion

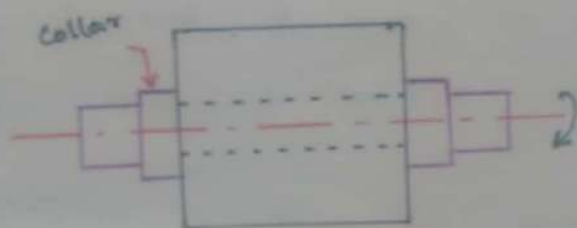
- (i) Completely constrained motion
- (ii) Incompletely constrained motion
- (iii) Successfully constrained motion

### (i) Completely constrained Motion:

When the motion between two elements of a pair is in a definite direction irrespective of the direction of the force applied, it is known as completely constrained motion.



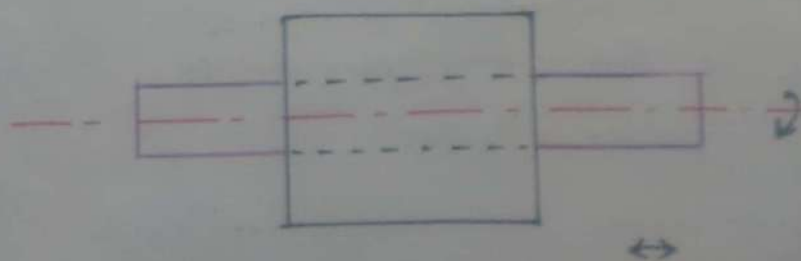
Sliding pair



Turning pair.

### (ii) Incompletely constrained motion:

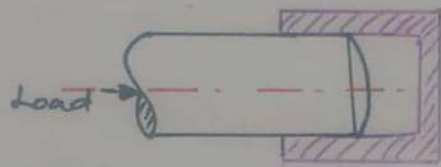
When the motion between the two elements of a pair is possible in more than one direction and depends upon the direction of force applied, it is known as incompletely constrained motion.





### (iii) Successfully Constrained Motion:

When the motion between the two elements of a pair is possible in more than one direction but is made to have motion in only one direction by using some external means (or force), it is known as successfully constrained motion.



Here, rotary & linear motion is possible. But linear is restricted, only rotary motion is possible.  
Ex: Foot step bearing.

### Rigid & Resistant bodies:

A body is said to be rigid if under the action of forces, it doesn't suffer any distortion.

In a mechanism, a link or element need not to be a rigid, it must be resistant body. If a body is capable of transmitting the required force with negligible deformation, then the body is said to be resistant body.

### Kinematic link (or) element:

Each part of the machine which moves to some other part, is known as kinematic link or element. A link may consist of several parts which are rigidly fastened together.

Types of link:

Rigid link

Flexible link - Belt, ropes, chains

Fluid link - Hydraulic press, Jack

Kinematic pair:

Two links or elements of a machine, when in contact with each other are said to form a pair.

If the relative motion between them is completely or successfully constrained, then the pair is said to be kinematic pair.

Kinematic chain:

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion, it is called kinematic chain.

Structure:

It is an assemblage of number of resistant bodies having no relative motion between them & meant for carrying loads having restraining action.

Ex:

Railway bridge

Roof Truss

Machine frames

## Machine

Parts move relative to each other

Transforms available energy into useful work

Links of a machine may transmit both power & motion.

## Structure

Parts (or member of a structure) do not move

No energy transformation.

Members of a structure transmit force only.

## Classification of kinematic pairs

\* According to the type of relative motion:

- a) Sliding pair
- b) Turning pair
- c) Rolling pair
- d) screw pair
- e) Spherical pair

\* According to the type of contact:

- (a) Lower pair
- (b) Higher pair

\* According to the type of closure:

- (a) self closed pair
- (b) force-closed pair

## Types of joints:

- \* Binary joints
- \* Ternary joints
- \* Quaternary joints

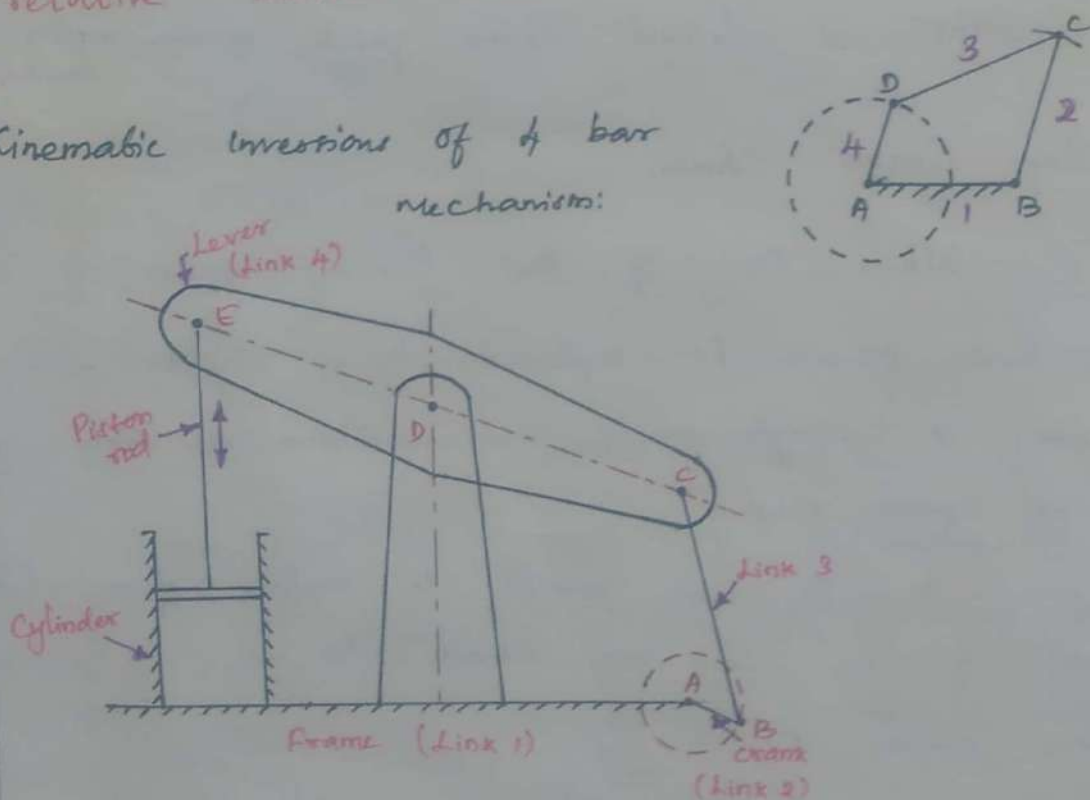
# Inversion of Mechanisms:

