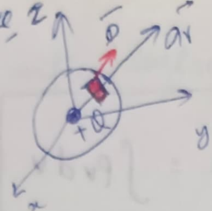
Flux density at any point on the surface is

$\vec{D} = \frac{Q}{4\pi r^2} \hat{a_r}$ c/m²



vector form

$\bar{D} = \frac{d\psi}{ds} \hat{a_n}$ c/m²

$S = 4\pi r^2$.

Relation between D and E

$\vec{E} = \frac{Q}{4\pi \varepsilon r^2} \hat{a_r}$

$d\psi \Rightarrow$ Total flux lines crossing normal thro' the diff surface area ds

$\vec{E} = \vec{D}/\varepsilon$

$\boxed{\vec{D} = \varepsilon \vec{E}}$

Electric flux density is also called as electric displacement.

Gauss's law

Objective:- To learn one of the fundamental law of electromagnetics

To find fields due to symmetrical charge distributions

Gauss's law constitutes one of the fundamental laws of electromagnetism.

Gauss law states that the total electric flux ψ through any closed surface is equal to the total charge enclosed by that surface.

$\psi = Q_{enc}$

$\psi = \oint_s d\psi$

$\psi = \oint_s \vec{D} \cdot ds$ = total charge enclosed

$Q = \int_v \rho_v \, dv$

or

$$\boxed{Q = \oint_S \vec{D}.ds = \int_V \rho_v dv}$$ Integral form

Applying divergence theorem

$$\oint \vec{D}.ds = \int_V \nabla.\vec{D}\, dv$$

Comparing two equations

$$\therefore \boxed{\rho_v = \nabla.\vec{D}}$$ Differential or point form.

which is first of the four Maxwell's equations. Equation states that the volume charge density is the same as the divergence of electric flux density.

It relates the definition of divergence & also
(WKT ρ_v is simply at charge per unit volume)

Advantages :-

Gauss' law provides an easy means of finding E or D for symmetrical charge distributions, such as a point charge, an infinite line charge, an infinite cylindrical surface charge & a spherical distribution of charge.

Proof of Gauss's law :-

Consider a point charge Q kept at the origin.

{ Consider a small area 'ds' on the surface of the sphere

Let $d\psi$ be the flux crossing the surface.

$$d\psi = \vec{D}.\vec{ds}$$

$$= D ds \cos 0^\circ$$

$$d\psi = D\, ds$$

Total flux leaving the entire surface $\quad \psi = \oint \vec{D}\cdot \vec{ds}$

$$\psi = \iint D\, ds \qquad \psi = \oint D_r \hat{a}_r \cdot r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r$$

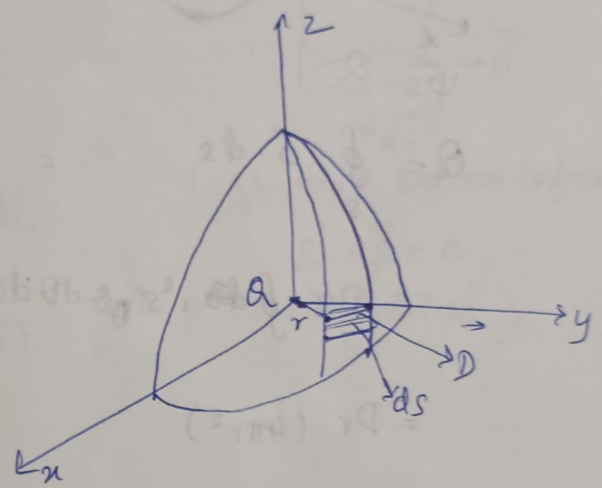
$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} D\cdot(r^2 \sin\theta\, d\theta\, d\phi)$$

$$= D r^2 \int_{\phi=0}^{2\pi} (-\cos\theta)_0^{\pi}\, d\phi$$

$$= D r^2 \,(-\cos\pi + \cos 0)\,(2\pi)$$

$$= D r^2 (2\pi)(2)$$

$$\psi = 4\pi r^2 D$$



WKT $\quad D = \dfrac{Q}{4\pi r^2}$

$$\psi = 4\pi r^2 \cdot \frac{Q}{4\pi r^2}$$

$$\underline{\underline{\psi = Q}}$$

Applications of Gauss's law:

