Data Structures and Algorithms

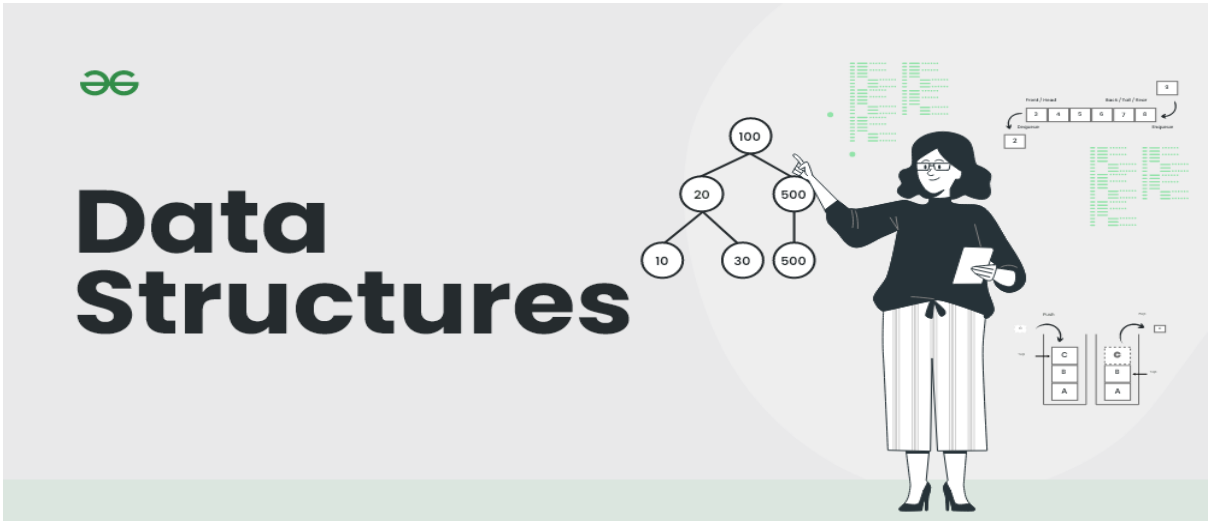
Unit – I

Elementary Data Structures

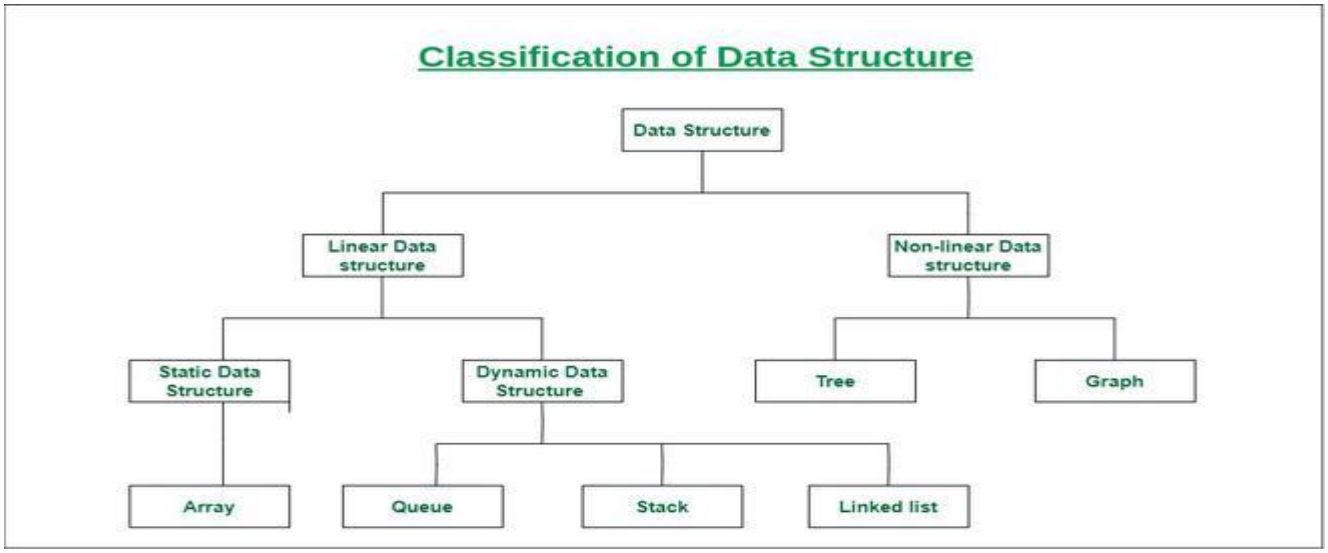
What is Data Structure:

A data structure is a storage that is used to store and organize data. It is a way of arranging data on a computer so that it can be accessed and updated efficiently.

