Ampere's Circuit law

States that the line integral of $\vec{H}$ around a closed path is the same as the net current I_{enc} enclosed by the path.

ie. the circulation of $\vec{H}$ equals I_{enc};

$$\oint \vec{H}\cdot d\vec{l} = I_{enc}$$

By applying stoke's theorem

$$I_{enc} = \oint_L \vec{H}\cdot d\vec{l} = \int_S (\nabla \times \vec{H})\cdot d\vec{S} \rightarrow ①$$

But,

$$I_{enc} = \int_S J\cdot d\vec{s} \rightarrow ②$$

Comparing ① & ②

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$

Ampere's law in point form or differential form.

$\nabla \times H = J \neq 0$ magnetostatic field is not conservative.

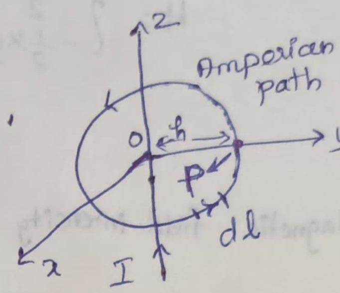
Applications of Ampere's law:

To Determine $\vec{H}$ for some symmetrical current distributions.

i) Infinite line current :-

For an infinitely long conductor carrying a current I along z-axis, consider a closed path (Amperian path)

From Ampere's law,



$$I = \int H \cdot dl$$

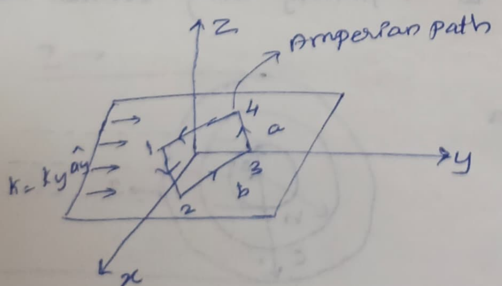
$$= H \int dl$$

$$I = H(2\pi h)$$

$$H = \frac{I}{2\pi h} \quad A/m$$

ii) Infinite sheet of current:

uniform current density $k = k_y \hat{a_y}$ A/m.



Applying ampere's law to the path 1-2-3-4-1

$$\oint H \cdot dl = I_{enc} = k_y b \quad \rightarrow (1)$$

$$H = \begin{cases} H_o a_x & z > 0 \\ -H_o a_x & z < 0 \end{cases} \quad \longrightarrow (2)$$

$$\oint H \cdot dl = \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \right) A \cdot dl$$

$$= 0(-a) + (-H_0)(-b) + (0)(a) + (H_0)(b)$$

$$\oint H \cdot dl = 2H_0 b$$

from ① ∴ $2H_0 b = K_y b$

$$H_0 = \frac{1}{2} K_y \longrightarrow ③$$

sub in eqn ②

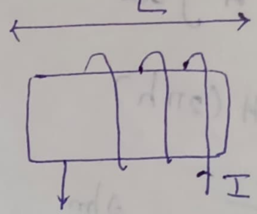
$$H = \begin{cases} \frac{1}{2} K_y \, a_x, & z > 0 \\ -\frac{1}{2} K_y \, a_x, & z < 0 \end{cases}$$

In general, for an infinite sheet of current density K A/m

$$H = \frac{1}{2} K \times a_n$$

* Magnetic field intensity due to solenoid:

Here N - number of turns



Current enclosed $= NI$

By ampere's law $\oint H \cdot dl = I_{enc}$

$$HL = NI$$

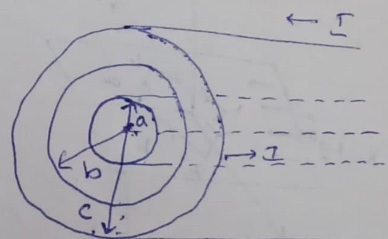
$$\boxed{H = \frac{NI}{L}} \text{ A/m}$$

* Magnetic field intensity due to infinitely long coaxial cable

Radius of the inner conductor - a

Inner radius of the outer conductor - b

outer radius - c



Current thro' inner conductor $= I$

Current thro' outer conductor $= -I$

case i) when $r<a$ (inside the inner conductor)

Since current is uniformly distributed over the cross section

Current density $J = \frac{I}{\pi a^2}$

current enclosed by the circular path with radius, $r<a$

$$I_{enc} = \frac{I}{\pi a^2} \times \pi r^2$$

$$I_{enc} = \frac{I r^2}{a^2}$$

from ampere's law.

$$\oint H \cdot dl = I_{enc}$$

$$H(2\pi r) = \frac{I r^2}{a^2}$$

$$H = \frac{I r^2}{a^2 \times 2\pi r}$$

$$H = \frac{I r}{2\pi a^2} \quad \text{when } r<a$$

case ii) when $a<r<b$ (inbetween the conductor)

Current enclosed $= I$

$$\oint H \cdot dl = I_{enc}$$

$$\oint H \cdot dl = I$$

$$H \cdot (2\pi r) = I$$

$$H = \frac{I}{2\pi r} \quad \text{when } a<r<b$$

Case iii) when $b < r < c$ (inside the outer conductor)

current density $\quad J = \dfrac{-I}{\pi c^2 - \pi b^2}$

current enclosed by the circular path with radius $b < r < c$

$$I_{enc} = \frac{-I}{\pi c^2 - \pi b^2}(\pi r^2 - \pi b^2) + I$$

$$= \frac{-I\pi r^2 + \cancel{I\pi b^2} + I\pi c^2 - \cancel{I\pi b^2}}{\pi c^2 - \pi b^2}$$

$$= \frac{I\pi(c^2 - r^2)}{\pi(c^2 - b^2)}$$

$$I_{enc} = \frac{I(c^2 - r^2)}{c^2 - b^2}$$

from ampere's law,

$$\oint H \cdot dl = \frac{I(c^2 - r^2)}{c^2 - b^2}$$

$$H(2\pi r) = \frac{I(c^2 - r^2)}{c^2 - b^2}$$

$$H = \frac{I(c^2 - r^2)}{2\pi r(c^2 - b^2)}$$

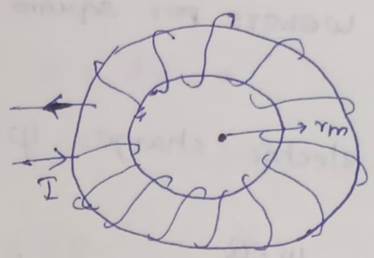
Case (iv) When $r > c$ (outside the outer conductor)

Current enclosed $= I + (-I)$

$$I_{enc} = 0$$

$$H = 0.$$

* Magnetic field intensity due to toroid



Toroidal coil carrying a current I with mean radius r_m.

No. of turns - N

Current enclosed $I_{enc} = NI$

From ampere's law,

$$\oint H \cdot dl = NI$$

$$H(2\pi r_m) = NI$$

$$H = \frac{NI}{2\pi r_m}$$

Stoke's theorem :

$$\oint H \cdot dl = \int_S (\nabla \times H) \cdot ds$$

Magnetic flux density - Maxwell's equation

Magnetic flux density B is related to the magnetic field intensity H

$$B = \mu_0 H$$

μ_0 - permeability of free space. - henrys per meter (H/m)

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

magnetic flux through the surface S is given by

$$\psi = \int B \cdot ds$$