SNS COLLEGE OF TECHNOLOGY

Coimbatore-20
An Autonomous Institution

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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT I – CONTROL SYSTEM MODELING

TOPIC 5- MODELING OF ELECTRIC SYSTEMS

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS
- TYPES OF SYSTEMS
- DYNAMIC SYSTEMS
- WAYS TO STUDY A SYSTEM
- MODEL & ITS NEEDS,TYPES
- ACTIVITY
- MODELING OF ELECTRICAL SYSTEMS(R,L,C)
- V-I AND I-V RELATIONS
- EXAMPLES
- SUMMARY

TYPES OF SYSTEMS

- **Static System:** If a system does not change with time, it is called a static system.
- **Dynamic System:** If a system changes with time, it is called a dynamic system.

DYNAMIC SYSTEMS

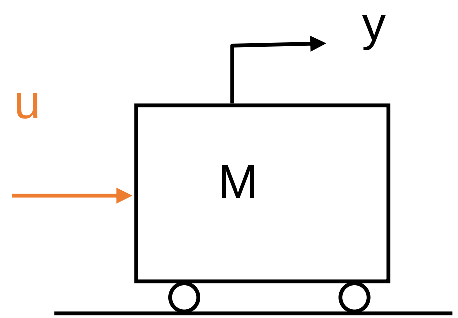
- A system is said to be dynamic if its current output may depend on the past history as well as the present values of the input variables.
- Mathematically,

$$y(t) = \varphi[u(\tau), 0 \le \tau \le t]$$

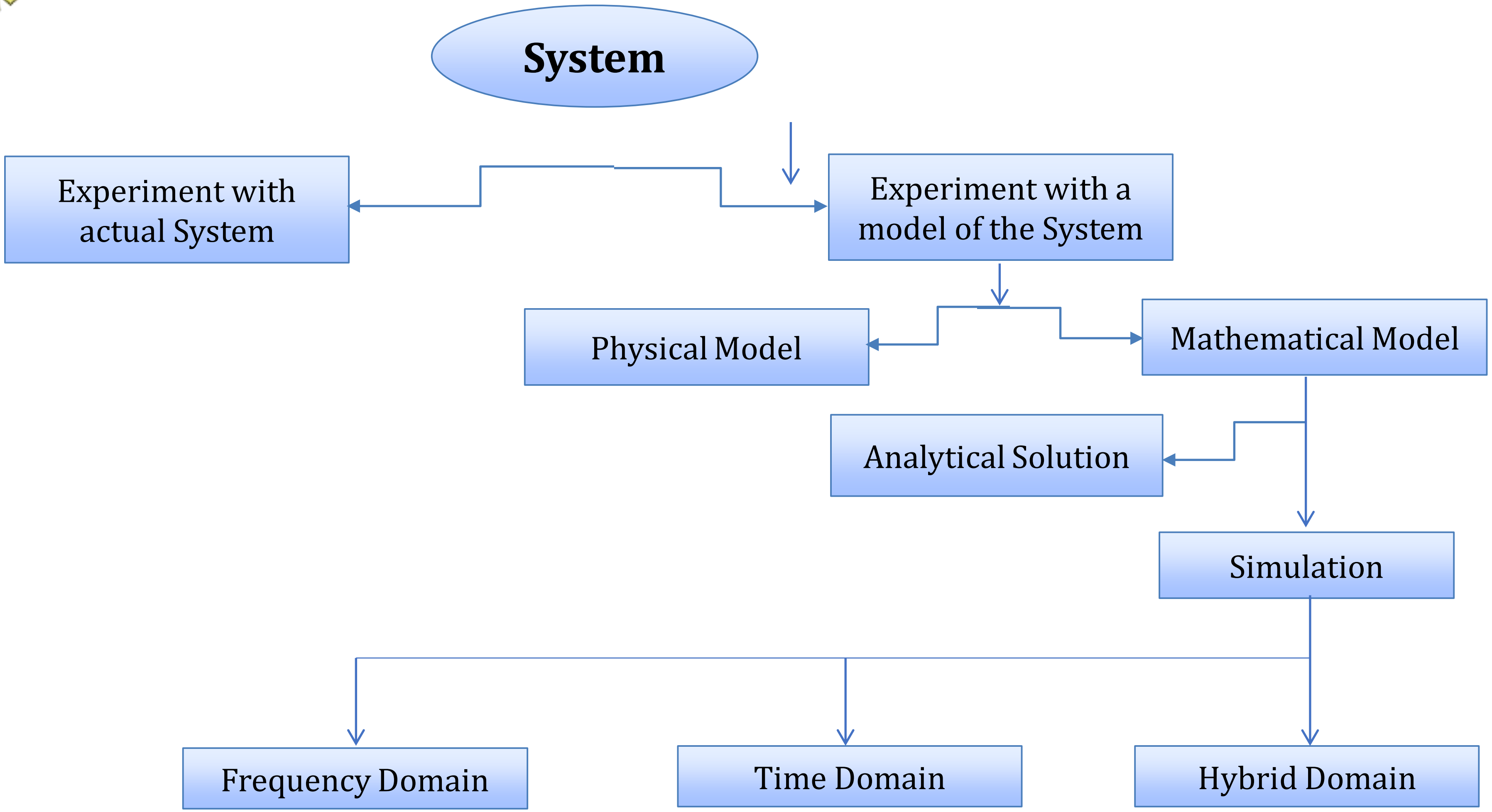
$$u : \text{Input}, \; t : \text{Time}$$

