Principle of Superposition

To determine the force on a particular charge, when we have more than two point charges.

It states that if there are N charges $Q_1, Q_2, \ldots Q_N$ located respectively, at points with position vectors $r_1, r_2 \ldots r_N$, the resultant force F on a charge Q located at point r is the vector sum of the forces exerted on Q by each of the charges $Q_1, Q_2 \ldots Q_N$.

$$\vec{F} = \frac{Q Q_1 \vec{R_r}}{4\pi\varepsilon_0 |\vec{R_1}|^3} + \frac{Q Q_2 (\vec{R_2})}{4\pi\varepsilon_0 |\vec{R_2}|^3} + \cdots + \frac{Q Q_N (\vec{R_N})}{4\pi\varepsilon_0 |\vec{R_N}|^3}$$

or

$$\vec{F} = \frac{Q}{4\pi\varepsilon_0} \sum_{k=1}^{N} \frac{Q_k (\vec{R_k})}{|\vec{R_k}|^3}$$

$$\frac{(r - r_k)}{|r - r_k|^3}$$

Electric field Intensity

Electric field intensity (or electric field strength) E is the force per unit charge when placed in an electric field.

$$E = \frac{F}{Q}$$

$$\vec{E} = \frac{\vec{F}}{Q}$$

$\vec{E}$ is obviously in the direction of the force $\vec{F}$.

<u>Unit:</u> Newtons per Coulomb

or Volts per meter

Electric field intensity at point r due to a point charge located at r'

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R \quad \text{or} \quad \frac{Q(r-r')}{4\pi\varepsilon_0 |r-r'|^3}$$

For N point charges $Q_1, Q_2 \ldots Q_N$ located at $r_1, r_2 \ldots r_N$, the electric field intensity at point r is

$$\vec{E} = \frac{Q_1 \vec{R_1}}{4\pi\varepsilon_0 |\vec{R_1}|^3} + \frac{Q_2 \vec{R_2}}{4\pi\varepsilon_0 |\vec{R_2}|^3} + \cdots + \frac{Q_N (\vec{R_N})}{4\pi\varepsilon_0 |\vec{R_N}|^3}$$

(or)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{N} \frac{Q_k \vec{R_k}}{|\vec{R_k}|^3} \qquad \boxed{\frac{(r-r_k)}{|r-r_k|^3}}$$

Problems:

1. Point charges 1 mC and -2 mC are located at (3, 2, -1) and (-1, -1, 4) respectively. Calculate the electric force on a 10 nC charge located at (0, 3, 1) and the electric field intensity at that point.

Solution:

$$\vec{F} = \sum_{k=1}^{2} \frac{Q\, Q_k \overbrace{(r-r_k)}^{\vec{R_{12}}}}{4\pi\varepsilon_0 |r-r_k|^3 \; |R_{12}|^3}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{Q_1 (r-r_1)}{|r-r_1|^3} + \frac{Q_2 (r-r_2)}{|r-r_2|^3}\right]$$

If the point 'p' is along the perpendicular bisector of wire then $\alpha_1 = \alpha_2$

Let $\alpha_1 = \alpha_2 = \alpha$

$$\therefore E = \frac{\rho_l}{4\pi\epsilon_0 h} 2\cos\alpha \, \hat{a_x}$$

$$\vec{E} = \frac{\rho_l}{2\pi\epsilon_0 h} \cos\alpha \, \hat{a_x}$$

If the line is infinitely long, then $\alpha = 0$

$$\therefore E = \frac{\rho_l}{4\pi\epsilon_0 h} (2) \hat{a_x}$$

$$\boxed{\vec{E} = \frac{\rho_l}{2\pi\epsilon_0 h} \hat{a_x}}$$

—x—

Electric field due to circular sheet of charge:

Consider a circular sheet of charge of radius 'a' with charge density ρ_s C/m². Let 'p' be a point, 'h' meters from the disc along its axis at which field has to be determined.

Consider a small elemental area.

$$ds = 2\pi r \, dr.$$

Let dE be the field at Point 'p' due to small area ds.