→ To calculate the electric field due to different charge distributions:-

If symmetric charge distribution exists, gaussian surface is constructed such that $\vec{D}$ is normal or tangential to the Gaussian surface

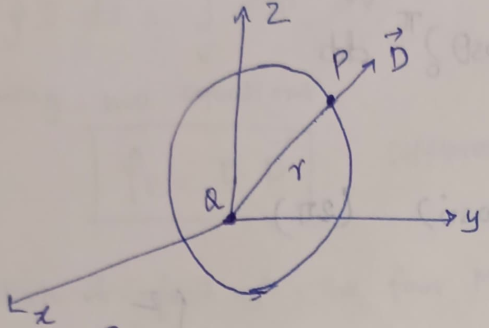
When D is normal to the surface $\quad \vec{D}\cdot \vec{ds} = D\, ds$

" tangential $\quad \vec{D}\cdot ds = 0$

i) Point Charge

A point charge Q is located at the origin.

To determine $\vec{D}$ at a point P, it is easy to choose a spherical surface with symmetry conditions. (Gaussian surface)



$\psi = Q$

$Q = \oint_S \vec{D} \cdot ds \quad = \oint_S D_r \hat{a_r} \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$

$= D_r \oint r^2 \sin\theta \, d\theta \, d\phi \quad = D_r \int_0^{2\pi} \int_0^{\pi} r^2 \sin\theta \, d\theta \, d\phi$

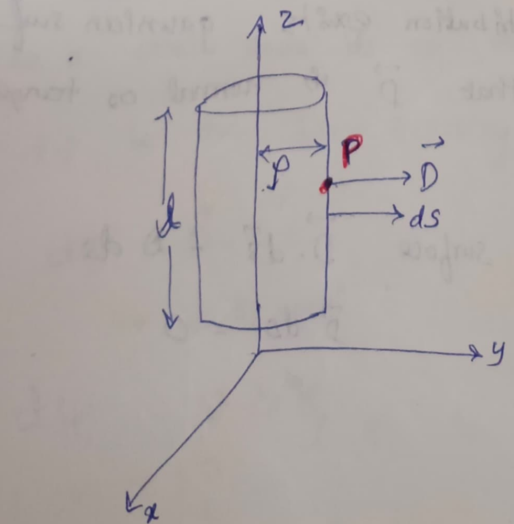
$= D_r \, (4\pi r^2)$

$Q = D_r \, 4\pi r^2$

$D_r = Q/4\pi r^2$

Flux density $\boxed{\vec{D} = \frac{Q}{4\pi r^2} \hat{a_r}}$ $\qquad \vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2} \hat{a_r}$

ii) Infinite line charge:



Apply Gauss law

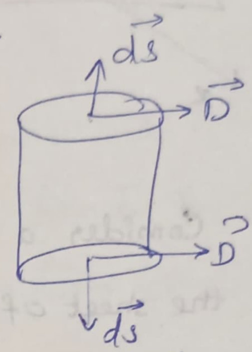
$$Q = \oint_S \vec{D} \cdot \vec{ds}$$

$d\rho$
$\rho\, d\phi$
dz

$$\rho_l \, l = \oint_S \vec{D} \cdot \vec{ds}$$

here $\boxed{\vec{D} = D_\rho \hat{a_\rho}}$ since $\vec{D}$ is normal to the cylindrical surface.

