They are said to be inhomogeneous when ε depend on the space coordinates.

Materials for which D and E are in the same direction are said to be isotropic.

For conductors:

The same idea holds for a conducting material in which $J = \sigma E$ applies. The material is linear if σ does not vary with E, homogeneous if σ is same at all points, and isotropic if σ does not vary with direction.

Boundary Conditions:

If the electric field exists in the region consisting of two different media, the conditions that the field must satisfy at the interface separating the media are called boundary conditions.

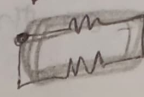
Boundary conditions at an interface separating

- Dielectric (ε_{r1}) & Dielectric (ε_{r2})
- Conductor & dielectric
- Conductor & free space

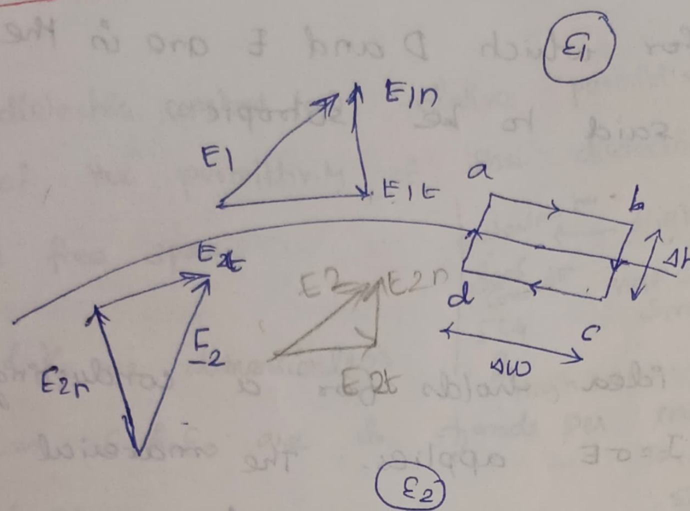
require high capacitance to store charge.

~~1) Conductor~~

Voltage is same in closed path
V_L



1) Dielectric - Dielectric boundary conditions:



Dielectric - Dielectric boundary $(E_{1t} = E_{2t})$

The fields E_1 and E_2 are decomposed into two components

$$E_1 = E_{1t} + E_{1n}$$

$$E_2 = E_{2t} + E_{2n}$$

$V=0$

Apply $\oint E \cdot dl = 0$ to the closed path abcda,

$$\therefore \quad E_{1t}\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w + E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2} = 0$$

$$E_{1t}\Delta w - E_{2t}\Delta w = 0$$

$$(E_{1t} - E_{2t})(\Delta w) = 0$$

$$\boxed{E_{1t} = E_{2t}} \longrightarrow (1)$$

* Thus the tangential components of E are the same on the two sides of the boundary.

In other words, E_t undergoes no change on the boundary and it is said to be continuous across the boundary.

WKT $D = \varepsilon E$

$\therefore$ $E_{1t} = E_{2t} \Rightarrow \frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$ —

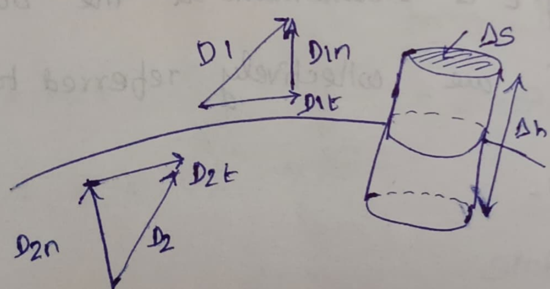
$$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$$

$$\frac{D_{1t}}{D_{2t}} = \frac{\varepsilon_1}{\varepsilon_2}$$

D_t undergoes some change across the interface.

D_t is said to be discontinuous across the interface.

ε_1 ①



② ε_2

Consider the pillbox (cylindrical gaussian surface)

Apply $\oint_s D \cdot ds = Q$

$D_{1n} \Delta s - D_{2n} \Delta s = \rho_s \Delta s$

The contribution due to sides vanishes.

[Allowing $\Delta h \to 0$]

$$(D_{1n} - D_{2n})\,\Delta S = \rho_s\,\Delta S$$

$$D_{1n} - D_{2n} = \rho_s \longrightarrow (2)$$

ρ_s is the charge density placed at the boundary.

