Electrostatic boundary-value problems :

Consider practical electrostatic problems where only electrostatic conditions (charge & potential) at some boundaries are known and it is desired to find E and V throughout the region. Such problems are usually referred to as boundary value problems. These problems are usually tackled using poisson's and Laplace's eqn.

Poisson's and Laplace's equations

From Gauss's law

$$\nabla \cdot D = \nabla \cdot \varepsilon E = \rho_v$$

and

$$E = -\nabla V$$

$$\nabla \cdot (-\varepsilon \nabla V) = \rho_v$$

(if ε is independent of x, y, z) (homogeneous)

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$$

This is known as Poisson's equation.

A special case of this equation ($\rho_v = 0$) charge-free region,

$$\boxed{\nabla^2 V = 0}$$

This is known as Laplace's equation.

Scalar field V is said to harmonic in a given region if its laplacian vanishes.

$$\nabla^2 V = 0$$

If it is satisfied, the solution for V is harmonic (it is of the form of sine or cosine)

Laplace's equation in Cartesian, cylindrical or spherical Coordinates

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

Poisson's eqn is obtained by simply replacing zero by $-\rho_v/\varepsilon$.

Appl:

The xerographic copying machine is an important application of electrostatics. The surface of the photoconductor is initially charged uniformly. When light from the document to be copied is focused on the photoconductor (PC), the charges on the lower surface combine with those on the upper surface to neutralize each other.

The image is developed by pouring a charged black powder over the surface of the P.C. The electric field attracts the charged powder, which is later transferred to paper & melted to form a permanent image.