SNS COLLEGE OF TECHNOLOGY
Coimbatore – 35

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

16EC401 / Wireless Communication

IV ECE/ VII SEMESTER

Unit IV **- MULTIPATH MITIGATION TECHNIQUES**

Topic 3 : Zero forcing Algorithm, LMS Algorithms

Algorithms for Adaptive Equalization

Practical considerations for choice of an equalizer structure and its algorithm

- The cost of the computing platform (affordable or not?)
  - Especially when used in user equipments
- The power budget (power limited applications or else?)
  - In portable radio applications, battery drain at the subscriber unit is a paramount consideration
- The radio propagation characteristics (fast fading & time delay spread?)
  - The speed of the mobile unit determines the channel fading rate and the Doppler spread, which is directly related to the coherence time of the channel

Three basic equalization methods

Linear equalization (LE):

Performance is not very good when the frequency response of the frequency selective channel contains deep fades.

Zero-forcing algorithm aims to eliminate the intersymbol interference (ISI) at decision time instants (i.e. at the center of the bit/symbol interval).

Least-mean-square (LMS) algorithm

Recursive least-squares (RLS) algorithm offers faster convergence, but is computationally more complex than LMS (since matrix inversion is required).

zero-forcing algorithm

Criterion:

To force the samples of the combined channel and equalizer impulse response to zero at all but one of sample points in the tapped delay line filter.

Disadvantage:

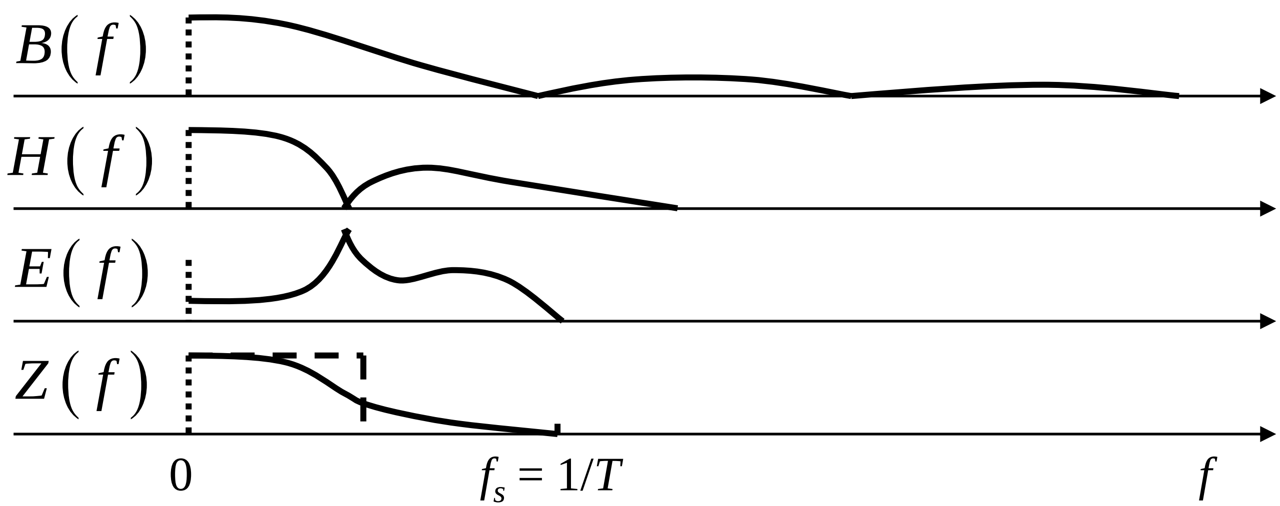
May excessively amplify noise at frequencies where the folded channel spectrum has high attenuation.