If no free charges exist at the interface

$\rho_s = 0$.

$$\therefore \boxed{D_{1n} = D_{2n}} \longrightarrow (3)$$

* Thus the normal component of D is continuous across the interface. $\therefore$ D_n undergoes no change at the boundary

WKT $D = \varepsilon E$

$$\therefore \varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

$$\frac{E_{1n}}{E_{2n}} = \frac{\varepsilon_2}{\varepsilon_1}$$

* normal comp of E is discontinuous at the boundary.

Eqn (1), (2) and (3) are collectively referred to as boundary conditions.

Applied in

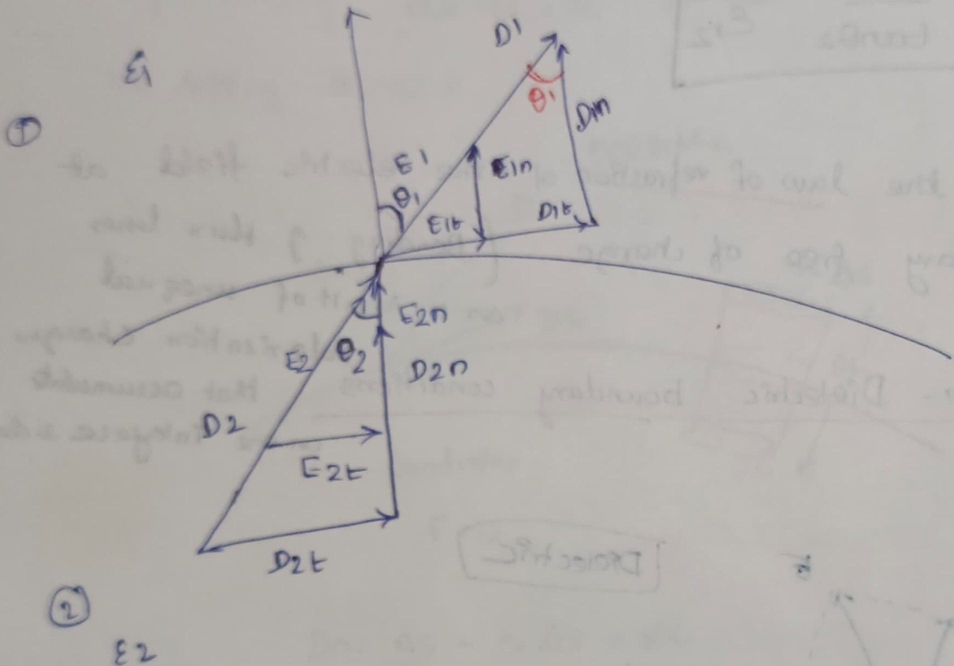
* finding the electric field on one side of the boundary given the field on the other side.

* To Determine the "refraction" of the electric field across the interface.

Consider D_1 or E_1 and D_2 or E_2 making angles θ_1 an... 2

with the normal to the interface.

Refraction of D or E at a dielectric - dielectric boundary



from figure

from ① $E_{1t} = E_{2t}$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2 \rightarrow ④$$

$$\sin\theta_1 = \frac{E_{1t}}{E_1}$$

$$\sin\theta_2 = \frac{E_{2t}}{E_2}$$

from eqn ③

$$D_{1n} = D_{2n}$$

$$\cos\theta_1 = \frac{E_{1n}}{E_1}$$

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

$$\cos\theta_2 = \frac{E_{2n}}{E_2}$$

$$\varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2 \rightarrow ⑤$$

Dividing eq ④ by ⑤ gives

$$\frac{\cancel{E_1} \sin\theta_1}{\varepsilon_1 \cancel{E_1} \cos\theta_1} = \frac{\cancel{E_2} \sin\theta_2}{\varepsilon_2 \cancel{E_2} \cos\theta_2}$$

