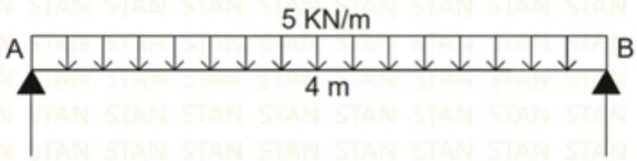
A beam 4m in length is simply supported at its ends and carries uniformly distributed load of 5KN/m over the entire length. Determine strain energy stored in the beam.

$E = 200$ GPA and $I = 1440\ cm^4$



$$I = 1440\ cm^4 = \frac{1440}{[100]^4} m^4 = 1440 \times 10^{-8}\ m^4$$

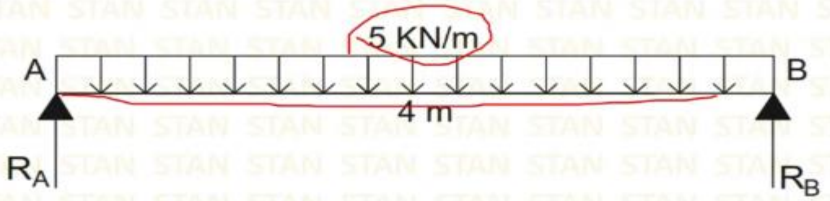
$$E = 200\ GPA = 200 \times 10^9 \frac{N}{m^2} = 200 \times 10^6 \frac{KN}{m^2}$$

$1\ m = \frac{1}{100}$ cm

$Pascal = \frac{N}{m^2}$

$1\ Giga = 10^9$

$1\ Kilo = 10^3$



Total load in the beam = 5 × 4 = 20 KN

$$R_A = R_B = \frac{20}{2} = 10\ KN$$

$$\text{Strain Energy due to bending, } U = \frac{1}{2EI}\int_0^L M^2 dx$$

Strain energy due to bending , $U = \frac{1}{2EI}\int_0^L M^2 dx$

$M = 10x - 5 \times x \times \frac{x}{2}$

$M = 10x - 2.5x^2$

$U = \frac{1}{2EI}\int_0^4 [10x - 2.5x^2]^2 dx$

$$U = \frac{1}{2EI}\int_0^4 [10x - 2.5x^2]^2 dx$$

$$= \frac{1}{2EI}\int_0^4 [100x^2 + 6.25x^4 - 50x^3]\, dx$$

$$= \frac{1}{2EI}\left[\frac{100x^3}{3} + \frac{6.25x^5}{5} - \frac{50x^4}{4}\right]_0^4$$

$$= \frac{1}{2EI}\left[\frac{100 \times 4^3}{3} + \frac{6.25 \times 4^5}{5} - \frac{50 \times 4^4}{4}\right]$$

$$= \frac{1}{2EI}[2133.33 + 1280 - 3200]$$

$$= \frac{1}{2EI}[213.33]$$

$$= \frac{1}{2EI} \times 213.33$$

$$= \frac{1}{2 \times 200 \times 10^6 \times 1440 \times 10^{-8}} \times 213.33$$

$$= \frac{213.33}{5760}$$

$U = 0.03703$ KNm

Or $U = 37.03$ Nm

Result:

Strain Energy stored in the beam, $U = 37.03$ Nm or 37.03 j

The external diameter of a hollow shaft is twice the internal diameter. It is subjected to a pure torque and it attains a maximum shear stress τ. Show that the strain energy stored per unit volume of the shaft is $\frac{5\tau^2}{16C}$. Such a shaft is required to transmit 5400 kW at 110 rpm, with uniform torque, the maximum stress not exceeding 84 MN/m². Determine:

(i) The shaft diameters

(ii) The energy stored per m³.

Take: $C = 90\ GN/m^2$.

Sol :- Given data :-

$$\frac{U}{\text{Unit volume}} = \frac{5\tau^2}{16C}$$

$$P = 5400\ kW, \quad N = 110\ rpm$$

$$\tau_{max} = 84\ MN/m^2$$

$$P = 5400\ kW, \quad N = 110\ rpm$$

$$\tau_{max} = 84\ MN/m^2 = \frac{84\times10^6}{(10^3)^2}$$

$$= \frac{84\times10^6}{\times10^6} = 84\ N/mm^2$$

$$\boxed{\tau_{max} = 84\ N/mm^2}$$

$$P = 5400\ kW = 5400\times10^3\ W$$

$$\boxed{1\ kW = 1000\ W}$$

diameter :-

$$P = \frac{2\pi NT}{60} \qquad T = ?$$

$$T = \frac{P\times60}{2\pi N}$$

$$= \frac{5400\times10^3\times60}{2\times\pi\times110}$$

$$T = 468783\ Nm$$

$$T = 468783\times10^3\ Nmm$$

$$D = ?$$

By using torsional equation,

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{l}$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$T = J \times \frac{\tau}{R}$$

