Coulomb's law & Electric field intensity

Objectives:

* To learn the basic law used for finding force and electric field due to different charge configurations.

* Two fundamental laws governing electrostatic fields:

1) Coulomb's law (2) Gauss's law.

Although coulomb's law is applicable in finding the electric field due to any charge configuration, it is easier to use gauss's law for symmetrical charge distribution.

* Assume electricfield is in a vacuum or free space.

* Based on coulomb's law, the concept of electric field intensity is introduced and applied to point, line, surface and volume charges.

Coulomb's law: (Inverse square law)

Is an experimental law formulated in 1785 by Charles Augustin de coulomb.

It deals with the force a point charge exerts on another point charge.

point charge ⇒ smaller dimensions.

Charge ⇒ Coulombs (C) ⇒ One Coulomb = 6×10^{18} elect

one electron charge $e = -1.6019 \times 10^{-19}$ C

Coulomb's law states that the force F between two point charges Q_1 and Q_2 is :

along the line joining them

directly proportional to the product $Q_1 Q_2$ of the charges

inversely proportional to the square of distance R between them.

Expressed mathematically,

$$F = k \frac{Q_1 Q_2}{R^2} \quad \text{Newtons}$$

where k is the proportionality constant

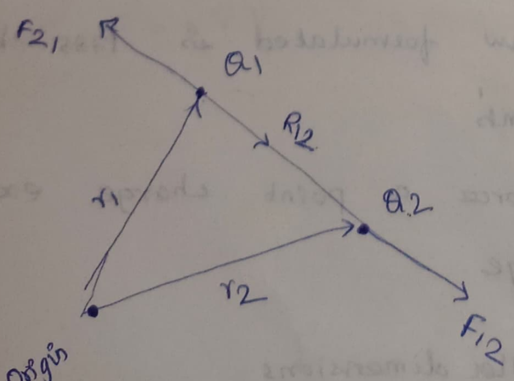
$$k = \frac{1}{4\pi\varepsilon_0}$$

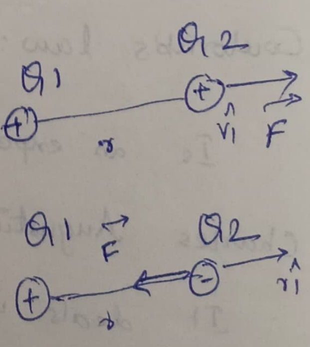
ε_0 is the permittivity of free space. (farads per meter)

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9} \text{ m/F}$$

Coulomb vector force on point charges Q_1 and Q_2





$$F = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R^2}$$

Force on Q_2 due to Q_1;

$$\boxed{\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R_{12}^2} \hat{a}_{R12}}$$

where $\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$

$R = |\vec{R}_{12}|$ $\qquad |\vec{R}| = R$

$$\hat{a}_{R12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|}$$

$$\therefore \boxed{\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R^3} \vec{R}_{12}}$$

$$\vec{F}_{12} = \frac{Q_1 Q_2 (r_2 - r_1)}{4\pi \varepsilon_0 |r_2 - r_1|^3}$$

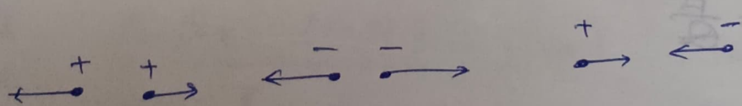
Force on Q_1 due to Q_2

$$\vec{F}_{21} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R^2} \hat{a}_{R21}$$

$$= |\vec{F}_{12}| (-\hat{a}_{R12})$$

$$\vec{F}_{21} = -\vec{F}_{12}$$

* Like charges repel each other, while unlike charges attract.



* Q_1 & Q_2 must be point charges
* Q_1 and Q_2 must be static (rest)
* For like charges $Q_1 Q_2 > 0$, for unlike charges $Q_1 Q_2 < 0$.

Principle of Superposition

To determine the force on a particular charge, when we have more than two point charges.