Data Structure=Organized data +Allowed operations



Classification of Data Structure:



Operations on the Data Structures:

Following operations can be performed on the data structures:

1. Traversing
2. Searching
3. Inserting
4. Deleting
5. Sorting
6. Merging

1. Traversing- It is used to access each data item exactly once so that it can be processed.
2. Searching- It is used to find out the location of the data item if it exists in the given collection of data items.
3. Inserting- It is used to add a new data item in the given collection of data items.
4. Deleting- It is used to delete an existing data item from the given collection of data items.
5. Sorting- It is used to arrange the data items in some order i.e. in ascending or descending order in case of numerical data and in dictionary order in case of alphanumeric data.
6. Merging- It is used to combine the data items of two sorted files into single file in the sorted form.

- **Linear data structure:** Data structure in which data elements are arranged sequentially or linearly, where each element is attached to its previous and next adjacent elements, is called a linear data structure.
  Examples of linear data structures are array, stack, queue, linked list, etc.
  - **Static data structure:** Static data structure has a fixed memory size. It is easier to access the elements in a static data structure.
    An example of this data structure is an array.
  - **Dynamic data structure:** In dynamic data structure, the size is not fixed. It can be randomly updated during the runtime which may be considered efficient concerning the memory (space) complexity of the code.
    Examples of this data structure are queue, stack, etc.
- **Non-linear data structure:** Data structures where data elements are not placed sequentially or linearly are called non-linear data structures. In a non-linear data structure, we can't traverse all the elements in a single run only.
  Examples of non-linear data structures are trees and graphs.

Array

- Linear Data Structures using C
- Elements are stored in contiguous memory locations
- Can access elements randomly using index
- Stores homogeneous elements i.e, similar elements
- Syntax:
- Array declaration
  - Datatype varname [size] ;
- Can also do declaration and initialization at once
  - Datatype varname [] = {ele1, ele2, ele3, ele4};

Advantages

- Random access
- Easy sorting and iteration
- Replacement of multiple variables

Disadvantages

- Size is fixed
- Difficult to insert and delete
- If capacity is more and occupancy less, most of the array gets wasted
- Needs contiguous memory to get allocated

Applications

- For storing information in a linear fashion
- Suitable for applications that require frequent searching

Linked List

- Linear Data Structure
- Elements can be stored as per memory availability
- Can access elements on linear fashion only
- Stores homogeneous elements i.e, similar elements
- Dynamic in size
- Easy insertion and deletion
- Starting element or node is the key which is generally termed as the head.

Advantages

- Dynamic in size
- No wastage as capacity and size is always equal
- Easy insertion and deletion as 1 link manipulation is required
- Efficient memory allocation

Disadvantages

- If the head node is lost, the linked list is lost
- No random access possible

Applications

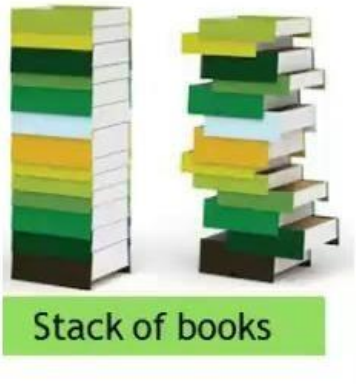
- Suitable where memory is limited
- Suitable for applications that require frequent insertion and deletion
-

Stack

A Stack is linear data structure. A stack is a list of elements in which an element may be inserted or deleted only at one end, called the top of the stack. Stack principle is LIFO (last in, first out). Which element inserted last on to the stack that element deleted first from the stack.

As the items can be added or removed only from the top i.e. the last item to be added to a stack is the first item to be removed.

Real life examples of stacks are:



Operations on stack:

The two basic operations associated with stacks are:
1. Push
2. Pop

While performing push and pop operations the following test must be conducted on the stack.

a) Stack is empty or not b) stack is full or not

1. Push: Push operation is used to add new elements in to the stack. At the time of addition first check the stack is full or not. If the stack is full it generates an error message "stack overflow".