Suitability: Wire line communications

Linear equalization, zero-forcing algorithm

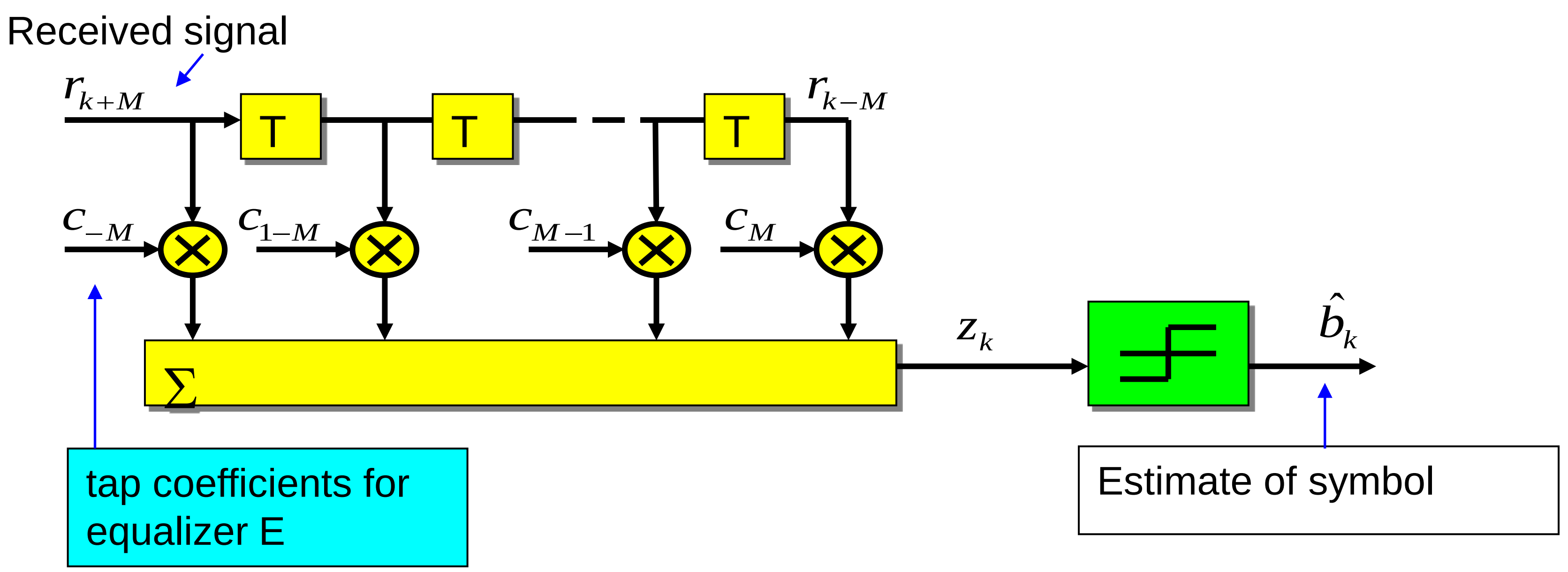
Basic idea: $$Z(f) = B(f)H(f)E(f)$$

Raised cosine spectrum	=	Transmitted symbol spectrum	Channel frequency response (incl. T & R filters)	Equalizer frequency response



Zero-Forcing Equalization

- Zero ISI at the receiver output
- B(f)H(f)E(f)=Z(f)
- Z(f): Nyquist spectrum e.g. raised cosine



Zero-forcing equalizer

Transmitted impulse sequence → Communication channel → $r(k)$ → Equalizer → $z(k)$ → Input to decision circuit

Overall channel

Communication channel: FIR filter contains $2N+1$ coefficients

Equalizer: FIR filter contains $2M+1$ coefficients

Channel impulse response

$$h(k) = \sum_{n=-N}^{N} h_n \, \delta(k-n)$$

Equalizer impulse response

$$c(k) = \sum_{m=-M}^{M} c_m \, \delta(k-m)$$

Coefficients of equivalent FIR filter

$$f_k = \sum_{m=-M}^{M} c_m h_{k-m} \quad (-M \le k \le M)$$

(in fact the equivalent FIR filter consists of $2M+1+2N$ coefficients, but the equalizer can only “handle” $2M+1$ equations)