It states that if there are N charges $Q_1, Q_2, \ldots Q_N$ located respectively, at points with position vectors $r_1, r_2 \ldots r_N$, the resultant force F on a charge Q located at point r is the vector sum of the forces exerted on Q by each of the charges $Q_1, Q_2 \ldots Q_N$.

$$\vec{F} = \frac{Q Q_1 \vec{R_1}}{4\pi\varepsilon_0 |\vec{R_1}|^3} + \frac{Q Q_2 (\vec{R_2})}{4\pi\varepsilon_0 |\vec{R_2}|^3} + \cdots + \frac{Q Q_N (\vec{R_N})}{4\pi\varepsilon_0 |\vec{R_N}|^3}$$

or

$$\vec{F} = \frac{Q}{4\pi\varepsilon_0} \sum_{k=1}^{N} \frac{Q_k (\vec{R_k})}{|\vec{R_k}|^3}$$

$$\boxed{\frac{(r - r_k)}{|r - r_k|^3}}$$

Electric field Intensity

Electric field intensity (or electric field strength) E is the force per unit charge when placed in an electric field.

$$E = \frac{F}{Q}$$

$$\vec{E} = \frac{\vec{F}}{Q}$$

$\vec{E}$ is obviously in the direction of the force $\vec{F}$.

<u>Unit:</u> Newtons per Coulomb

or Volts per meter

Electric field intensity at point r due to a point charge located at r'

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R \quad \text{or} \quad \frac{Q\,(r-r')}{4\pi\varepsilon_0\,|r-r'|^3}$$

For N point charges $Q_1, Q_2 \ldots Q_N$ located at $r_1, r_2 \ldots r_N$, the electric field intensity at point r

is

$$\vec{E} = \frac{Q_1 \vec{R_1}}{4\pi\varepsilon_0|\vec{R_1}|^3} + \frac{Q_2 \vec{R_2}}{4\pi\varepsilon_0|\vec{R_2}|^3} + \cdots + \frac{Q_N (\vec{R_N})}{4\pi\varepsilon_0|\vec{R_N}|^3}$$

(or)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{K=1}^{N}\frac{Q_K \vec{R_K}}{|\vec{R_K}|^3} \qquad \boxed{\frac{(r-r_k)}{|r-r_k|^3}}$$

Problems:

1. Point charges 1 mC and −2 mC are located at (3, 2, −1) and (−1, −1, 4) respectively. Calculate the electric force on a 10 nC charge located at (0, 3, 1) and the electric field intensity at that point.

Solution:

$$\vec{F} = \sum_{k=1}^{2}\frac{Q\,Q_K\,\overbrace{(r-r_k)}^{\vec{R_{12}}}}{4\pi\varepsilon_0\,|r-r_k|^3\,|R_{12}|^3}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{Q_1\,(r-r_1)}{|r-r_1|^3} + \frac{Q_2\,(r-r_2)}{|r-r_2|^3}\right]$$

$$= \cancel{10}\ \frac{10\times10^{-9}}{(1/9\times10^{9})}\left[\frac{10^{-3}(-3,1,2)}{14\sqrt{14}} + \frac{2\times10^{-3}(1,4,-3)}{26\sqrt{26}}\right]$$

$$\vec{F} = (-6.507\,\hat{a_x} - 3.817\,\hat{a_y} + 7.506\,a_z)\,mN$$

At that point,

$$\vec{E} = \vec{F}/Q$$

$$= \frac{(-6.507\,\hat{a_x} - 3.817\,\hat{a_y} + 7.506\,\hat{a_z})\times10^{-3}}{10\times10^{-9}}$$

$$\vec{E} = -650.7\,\hat{a_x} - 381.7\,\hat{a_y} + 750.6\,\hat{a_z}\ \ kV/m$$

Electric Fields due to continuous charge distributions:

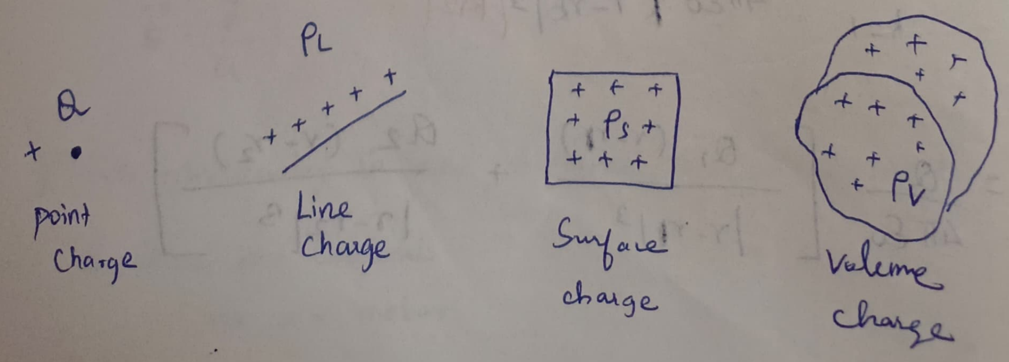
* So far we have considered only forces & electric fields due to point charges,