<u>Example</u>: A moving mass

<u>Model</u>: Force=Mass x Acceleration



WAYS TO STUDY A SYSTEM



MODEL

- A *model* is a simplified representation or abstraction of reality.

- Reality is generally too complex to model exactly.

A set of mathematical equations (e.g., differential eqs.) that describes the input-output behavior of a system.

What is a model used for?

- Simulation
- Prediction/Forecasting
- Prognostics/Diagnostics
- Design/Performance Evaluation
- Control System Design

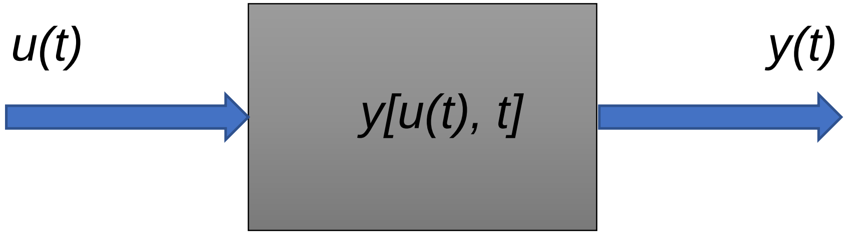
BLACK BOX MODEL

- When only input and output are known.
- Internal dynamics are either too complex or unknown.

- Easy to Model

GREY BOX MODEL

- When input and output and some information about the internal dynamics of the system is known.



- Easier than white box Modelling.

WHITE BOX MODEL

- When input and output and internal dynamics of the system is known.

$$u(t) \longrightarrow \boxed{\frac{dy(t)}{dt} = 3\frac{du(t)}{dt} - \frac{d^2y(t)}{dt^2}} \longrightarrow y(t)$$

- One should know complete knowledge of the system to derive a white box model.

ACTIVITY

Fill the empty circle

1 2 3

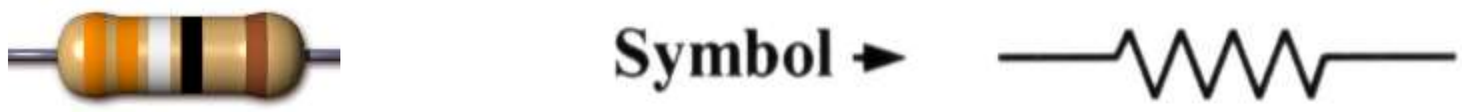
4 5 4 6 7

2 1 0 10 4 6 3

6 7 ○ 8 9

2 6 8

BASIC ELEMENTS OF ELECTRICAL SYSTEMS



- The time domain expression relating voltage and current for the resistor is given by Ohm's law

$$v_R(t) = i_R(t)R$$

- The Laplace transform of the above equation is

$$V_R(s) = I_R(s)R$$

BASIC ELEMENTS OF ELECTRICAL SYSTEMS

Capacitor

- The time domain expression relating voltage and current for the Capacitor is given as:

$$v_c(t) = \frac{1}{C}\int i_c(t)dt$$

- The Laplace transform of the above equation (assuming there is no charge stored in the capacitor) is

$$V_c(s) = \frac{1}{Cs} I_c(s)$$

BASIC ELEMENTS OF ELECTRICAL SYSTEMS

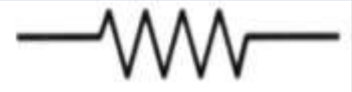
- The time domain expression relating voltage and current for the inductor is given as:

$$v_L(t) = L\frac{di_L(t)}{dt}$$

- The Laplace transform of the above equation (assuming there is no energy stored in inductor) is