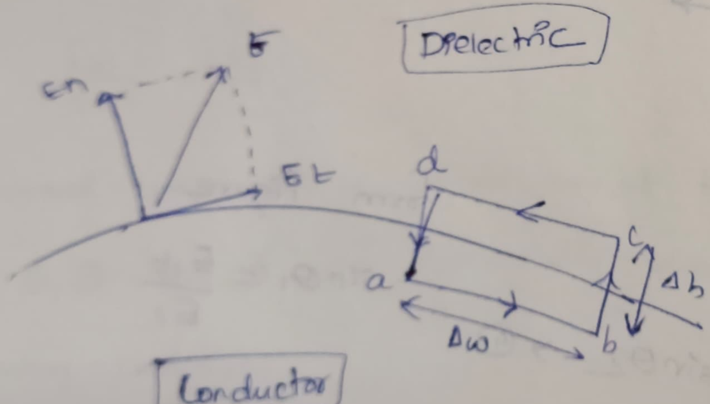
$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_0\varepsilon_{r1}}{\varepsilon_0\varepsilon_{r2}}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$$

This is the law of refraction of the electric field at a boundary free of charge. (Bending of flux lines as a result of unequal polarization charges that accumulate on the interface sides)

2) Conductor - Dielectric boundary conditions.



Conductor - dielectric boundary

The conductor is assumed to be perfect i.e $\sigma \to \infty$, or $\rho_c \to 0$

Applying $\oint E \cdot dl = 0$ to the closed path abcda gives,

$$\int_{a}^{b} E \cdot dl + \int_{b} = 0$$

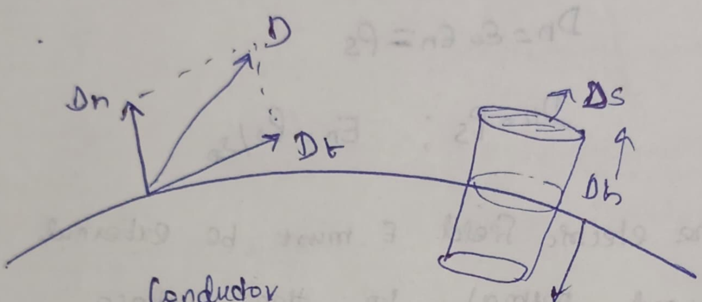
$$0 \cdot \Delta w + 0 \cdot \frac{\Delta h}{2} + E_n \frac{\Delta h}{2} - E_t \Delta w - E_n \frac{\Delta h}{2} - 0 \cdot \frac{\Delta h}{2} = 0$$

$$E_t \cdot \Delta w = 0$$

Tangential comp of E is zero at the boundary.

$$\boxed{E_t = 0} \rightarrow (1)$$

By applying $\oint D \cdot ds = Q$ to the pill box and setting $\Delta h \rightarrow 0$,



$$D_n \cdot \Delta S - 0 \cdot \Delta S = \Delta Q$$

Since $D = \varepsilon E = 0$ inside the conductor.

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_s$$

$$D_n = \rho_s \rightarrow (2)$$

And also, $E_n = \frac{\rho_s}{\varepsilon}$

$$\boxed{D_t = 0}$$

* No electric field may exist within a conductor, $\rho_v = 0$ and $E = 0$
* An electric field E must be external to the conductor and must be normal to the surface

$D_t = 0$, $D_n = \rho_s = \varepsilon E_n$

An important application of the fact that $E=0$ inside a conductor is in electrostatic screening or shielding.

3) Conductor - free space boundary conditions:

replacing ε_r by 1 (for free space)

Boundary conditions are

$$D_t = \varepsilon_0 E_t = 0$$

$$D_n = \varepsilon_0 E_n = \rho_s$$

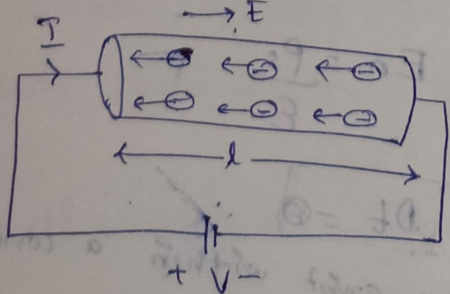
$$D_n = \rho_s \; ; \quad E_n = \rho_s / \varepsilon_0$$

The electric field E must be external to the conductor and normal to the surface.

Resistance and Capacitance

Resistance

A conductor of uniform cross section under an applied E field



Uniform cross section of area - S

Length - l