Zero-forcing equalizer

Overall filter response to be non-zero at decision time $k = 0$ and zero at all other sampling times $k \neq 0$:

$$f_k = \sum_{m=-M}^{M} c_m h_{k-m} = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

This leads to a set of $2M+1$ equations:

$$
\begin{aligned}
h_0 c_{-M} + h_{-1} c_{-M+1} + \ldots + h_{-2M} c_M &= 0 \quad (k = -M) \\
h_1 c_{-M} + h_0 c_{-M+1} + \ldots + h_{-2M+1} c_M &= 0 \\
&\vdots \\
h_M c_{-M} + h_{M-1} c_{-M+1} + \ldots + h_{-M} c_M &= 1 \quad (k = 0) \\
&\vdots \\
h_{2M-1} c_{-M} + h_{2M-2} c_{-M+1} + \ldots + h_{-1} c_M &= 0 \\
h_{2M} c_{-M} + h_{2M-1} c_{-M+1} + \ldots + h_0 c_M &= 0 \quad (k = M)
\end{aligned}
$$

Activity

- Imagine folding a paper in half once
- Then take the result and fold it in half again; and so on
- How many times can you do that?

Least-mean-square (LMS) algorithm

<u>Criterion:</u>

- To minimize the mean square error (MSE) between the desired equalizer output and the actual equalizer output. $\xi = E[e^{*}_{k} \cdot e_{k}]$
- Minimize must be solved iteratively
- Simplest algorithm, requires only 2N + I operations per iteration.
- The LMS equalizer maximizes the signal to distortion ratio at its output within the constraints of the equalizer filter length.
- A step size α is used to control the convergence rate and the stability

Least-mean-square (LMS) algorithm

Disadvantage: Low convergence rate because of the only one parameter α.

Especially when the eigen values of the input covariance matrix R_{NN} have a very large spread, i.e, $\lambda_{max} / \lambda_{min} >> 1$

- To prevent the adaptation from becoming unstable, the value of a is chosen from $$0 < \alpha < 2 / \sum_{i=1}^{N} \lambda_i$$

Where λ_i is the ith eigenvalue of the covariance matrix R_{NN}.

- The step size a can be controlled by the total input power in order to avoid instability in the equalizer $$\sum^{N} \lambda_i = y_N^T(n) y_N(n)$$

Least-mean-square (LMS) algorithm

For convergence towards minimum mean square error (MMSE)

Real part of n:th coefficient:

$$\mathrm{Re}\{c_n(i+1)\} = \mathrm{Re}\{c_n(i)\} - \Delta \frac{\partial |e_k|^2}{\partial[\mathrm{Re}\{c_n\}]}$$

Imaginary part of n:th coefficient:

$$\mathrm{Im}\{c_n(i+1)\} = \mathrm{Im}\{c_n(i)\} - \Delta \frac{\partial |e_k|^2}{\partial[\mathrm{Im}\{c_n\}]}$$

$$|e_k|^2 = e_k e_k^*$$

$2(2M+1)+1$ equations

Phase:

