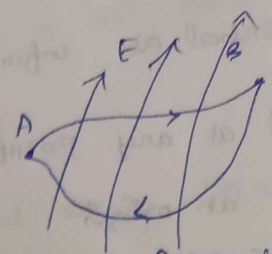
From the definition



Conservative Nature of Electrostatic field

$$V = -\int E \cdot dl$$

$$dV = -E \cdot dl$$

$$dV = -E_x dx - E_y dy - E_z dz$$

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz = -E \cdot dl$$

Comparing two expressions,

$$E_x = \frac{-\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

$$\therefore \boxed{E = -\nabla V}$$

Electric field intensity is the gradient of V.

Negative sign shows that the direction of E is opposite to the direction in which V increases. i.e. E is directed from higher to lower levels of V.

Potential due to different charge distributions.

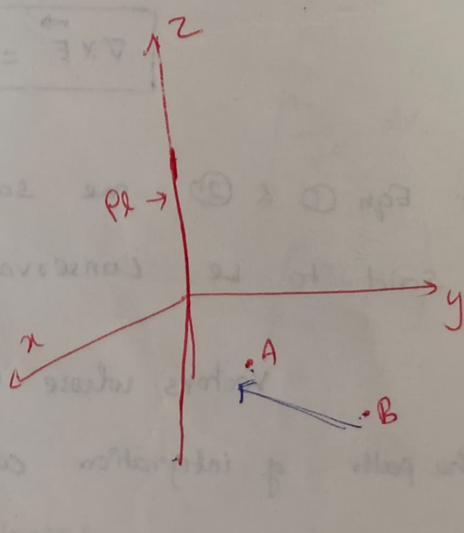
i) Uniformly charged line.

$$V_{ba} = -\int_b^a E \cdot dl$$

$$= -\int_b^a E \cdot dr$$

$$= -\frac{\rho_l}{2\pi\epsilon_0} \int_b^a \frac{1}{r} dr$$

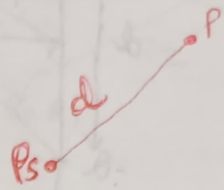
$$= -\frac{\rho_l}{2\pi\epsilon_0} \left[\ln(r)\right]_b^a$$



$$= \frac{-\rho_l}{2\pi\varepsilon_0} (\ln a - \ln b)$$

$$= \frac{\rho_l}{2\pi\varepsilon_0} (\ln b - \ln a)$$

$$\boxed{V_{ba} = \frac{\rho_l}{2\pi\varepsilon_0} \ln(b/a)}$$



ii) Charged disc

$$V = -\int E \cdot dl$$

$$= -\int_d^0 E \cdot dx$$

$$= -\int_d^0 \frac{\rho_s}{2\varepsilon} (1-\cos\alpha) \, dx$$

$$= \frac{-\rho_s}{2\varepsilon} (1-\cos\alpha) \int_d^0 dx$$

$$\int_R^0 dz$$

$$= \frac{-\rho_s}{2\varepsilon} (1-\cos\alpha) \, (0-d)$$

$$\frac{\rho_s}{2\varepsilon}\left[1-\frac{h}{\sqrt{h^2+a^2}}\right] R$$

$$= \frac{\rho_s}{2\varepsilon} (1-\cos\alpha) \, d$$

R

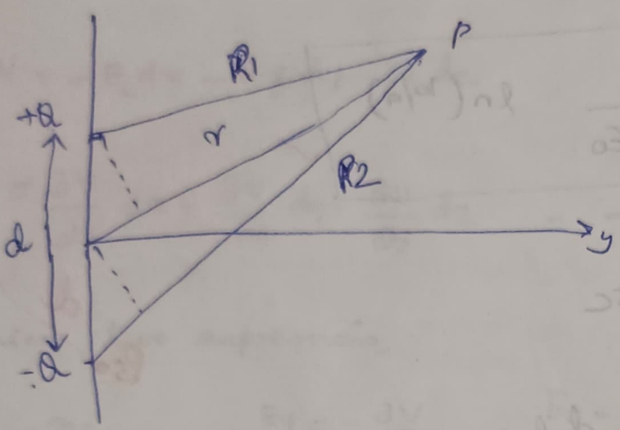
$$= \frac{\rho_s}{2\varepsilon} \left(1 - \frac{h}{\sqrt{h^2+R^2}}\right) \sqrt{h^2+R^2}$$

$$R = \sqrt{h^2+a^2}$$

$$\boxed{V = \frac{\rho_s}{2\varepsilon} \left(\sqrt{h^2+R^2} - h\right)}$$

Potential due to Electric dipole

Equal and opposite point charges separated by a small distance is known as electric dipole.



Potential at P due to +Q is

$$V_1 = \frac{Q}{4\pi\varepsilon_0 R_1}.$$

Potential at P due to -Q is

$$V_2 = \frac{-Q}{4\pi\varepsilon_0 R_2}$$

Total potential at P due to the dipole is

$$V = V_1 + V_2$$

$$= \frac{Q}{4\pi\varepsilon_0 R_1} - \frac{Q}{4\pi\varepsilon_0 R_2}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{R_2 - R_1}{R_1 R_2}\right]$$

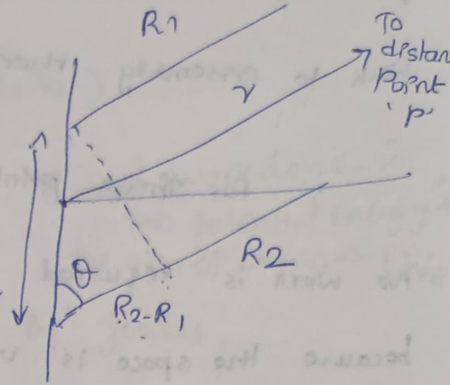
For a distant point

$R_1 = R_2 = r$

$R_1 R_2 = r^2$

(for the numerator) if R_1 & R_2 are assumed to parallel.

then $R_2 - R_1 = d\cos\theta$



$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{d\cos\theta}{r^2}\right)$$

$$\boxed{V = \frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2}}$$

Dipole moment $m = Qd$

$\oplus\!\!-$

x^2
nx^{n-1}
$2x$

$\left[r^{-2} \quad -2r^{-2-1} \quad -2r^{-3} \quad -\frac{2}{r^3} \right]$

Potential:

$$\boxed{V = \frac{m\cos\theta}{4\pi\varepsilon_0 r^2}}$$

Electric field intensity

$E = -\nabla V$

Assume spherical coordinate system

$$E = -\left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi\right]$$

$$= -\left[\frac{m\cos\theta}{4\pi\varepsilon_0}\left(\frac{-2}{r^3}\right)\hat{a}_r + \frac{1}{r}\frac{m}{4\pi\varepsilon_0 r^2}(-\sin\theta)\hat{a}_\theta + \frac{1}{r\sin\theta}(0)\right]$$

$$= \frac{m\cos\theta}{2\pi\varepsilon_0 r^3}\hat{a}_r + \frac{m\sin\theta}{4\pi\varepsilon_0 r^3}\hat{a}_\theta$$

$$E = \frac{m}{4\pi\varepsilon_0 r^3}\left[2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right]$$