$$\vec{E} = \frac{\vec{D}}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{a}_z$$

Gauss Divergence theorem

From gauss law

$$\iint D \cdot ds = Q$$

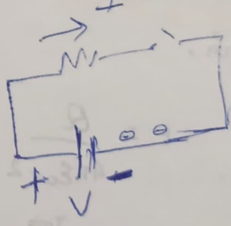
$$Q = \iiint \rho_v \, dv$$

$$\iint D \cdot ds = \iiint \rho_v \, dv$$

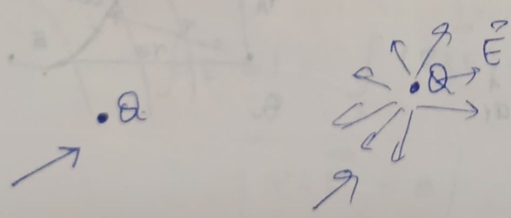
$$\boxed{\iint D \cdot ds = \iiint \nabla \cdot D \, dv}$$

from point form of gauss law,

$$\nabla \cdot D = \rho_v$$



Electric Potential:



We wish to move a point charge Q from Point A to Point B in an electric field E. From coulomb's law, force on Q is $F = QE$, so that the work done by external force in displacing the charge by dl

$$dW = -F \cdot dl$$

$$= -QE \cdot dl$$

Negative sign indicates that the work is being done by an external agent.

Thus, the total work done or potential energy required in moving Q from A to B is,

$$W = -Q \int_{A}^{B} E \cdot dl$$

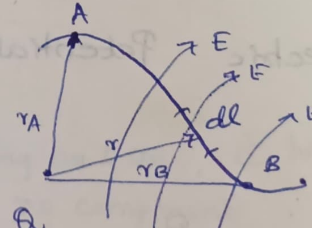
Dividing W by Q gives the pot. energy per unit charge. This is denoted by V_{AB}, is known as the potential difference between points A and B.

$$V_{AB} = \frac{W}{Q} = -\int_{A}^{B} E \cdot dl$$

V_{AB} unit is joules per coulomb, (or) volts (V)

Potential of point A wrt B is defined as the work done in moving a unit positive charge from B to A.

If the E field is due to a point charge Q located at the origin,



$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \hat{a_r}$$

$$V_{AB} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\varepsilon_0 r^2} \hat{a_r} \cdot dr \hat{a_r}$$

$$= \frac{-Q}{4\pi\varepsilon_0} \left[\frac{-1}{r}\right]_{r_A}^{r_B}$$

$$\int \frac{1}{r^2} dr$$

$$\left[\frac{-1}{r}\right]$$

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$V_{AB} = V_B - V_A.$$

where V_A & V_B are absolute potentials

If potential at infinity is zero, $V_A = 0$ as $r_A \to \infty$

potential at any point r_A due to point charge Q located at origin

$$V = \frac{Q}{4\pi\varepsilon_0 r} \Rightarrow \text{absoute potential}$$

Relationship between E & V

The potential difference between points A & B is independent of the path taken.

$$V_{BA} = -V_{AB}$$

$$\oint E \cdot dl = V_{BA} + V_{AB} = 0$$

$$\boxed{\oint \vec{E} \cdot dl = 0} \rightarrow (1)$$

The line integral of E along a closed path must be zero. This implies that no net work is done in moving a charge along a closed path in an electrostatic field.

Apply stoke's theorem

$$\oint \vec{E} \cdot dl = \int_S (\nabla \times \vec{E}) \cdot ds = 0$$

$$\boxed{\nabla \times \vec{E} = 0} \rightarrow (2)$$

If Eqn (1) & (2) are satisfied, then the vector field is said to be conservative, or irrotational.

Vectors whose line integral does not depend on the path of integration are called conservative ~~field~~. vectors.

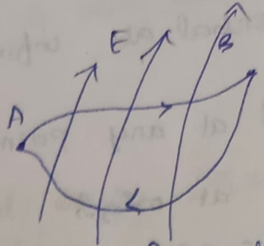
From the definition

$$V = -\int E \cdot dl$$

$$dV = -E \cdot dl$$

$$dV = -E_x\,dx - E_y\,dy - E_z\,dz$$

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz = -E \cdot dl$$



Conservative Nature of Electrostatic field

Comparing two expressions,

$$E_x = \frac{-\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

$$\therefore \boxed{E = -\nabla V}$$

Electric field intensity is the gradient of V.

Negative sign shows that the direction of E is opposite to the direction in which V increases. i.e. E is directed from higher to lower levels of V.

Potential due to different charge distributions.

i) Uniformly charged line.

$$V_{ba} = -\int_b^a E \cdot dl$$

$$= -\int_b^a E \cdot dr$$

$$= -\frac{\rho_l}{2\pi\varepsilon_0}\int_b^a \frac{1}{r}\,dr$$

$$= -\frac{\rho_l}{2\pi\varepsilon_0}\left[\ln(r)\right]_b^a$$