The method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as inversion of mechanism.

## Grashof's law:

For a four bar mechanism, the sum of the shortest & longest link lengths should not be greater than the sum of the remaining two links if there is to be continuous relative motion between the two links.

## Kinematic Inversions of 4 bar mechanism:



In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about a fixed centre D.

The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank.

This mechanism is to convert rotary motion into reciprocating motion.

Inversions of single slider crank chain:

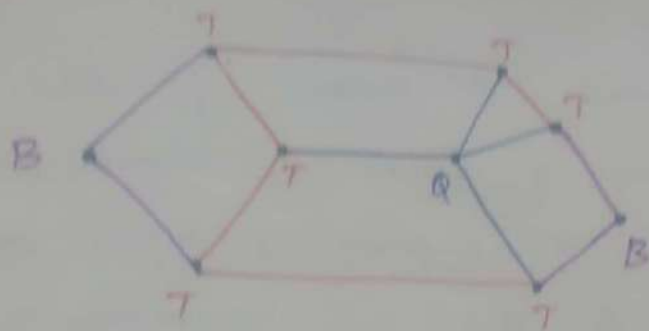
Types:

- \* Pendulum (or) Bull Engine
- \* Oscillating cylinder engine
- \* Rotary internal combustion engine (or) Gnome engine
- \* Crank & slotted lever quick return motion mechanism

Slider Crank Chain:

When one of the turning pairs of a four-bar chain is replaced by a sliding pair, it becomes a single slider-crank chain (or) simply a slider crank chain.

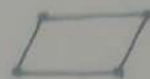
It is also possible to replace two sliding pairs of a four bar chain to get a double slider crank chain.



B- Binary link



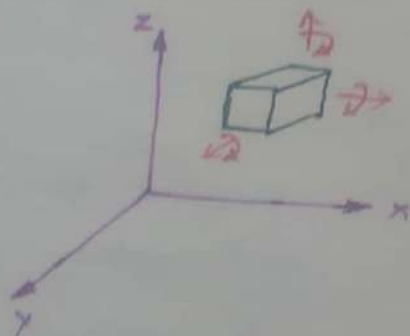
T- Ternary link



Q- Quaternary link

Degrees of Freedom:

It can be defined as the number of independent relative motions, both translational & rotational, a pair have.



6- D.O.F

Redundant chain does not allow any motion of a link relative to other.

A linkage is obtained if one of the links of a kinematic chain is fixed.

If one of the links of a redundant chain is fixed, it is known as structure or locked system.

The d.o.f of a locked system is zero.

A structure with negative d.o.f is known as a super structure.

Kutzbach criterion:

A mechanism with  $l$  number of links connected by  $j$  number of joints or lower pairs or binary joints and  $h$  number of higher pairs, then the number of d.o.f of mechanism is given by

$$n = 3(l-1) - 2j - h$$

$n$  = No. of d.o.f

$l$  = links

$j$  = Joints

$h$  = Higher pairs.

This equation is called Kutzbach criterion for the movability of a mechanism having plane motion.

Grubler's Criterion:

Grubler's criterion applies to mechanisms with only single d.o.f joints where the overall movability of the mechanism is unity.

Substituting  $n=1$  &  $h=0$  in Kutzbach criterion then

$$1 = 3(l-1) - 2j \quad \text{or} \quad 3l - 2j - 1 = 0$$

This equation is known as Grubler's criterion for the plane mechanisms with constrained motion.

$$J = \frac{3}{2} l - 2$$

$$3 = \frac{3}{2} (3) - 2$$

$$3 = 2.5 \Rightarrow \text{L.H.S} > \text{R.H.S.}$$

The given set of links belongs to locked chain.

- No relative motion is possible, as left hand side is greater than Right hand side - it is not a kinematic chain.

(2)

$$\text{No. of links } (l) = 4.$$

$$\text{No. of pairs } (p) = 4.$$

$$\text{No. of joints } (j) = 4.$$

$$l = 2p - 4.$$

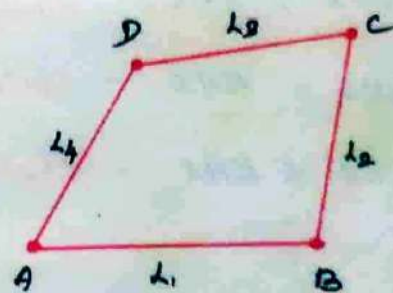
$$4 = 2(4) - 4$$

$$4 = 4 \Rightarrow \text{L.H.S} = \text{R.H.S.}$$

$$j = \frac{3}{2} l - 2$$

$$4 = \frac{3}{2} (4) - 2$$

$$4 = 4 \Rightarrow \text{R.H.S} = \text{L.H.S}$$



The given set of links belongs to constrained chain as it satisfies both the conditions it belongs to a kinematic chain.