Capacitance

The capacitance is defined as the ratio of the magnitude of the charge on one of the plates to the potential difference between them;

$$C = \frac{Q}{V} = \frac{\varepsilon \oint E \cdot ds}{\int E \cdot dl} \qquad \because D = \varepsilon E$$

To obtain C for any given two-conductor capacitance by following methods.

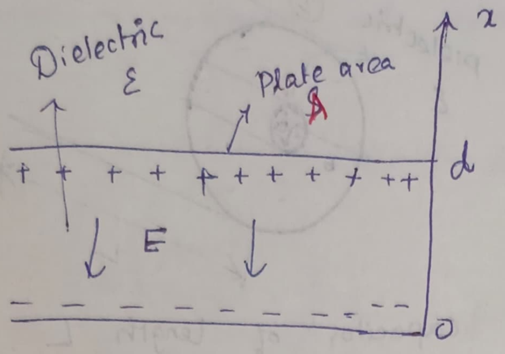
1. Assuming Q and determining V in terms of Q (involving Gauss's law)

2. Assuming V and determining Q in terms of V (involving Laplace's equation).

The former method involves following steps:

1. Choose a suitable coordinate systems
2. Let the two conducting plates carry +Q and -Q
3. Determine E by using Coulomb's or Gauss's law and find V.
4. C = Q/V

1) Parallel - Plate Capacitor



Each of plates has an area S.

Charge density

$$\rho_s = \frac{Q}{A}$$

Electric field intensity at any point between the plates

$$E = \frac{\rho_s}{\varepsilon}(-a_x)$$

$$= \frac{-Q}{\varepsilon A} a_x$$

∴ E-field b/w 2 plates

$$E = \frac{\rho_s}{2\varepsilon_0} a_n + \frac{(-\rho_s)}{2\varepsilon_0}(-a_n)$$

$$= \frac{\rho_s}{\varepsilon_0} a_n$$

$$\therefore V = -\int E \cdot dl$$

$$= -\int_0^d \frac{-Q}{\varepsilon A} a_x \cdot dx\, a_x$$

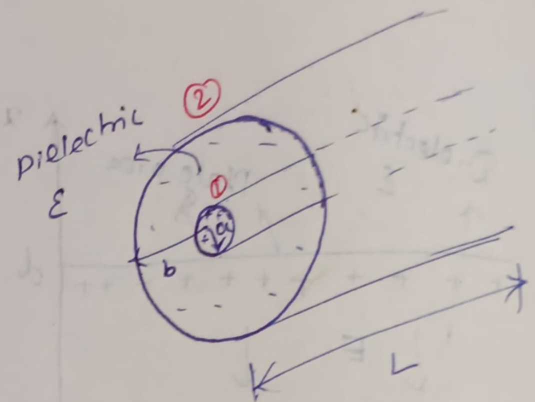
$$= \int_0^d \frac{Q}{\varepsilon A} dx$$

$$V = \frac{Q}{\varepsilon A} d$$

$$\therefore C = Q/V = \frac{\varepsilon A}{d}$$

$$\boxed{C = \frac{\varepsilon A}{d}}$$

2) Coaxial Capacitor



Coaxial cylindrical capacitor of length L.

inner radius a and outer radius b (b > a)

Electric field intensity for infinite line charge

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon r}\,\hat{a}_r$$

$$\int_2^1 E \cdot dl$$

$$\therefore V = -\int_b^a E \cdot dl = -\int_b^a \frac{\rho_L}{2\pi\varepsilon r}\,\hat{a}_r \cdot dr\,\hat{a}_r$$

$$= \frac{-\rho_L}{2\pi\varepsilon} \int_b^a \frac{1}{r}\,dr$$

$$= -\frac{\rho_L}{2\pi\varepsilon} \ln(r)_b^a = \frac{-\rho_L}{2\pi\varepsilon}\left[\ln(a) - \ln(b)\right]$$

$$V = \frac{\rho_L}{2\pi\varepsilon} \ln(b/a)$$

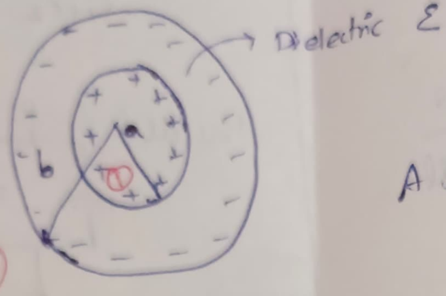
$$C = \frac{Q}{V}$$

$$= \frac{\rho_L \cdot L}{\frac{\rho_L}{2\pi\varepsilon}\ln(b/a)}$$

$$\boxed{C = \frac{2\pi\varepsilon L}{\ln(b/a)}}$$

3) spherical capacitor

It has two concentric spherical conductors.
Inner sphere of radius a and outer sphere of radius b (b > a)



A spherical capacitor

Electric field intensity at any point inbetween the shells is,

$$\bar{E} = \frac{Q}{4\pi \varepsilon r^2} \hat{a}_r$$ (Due to spherical charge distribution

$$\therefore V = -\int_b^a \frac{Q}{4\pi\varepsilon r^2} \hat{a}_r \cdot dr \hat{a}_r$$

$$= -\frac{Q}{4\pi\varepsilon} \int_b^a \frac{1}{r^2} dr$$

$$= \frac{-Q}{4\pi\varepsilon} \left[\frac{-1}{r}\right]_b^a = \frac{+Q}{4\pi\varepsilon} \left[\frac{1}{a} - \frac{1}{b}\right]$$

$$C = \frac{Q}{V}$$

$$\therefore \boxed{C = \frac{4\pi\varepsilon}{(1/a - 1/b)}}$$

If the outer sphere is ~~uniformly~~ infinitely large $b \to \infty$

$$C = \frac{4\pi\varepsilon}{1/a}$$