2. Pop: Pop operation is used to delete elements from the stack. At the time of deletion first check the stack is empty or not. If the stack is empty it generates an error message "stack underflow".

All insertions and deletions take place at the same end, so the last element added to the stack will be the first element removed from the stack. When a stack is created, the stack base remains fixed while the stack top changes as elements are added and removed. The most accessible element is the top and the least accessible element is the bottom of the stack.

Representation of Stack (or) Implementation of stack:

The stack should be represented in two ways:

1. Stack using array
2. Stack using linked list

1. Stack using array:

Let us consider a stack with 6 elements capacity. This is called as the size of the stack. The number of elements to be added should not exceed the maximum size of the stack. If we attempt to add new element beyond the maximum size, we will encounter a ***stack overflow*** condition. Similarly, you cannot remove elements beyond the base of the stack. If such is the case, we will reach a ***stack underflow*** condition.

1. push(): When an element is added to a stack, the operation is performed by push(). Below Figure shows the creation of a stack and addition of elements using push().

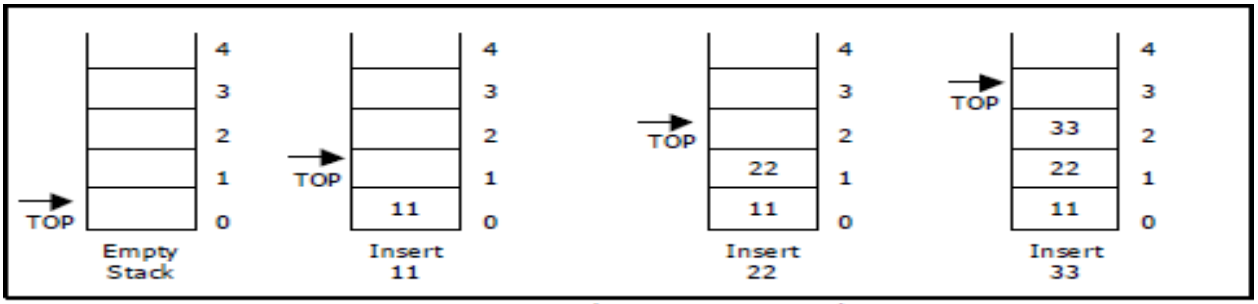


Figure . Push operations on stack

Initially **top=-1**, we can insert an element in to the stack, increment the top value i.e **top=top+1**. We can insert an element in to the stack first check the condition is stack is full or not. i.e **top>=size-1**. Otherwise add the element in to the stack.

```
void push()
{
        int x;
        if(top >= n-1)
        {
                printf("\n\nStack
        Overflow..");
                return;
        }
        else
        {
                printf("\n\nEnter data: ");
                scanf("%d", &x);
                stack[top] = x;
                top = top + 1;
                printf("\n\nData Pushed into
                the stack");
        }
}
```

Algorithm: Procedure for push():

Step 1: START
Step 2: if top>=size-1 then
        Write " Stack is Overflow"
Step 3: Otherwise
        3.1: read data value 'x'
        3.2: top=top+1;
        3.3: stack[top]=x;
Step 4: END

2. Pop(): When an element is taken off from the stack, the operation is performed by pop(). Below figure shows a stack initially with three elements and shows the deletion of elements using pop().