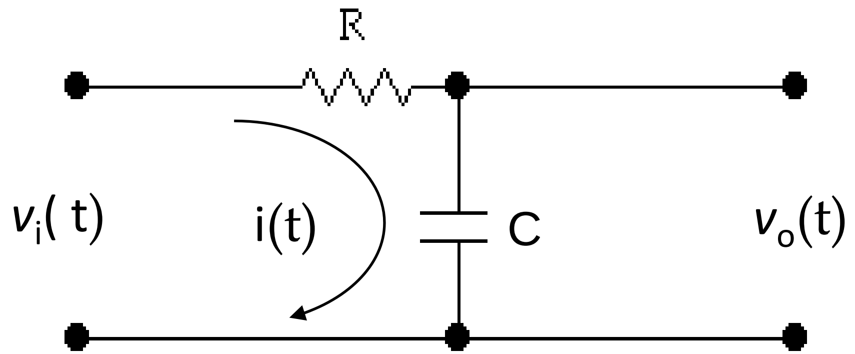
$$V_L(s) = LsI_L(s)$$

V-I AND I-V RELATIONS

Component	Symbol	V-I Relation	I-V Relation
Resistor		$v_R(t) = i_R(t)R$	$i_R(t) = \frac{v_R(t)}{R}$
Capacitor		$v_c(t) = \frac{1}{C}\int i_c(t)dt$	$i_c(t) = C\frac{dv_c(t)}{dt}$
Inductor		$v_L(t) = L\frac{di_L(t)}{dt}$	$i_L(t) = \frac{1}{L}\int v_L(t)dt$

EXAMPLE 1

- The two-port network shown in the following figure has $v_i(t)$ as the input voltage and $v_o(t)$ as the output voltage. Find the transfer function $V_o(s)/V_i(s)$ of the network.



$$v_i(t) = i(t)R + \frac{1}{C}\int i(t)dt \qquad v_o(t) = \frac{1}{C}\int i(t)dt$$

EXAMPLE 1

$$v_i(t) = i(t)R + \frac{1}{C}\int i(t)dt \qquad v_o(t) = \frac{1}{C}\int i(t)dt$$

- Taking Laplace transform of both equations, considering initial conditions to zero.

$$V_i(s) = I(s)R + \frac{1}{Cs}I(s) \qquad V_o(s) = \frac{1}{Cs}I(s)$$

- Re-arrange both equations as:

$$V_i(s) = I(s)(R + \frac{1}{Cs}) \qquad CsV_o(s) = I(s)$$

EXAMPLE 1

$$V_i(s) = I(s)(R + \frac{1}{Cs})$$

$$CsV_o(s) = I(s)$$

- Substitute I(s) in equation on left

$$V_i(s) = CsV_o(s)(R + \frac{1}{Cs})$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{Cs(R + \frac{1}{Cs})}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + RCs}}$$

EXAMPLE 1

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1+RCs}$$

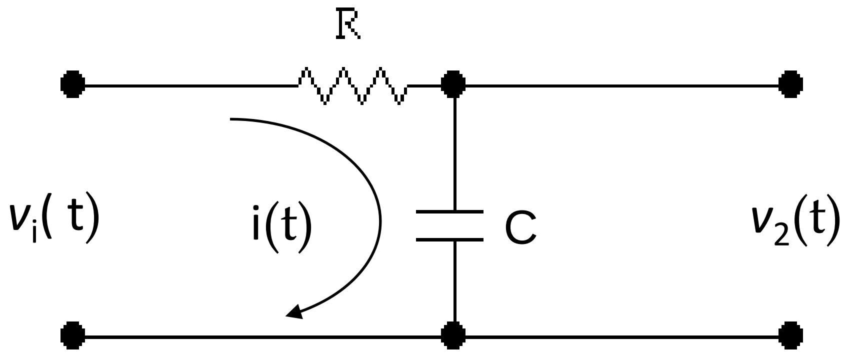
- The system has one pole at

$$1+RCs=0 \qquad \Rightarrow s=-\frac{1}{RC}$$

EXAMPLE 2

- Design an Electrical system that would place a pole at -3 if added to the other system.

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1+RCs}$$



- System has one pole at

$$s = -\frac{1}{RC}$$

- Therefore,

$$-\frac{1}{RC} = -3 \qquad if \qquad R = 1\ M\Omega \quad and \quad C = 333\ pF$$

SUMMARY