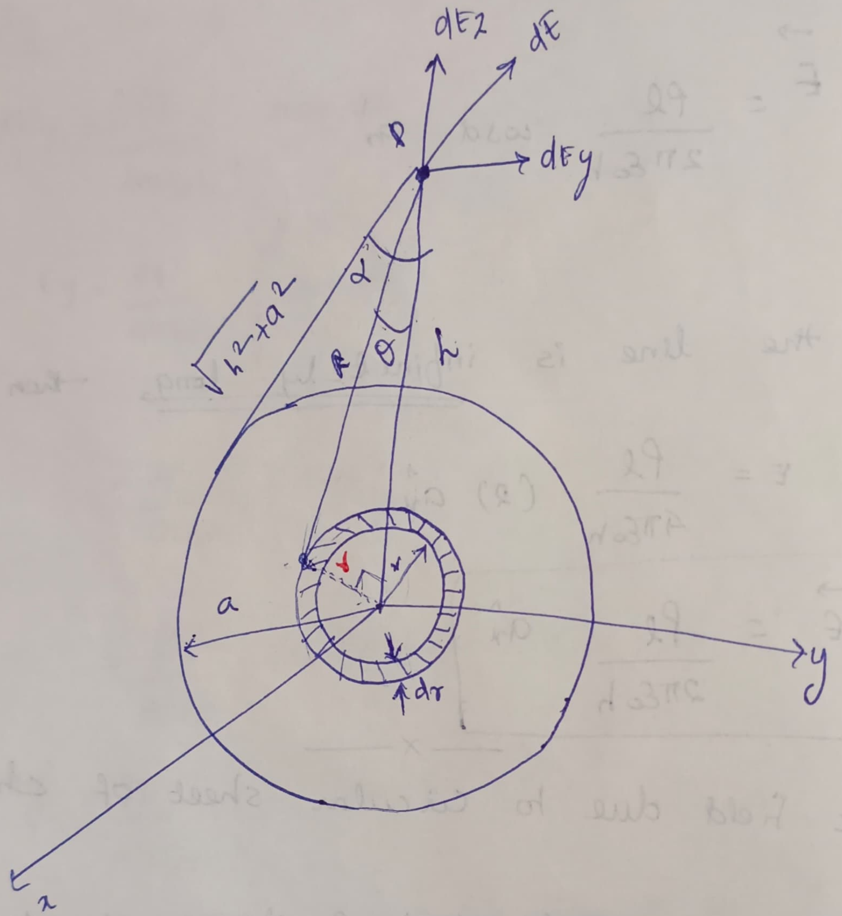
Owing to the symmetry of the charge distribution, for every element 1, there is corresponding element 2 where contribution along a_ρ cancels that of element 1

So E has only z-component.

E_y vanishes.

From figure

$dE_z = dE \cos\theta$



$$\underline{dE} = \frac{\rho_s \times ds}{4\pi \varepsilon_0 R^2} \hat{a}_z$$

$$\therefore dE_z = \frac{\rho_s \times 2\pi \underline{r}\, dr}{4\pi \varepsilon_0 \underline{R}^2} \cos\theta \; \hat{a}_z \rightarrow (1)$$

from fig. $\tan\theta = \frac{r}{h}$

$\boxed{r = h\tan\theta} \Rightarrow \frac{dr}{d\theta} = h\sec^2\theta$

$$\boxed{dr = h \sec^2\theta \, d\theta}$$

from
$$\cos\theta = \frac{h}{R}$$

$$R = \frac{h}{\cos\theta}$$

$$\boxed{R = h\sec\theta}$$

sub in ①

$$dE_z = \frac{\rho_s \times \cancel{2\pi} \times h\tan\theta \times \cancel{h}\sec^2\theta \, d\theta}{\cancel{4\pi}\varepsilon_0 \, h^2 \sec^2\theta}\cos\theta$$

$$= \frac{\rho_s \tan\theta \cos\theta \, d\theta}{2\varepsilon_0}\hat{a_z}$$

$$dE_z = \frac{\rho_s \frac{\sin\theta}{\cancel{\cos\theta}} \cancel{\cos\theta} \, d\theta}{2\varepsilon_0}\hat{a_z}$$

$$\boxed{dE_z = \frac{\rho_s \sin\theta \, d\theta}{2\varepsilon_0}\hat{a_z}}$$

Integrating

$$E = E_z = \int_0^{\alpha} \frac{\rho_s \sin\theta \, d\theta}{2\varepsilon_0}\hat{a_z}$$

$$= \frac{\rho_s}{2\varepsilon_0}\left[-\cos\theta\right]_0^{\alpha}\hat{a_z}$$

$$= \frac{\rho_s}{2\varepsilon_0}(-\cos\alpha + \cos 0)\hat{a_z}$$

$$\boxed{E = \frac{\rho_s}{2\varepsilon_0}\left[1 - \cos\alpha\right]\hat{a_z}}$$

$\sqrt{h^2+a^2}$

$$E = \frac{\rho_s}{2\varepsilon_0}\left[1 - \frac{h}{\sqrt{h^2+a^2}}\right]\hat{a_z}$$

Electric field due to infinite sheet

$\alpha = 90^\circ$

$$\therefore \boxed{E = \frac{\rho_s}{2\varepsilon_0}\hat{a_z}}$$

Electric flux density:

Electric flux: The lines drawn to trace the direction in which a positive test charge will experience force due to the main charge are called the lines of force. These lines of force are known as electric flux which is equal to the charge itself. The symbol is 'ψ' and its unit is Coulombs. The total number of lines of force in any particular E field is called the electric flux. It is represented by the symbol ψ. Unit is Coulomb C.

Electric flux density:

It is given by the ratio between number of flux lines crossing a surface normal to the lines and the surface area.

The direction of $\vec{D}$ is the direction of the flux lines at that point.

$$D = \frac{\psi}{S} \quad \frac{\text{Total flux}}{\text{Surface area}}$$

Units: Coulombs/m^2.

Consider imaginary sphere of radius ... placed at centre