UNIT - I : PROBABILITY & RANDOM VARIABLE

TOPIC 1 : AXIOMS OF PROBABILITY :

★ **Probability :**

$$\text{Probability of an event happening} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$

Example : the chance of rolling a "6" with a die.

$$\text{Probability} = \frac{1}{6}.$$

★ **Experiment or Trial :**

An action where the result is uncertain.

Example : Tossing a coin, Throwing a dice.

★ **Sample Space :**

All the possible outcomes of an experiment.

Example : Choosing a card from a deck.

★ **Event :**

A single result of an experiment.

Example :
- Getting a tail when tossing a coin
- Rolling a "5" in a die.

★ **Favourable Events :**

The number of outcomes of a random experiment which result in the happening of a particular events are called Favourable events.

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Example :

In a toss of two coins, the number of cases favourable to the event can be said " exactly one head" which are 2, namely {HT, TH}.

★ Independent events :

Two or more events are said to be independent if the occurence of one event does not affects the occurence of the other event.

Example : Tossing a coin : one trial is not affected by the result of the previous trial.

★ Dependent Events :

Two or more events are said to be dependent if the occurence of one event affects the occurence of the other event.

Ex : Out of 52 cards, if one is drawn, then only 51 are left. Unless the card is put back or replaced, the composition stands changed and the probability of the second card is affected.

★ Mutually exclusive events :

Two or more events are said to be mutually exclusive if the happening of any one of them excludes the happening of all the other events.

Example : In tossing a coin, we can't get head and tail simultaneously, any one event can occur at a time.

★ Equally likely events :

Two or more events are equally likely if each of them has an equal chance of happening.

Example : In tossing a coin, all the outcomes of head or tail are equally likely, if the coin is not biased.

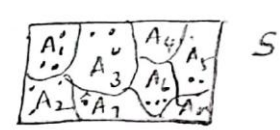
★ Mathematical Definition of probability :

If there are n equally likely, mutually exclusive and exhaustive outcomes and m of them are favourable to an event A, then the probability of the happening of A is given by,

$$P(A) = \frac{m}{n} = \frac{\text{Number of Favourable events}}{\text{Total number of events}}$$

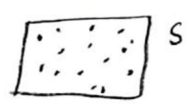
★ Axioms of probability :

Let S be the Sample Space and A be an event and let P be a real valued function defined on $P(S)$. Then $P(A)$ is called the probability of the event A, if P satisfies the following conditions :



(i) For every event A, $0 \le P(A) \le 1$

(ii) $P(S) = 1$



(iii) If $A_1, A_2, A_3, \cdots A_n$ are mutually exclusive events then,

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$$

(4)

Theorems of probability :

(i). Probability of an impossible event is zero.

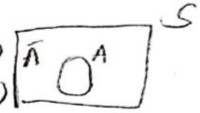
i.e., $P(\varphi) = 0$.

(ii) $P(\bar{A}) = 1 - P(A)$

$P(S) = P(A) + P(\bar{A})$

$1 = P(A) + P(\bar{A})$

$P(\bar{A}) = 1 - P(A)$

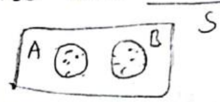


(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, if the events are not mutually exclusive.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B)$, if the events are mutually exclusive.

nothing in common



(iv) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.

(v) Suppose A and B are two events, if $B \subseteq A$ then $P(B) \leq P(A)$

(vi) $P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B)$

Problems :

(1) Four persons are chosen at random from a group containing 3 men, 2 women and 4 children. Show that the chance of exactly two of them being children is 10/21.

Solution:

$$P(A) = \frac{4C_2 \times 5C_2}{9C_4} = \frac{\frac{4 \times 3}{1 \times 2} \times \frac{5 \times 4}{1 \times 2}}{\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}} = \frac{60}{126} = \frac{10}{21}$$

$$\boxed{P(A) = \frac{10}{21}}$$