$$\phi(i+1) = \phi(i) - \Delta_\phi \frac{\partial |e_k|^2}{\partial \phi}$$

Iteration index

Step size of iteration

Least-mean-square (LMS) algorithm

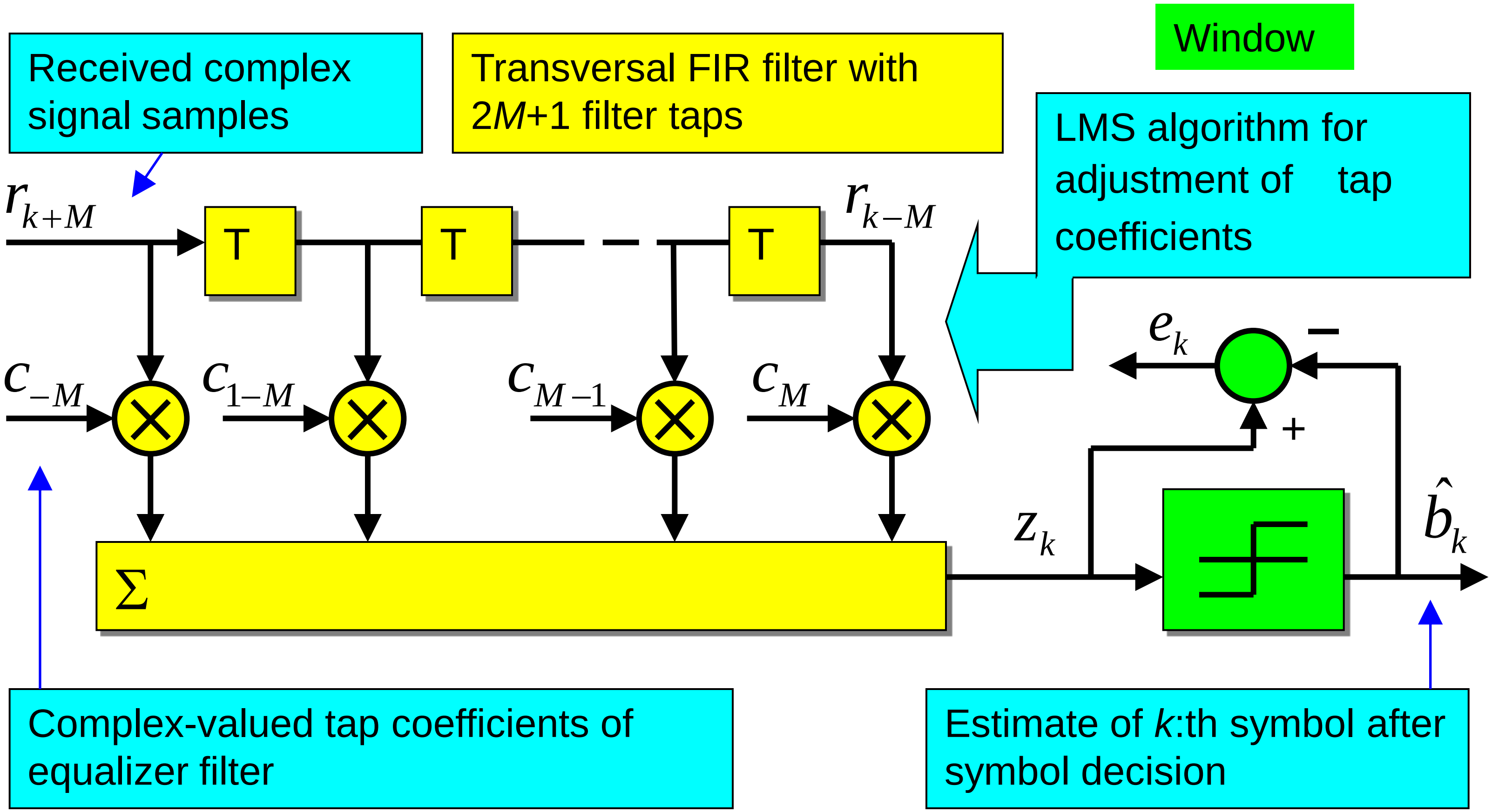
After some calculation, the recursion equations are obtained in the form

$$\mathrm{Re}\left\{c_n(i+1)\right\} = \mathrm{Re}\left\{c_n(i)\right\} - 2\Delta\,\mathrm{Re}\left\{\left(e^{j\phi}\sum_{m=-M}^{M} c_m\, r_{k-m} - \hat{b}_k\right) r_{k-n}^{\,*}\, e^{-j\phi}\right\}$$