We know that, $J = \frac{\pi d^4}{32}$, if bar is used as $\boxed{\text{Shaft}}$

$$T = \frac{\pi}{32} \times \left[D^4 - d^4\right] \times \frac{\tau}{D/2} \qquad \boxed{\because R = \frac{D}{2}}$$

$$= \frac{\pi}{16} \times \left[\frac{D^4 - \left(\frac{D^4}{16}\right)}{D}\right] \times$$

$$= \frac{\pi}{16} \left[\frac{\left(\frac{16D^4 - D^4}{16}\right)}{D}\right] \times \tau$$

$$= \frac{\pi}{16} \left[\frac{\left(\frac{15D^4}{16}\right)}{D/1}\right] \times \tau = \frac{\pi}{16}\left[\frac{15D^4}{16} \times \frac{1}{D}\right] \times \tau$$

$$= \frac{\pi}{16}\left[\frac{15\cancel{D^4}^{\,3}}{16\cancel{D}}\right] \times \tau = \frac{\pi}{16}\left[\frac{15D^3}{16}\right] \times \tau$$

$$= \frac{\pi \times 15 \times D^3 \times \tau}{256} \Rightarrow T = \frac{\pi \times 15 \times D^3 \times \tau}{256}$$

$$4.68783 \times 10^3 = \frac{\pi \times 15 \times D^3 \times \overset{84}{\tau_{max}}}{256}$$

$$\Rightarrow D^3 = \frac{4.68783 \times 10^3 \times 256}{\pi \times 15 \times 84}$$

$$D^3 = 30317361.45$$

$$D = \sqrt[3]{30317361.45}$$

$$D = 311.8 \text{ mm}$$

$$\Rightarrow \boxed{D = 311.8\text{mm}}$$

$$d = \frac{D}{2} = \frac{311.8}{2} = 155.9 \text{ mm}$$

$$\boxed{d = 155.9\text{mm}}$$

ii) $\frac{U}{\text{Volume}} = \frac{5\tau^2}{16C} \Rightarrow \frac{U}{\text{Volume}} = \frac{5}{16} \times \frac{\tau^2}{C}$ $\quad \boxed{C = 90\times10^3}$

$$= \frac{5}{16} \times \left(\frac{(84)^2}{90\times10^3}\right)$$

$$= 0.0245 \text{ Nmm}$$

$$\boxed{\frac{U}{\text{Volume}} = 24.5 \times10^{-6} \text{ Nm}}$$

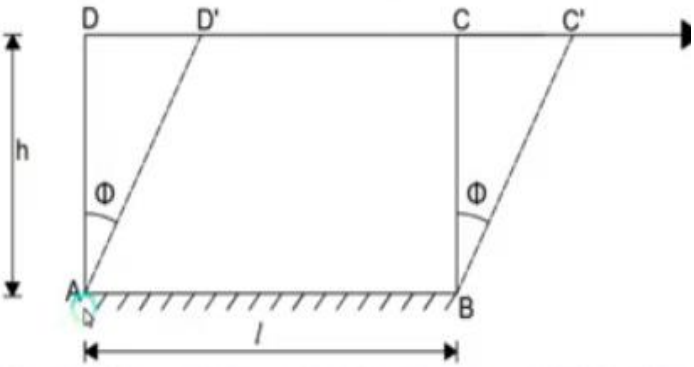
STRAIN ENERGY DUE TO SHEAR STRESS

Let

τ = Shear Stress

Φ = Shear Strain

S = Shear Force



$$\tau = \frac{\text{Shear Force}}{\text{Area}} = \frac{S}{b \times l}$$

$$S = \tau \times b \times l$$

$$\tan\Phi = \frac{CC'}{CB} = \Phi \text{ (because } \Phi \text{ is very small)}$$

$$CC' = CB\,\Phi$$

Work done by shear force $= \text{Average load} \times \text{distance}$

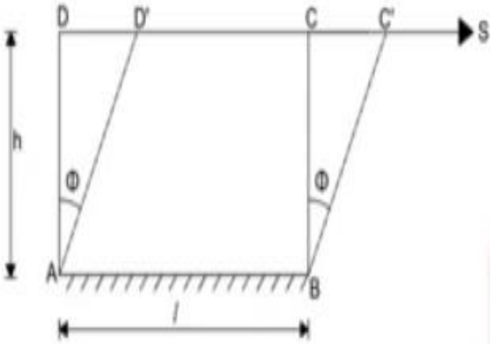
$$= \frac{S}{2} \times CC'$$

$$= \frac{1}{2} \times \tau \times b \times l \times CB \times \Phi$$

$$= \frac{1}{2} \times \tau \times b \times l \times h \times \frac{\tau}{C}$$

$$= \frac{\tau^2}{2C} \times b \times l \times h$$

$$= \frac{\tau^2}{2C} \times V$$



$S = \tau \times b \times l$

$CC' = CB\,\Phi$

$$\text{Shear Strain } \Phi = \frac{\text{Shear Stress } (\tau)}{\text{Modulus of Rigidity } (C)}$$

Strain energy stored = Work done by load

Strain energy stored due to shear $= \frac{\tau^2}{2C} \times V$