$$\rho_l \, l = \oint_S D_\rho \hat{a_\rho} \cdot (\rho \, d\phi \, dz) \hat{a_\rho}$$



$$= D_\rho \oint_S \rho \, d\phi \, dz$$

$$= D_\rho \rho \int_0^{l} \int_0^{2\pi} d\phi \, dz$$

(At top & bottom surfaces $\vec{D} \cdot \vec{ds} = 0$ tangential)

$$= D_\rho \cdot \rho \, (2\pi)(l)$$

$$\rho_l \cancel{l} = D_\rho \cdot \rho \, 2\pi \cancel{l}$$

$$D_\rho = \frac{\rho_l}{2\pi\rho}$$

$$\therefore \vec{D} = D_\rho \hat{a_\rho}$$

$$\boxed{\vec{D} = \frac{\rho_l}{2\pi\rho} \hat{a_\rho}}$$

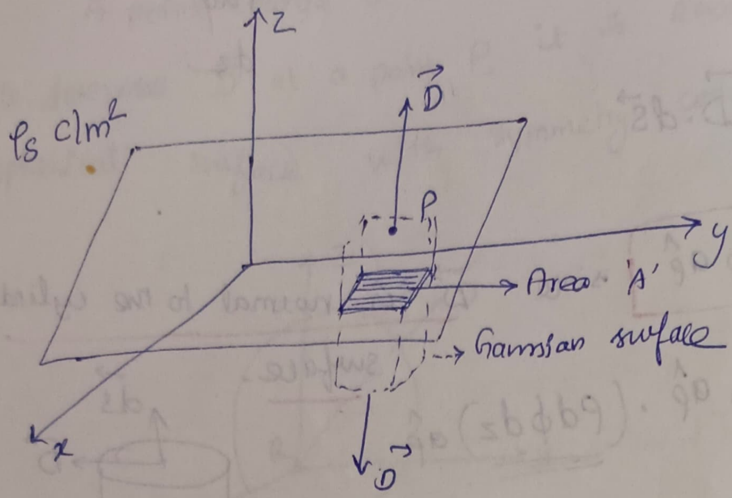
$\rho \rightarrow$ radius of cylinder

$\rho_l \rightarrow$ charge density

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho_l}{2\pi\varepsilon_0\rho} \hat{a_\rho}$$

iii) Infinite sheet of charge



dx dy d

Consider a rectangular box that is cut symmetrically by the sheet of charge and has two of its faces parallel to the sheet. Since $\vec{D}$ is normal to the sheet,

$$\boxed{\vec{D} = D_z \hat{a}_z}$$

Applying gauss law,

$$Q = \oint_S \vec{D} \cdot ds$$

$$\rho_s \cdot A = \oint_S D_z \hat{a}_z \cdot ds\, \hat{a}_z$$

$$= D_z \oint ds$$

$$\rho_s A = D_z \left(\oint_{top} ds_1 + \oint_{bottom} ds_2 \right)$$

along a_x & a_y, $\vec{D}$ has no component
$\vec{D} \cdot ds = 0$

$$\rho_s A = D_z \, (A + A)$$

$$\rho_s \cancel{A} = D_z \cdot 2\cancel{A}$$

$$D_z = \frac{\rho_s}{2}$$

$$\vec{D} = D_z \hat{a}_z$$

$$\therefore \boxed{\vec{D} = \rho_s/_2 \; \hat{a}_z}$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\,\hat{a}_z$$

Gauss Divergence theorem

From gauss law

$$\iint D\cdot ds = Q$$

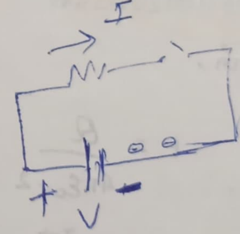
$$Q = \iiint \rho_v \, dv$$

$$\iint D\cdot ds = \iiint \rho_v \, dV$$

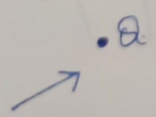
$$\boxed{\iint D\cdot ds = \iiint \nabla\cdot D \, dv}$$

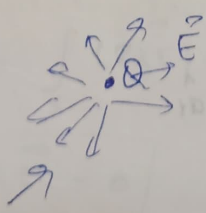
from point form of gauss law,

$$\nabla\cdot D = \rho_v$$



Electric Potential:





We wish to move a point charge Q from Point A to Point B in an electric field E. From coulomb's law, force on Q is $F = QE$, so that the work done by external force in displacing the charge by dl

$$dw = -F\cdot dl$$

$$= -QE\cdot dl$$