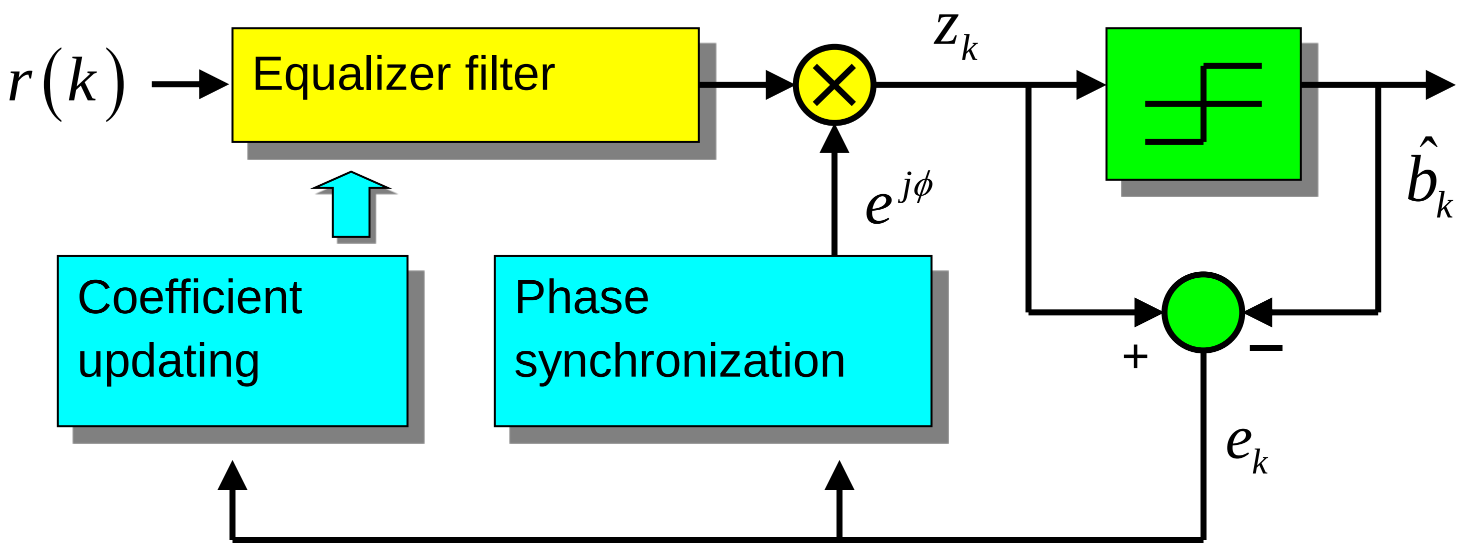
$$\mathrm{Im}\left\{c_n(i+1)\right\} = \mathrm{Im}\left\{c_n(i)\right\} - 2\Delta\,\mathrm{Im}\left\{\underbrace{\left(e^{j\phi}\sum_{m=-M}^{M} c_m\, r_{k-m} - \hat{b}_k\right)}_{e_k} r_{k-n}^{\,*}\, e^{-j\phi}\right\}$$

$$\phi(i+1) = \phi(i) - 2\Delta_\phi\,\mathrm{Im}\left\{\hat{b}_k^{\,*} e^{j\phi}\sum_{m=-M}^{M} c_m\, r_{k-m}\right\}$$

Conventional linear equalizer of LMS type



Joint optimization of coefficients and phase



Minimize:

$$J = E\left|e_k\right|^2 \qquad e_k = z_k - \hat{b}_k = \left(\sum_{m=-M}^{M} c_m\, r_{k-m}\right)\exp(j\phi) - \hat{b}_k$$

Effect of iteration step size

smaller ← Δ_{ϕ} → larger

smaller	larger
Slow acquisition	Poor stability
Poor tracking performance	Large variation around optimum value

Convergence condition

$$0<\Delta<2/\lambda_{max}$$

λ : eigenvalue of autocorrelation matrix of **r**

Assessment

- **The decision feedback equalizer has a linear traversal filter which is**
  a) Feed forward section
  b) Feedback section
  c) Both of the mentioned
  d) None of the mentioned

- **Choice of equalizer structure and its algorithm is not dependent on ________**
  a) Cost of computing platform
  b) Power budget
  c) Radio propagation characteristics
  d) Statistical distribution of transmitted power

THANK YOU