* Continuous charge distributions along a line, on a surface or in a volume are also possible.

Line charge density : ρ_L in C/m

Surface charge density : ρ_s in C/m2

Volume charge density : ρ_v in C/m^3.



$$dQ = \rho_L \, dL$$

$$Q = \int_L \rho_L \, dl.$$ (line charge)

$$dQ = \rho_s \, ds$$

$$Q = \int_S \rho_s \, ds$$ (surface charge)

$$dQ = \rho_v \, dv$$

$$Q = \int_V \rho_v \, dv$$ (volume charge)

Electric field intensity

$$E = \int_L \frac{\rho_L \, dl}{4\pi \varepsilon_0 R^2} \hat{a_R}$$ (line charge)

$$E = \int_S \frac{\rho_s \, ds}{4\pi \varepsilon_0 R^2} \hat{a_R}$$ (surface charge)

$$E = \int_V \frac{\rho_v \, dv}{4\pi \varepsilon_0 R^2} (\hat{a_R})$$ (volume charge)

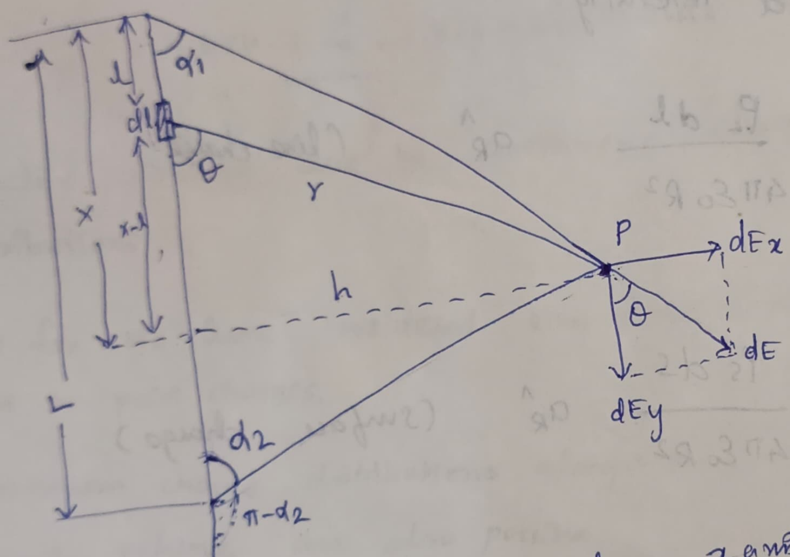
$$\rho_L = \frac{Q}{L}$$

$$\rho_L \cdot L = Q$$

Electric field intensity due to line charge

Assume a straight uniformly charged wire of length L at a linear density of ρ_L Coulombs/m. Let P be any point at which field intensity has to be determined.

Consider a small elemental length dl. Let dE be the Electric field due to charge element $\rho_L\,dl$.



dE has two components along x axis and y axis.

$$dE = \frac{\rho_L\,dl}{4\pi\varepsilon_0 r^2} \rightarrow (1)$$

$$\sin\theta = \frac{dE_x}{dE}$$

$$\cos\theta = \frac{dE_y}{dE}$$

$$dE_x = dE\sin\theta$$

$$dE_y = dE\cos\theta$$

$$\therefore dE_x = \frac{\rho_L\,dl}{4\pi\varepsilon_0 r^2}\sin\theta \rightarrow (2)$$

from figure

$$\tan\theta = \frac{h}{x-l}$$

$$x-l = \frac{h}{\tan\theta} = h\cot\theta$$

diff wrt θ

$$0 - \frac{dl}{d\theta} = h(-\mathrm{cosec}^2\theta)$$

$$\boxed{dl = h\,\mathrm{cosec}^2\theta\, d\theta}$$

$$\sin\theta = \frac{h}{r}$$

$$r = \frac{h}{\sin\theta}$$

$$\boxed{r = h\,\mathrm{cosec}\,\theta}$$

Sub in (2)

$$dE_x = \frac{\rho_l \cdot h\,\mathrm{cosec}^2 d\theta}{4\pi\varepsilon_0 r^2} \sin\theta$$

$$dE_x = \frac{\rho_l\, \cancel{h}\,\cancel{\mathrm{cosec}^2}\theta\, d\theta\, \sin\theta}{4\pi\varepsilon_0\, h^{\cancel{2}}\, \cancel{\mathrm{cosec}^2\theta}}$$

$$\boxed{dE_x = \frac{\rho_l \sin\theta\, d\theta}{4\pi\varepsilon_0 h}}$$

Integrating

$$E_x = \frac{\rho_l}{4\pi\varepsilon_0 h}\int_{\alpha_1}^{\pi-\alpha_2} \sin\theta\, d\theta$$

$$= \frac{\rho_l}{4\pi\varepsilon_0 h}\left[-\cos\theta\right]_{\alpha_1}^{\pi-\alpha_2}$$

$$= \frac{-\rho_l}{4\pi\varepsilon_0 h}\left[\underbrace{\cos(\pi-\alpha_2)}_{-\cos\alpha_2} - \cos\alpha_1\right]$$

$$E_x = \frac{-\rho l}{4\pi\varepsilon_0 h}\left[-\cos\alpha_2 - \cos\alpha_1\right]$$

$$E_x = \frac{\rho l}{4\pi\varepsilon_0 h}\left(\cos\alpha_1 + \cos\alpha_2\right)$$

Similarly

$$dE_y = \frac{\rho l}{4\pi\varepsilon_0 h}\cos\theta \, d\theta$$

$$E_y = \frac{\rho l}{4\pi\varepsilon_0 h}\int_{\alpha_1}^{\pi-\alpha_2}\cos\theta \, d\theta$$

$$= \frac{\rho l}{4\pi\varepsilon_0 h}\left[\sin\theta\right]_{\alpha_1}^{\pi-\alpha_2}$$

$$= \frac{\rho l}{4\pi\varepsilon_0 h}\left(\sin(\pi-\alpha_2) - \sin\alpha_1\right)$$

$$E_y = \frac{\rho l}{4\pi\varepsilon_0 h}\left(\sin\alpha_2 - \sin\alpha_1\right)$$

Total electric field intensity at point 'P' is

$$E = E_x \hat{a_x} + E_y \hat{a_y}$$

$$E = \frac{\rho l}{4\pi\varepsilon_0 h}\left(\cos\alpha_1 + \cos\alpha_2\right)\hat{a_x} + \frac{\rho l}{4\pi\varepsilon_0 h}\left(\sin\alpha_2 - \sin\alpha_1\right)\hat{a_y}$$

If the point 'p' is along the perpendicular bisector of wire then $\alpha_1 = \alpha_2$

Let $\alpha_1 = \alpha_2 = \alpha$

$$\therefore E = \frac{\rho_l}{4\pi\varepsilon_0 h} 2\cos\alpha \, \hat{a_x}$$

$$\vec{E} = \frac{\rho_l}{2\pi\varepsilon_0 h} \cos\alpha \, \hat{a_x}$$

If the line is infinitely long, then $\alpha = 0$

$$\therefore E = \frac{\rho_l}{4\pi\varepsilon_0 h} (2) \hat{a_x}$$

$$\boxed{\vec{E} = \frac{\rho_l}{2\pi\varepsilon_0 h} \hat{a_x}}$$

—x—

Electric field due to circular sheet of charge:

Consider a circular sheet of charge of radius 'a' with charge density ρ_s C/m^2. Let 'p' be a point, 'h' meters from the disc along its axis at which field has to be determined.

Consider a small elemental area.

$$ds = 2\pi r \, dr.$$

Let dE be the field at Point 'p' due to small area ds.

Owing to the symmetry of the charge distribution, for every element 1, there is corresponding element 2