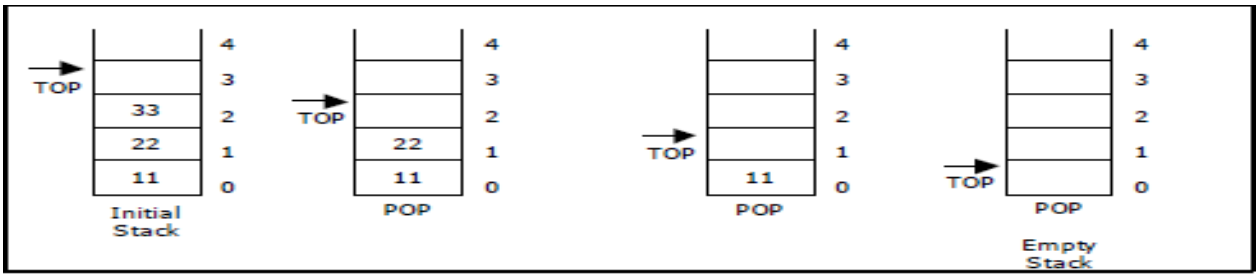


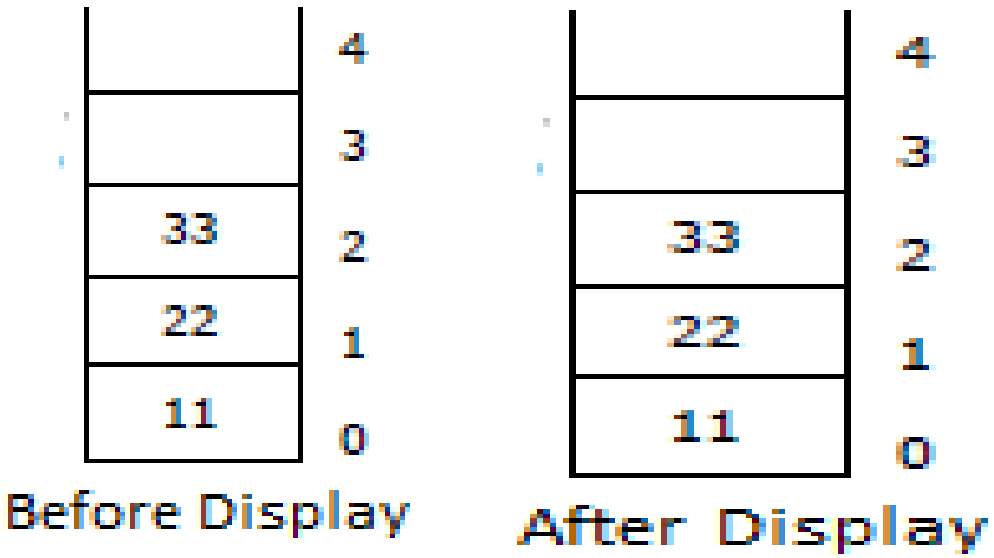
Figure Pop operations on stack

We can insert an element from the stack, decrement the top value i.e **top=top-1**.
We can delete an element from the stack first check the condition is stack is empty or not. i.e **top==-1**. Otherwise remove the element from the stack.

```
Void pop()
{
   If(top==-1)
   {
      Printf("Stack is Underflow");
   }
   else
   {
      printf("Delete data %d",stack[top]);
      top=top-1;
   }
}
```

Algorithm: procedure pop():
Step 1: START
Step 2: if top==-1 then
Write "Stack is Underflow"
Step 3: otherwise
3.1: print "deleted element"
3.2: top=top-1;
Step 4: END

3. display(): This operation performed display the elements in the stack. We display the element in the stack check the condition is stack is empty or not i.e top==-1.Otherwise display the list of elements in the stack.



void display()
{
If(top==-1)
{
Printf(“Stack is Underflow”);
}
else
{
printf(“Display elements are:);
for(i=top;i>=0;i--)
printf(“%d”,stack[i]);
}
}
	Algorithm: procedure pop():
	Step 1: START
	Step 2: if top==-1 then
	Write “Stack is Underflow”
	Step 3: otherwise
	3.1: print “Display elements are”
	3.2: for top to 0
	Print ‘stack[i]’
	Step 4: END

Source code for stack operations, using array:

```
#include<stdio.h>
#inlcude<conio.h>
int stack[100],choice,n,top,x,i;
void push(void);
void pop(void);
void display(void);
int main()
{
  //clrscr();
  top=-1;
  printf("\n Enter the size of STACK[MAX=100]:");
  scanf("%d",&n);
  printf("\n\t STACK OPERATIONS USING ARRAY");
  printf("\n\t----------");
  printf("\n\t 1.PUSH\n\t 2.POP\n\t 3.DISPLAY\n\t 4.EXIT");
  do
  {
    printf("\n Enter the Choice:");
    scanf("%d",&choice);
    switch(choice)
    {
      case 1:
      {
        push();
        break;
      }
      case 2:
      {
        pop();
        break;
      }
      case 3:
      {
```

```
            display();
            break;
         }
         case 4:
         {
            printf("\n\t EXIT POINT ");
            break;
         }
         default:
         {
            printf ("\n\t Please Enter a Valid Choice(1/2/3/4)");
         }

      }
   }
   while(choice!=4);
   return 0;
}
void push()
{
   if(top>=n-1)
   {
      printf("\n\tSTACK is over flow");

   }
   else
   {
      printf(" Enter a value to be pushed:");
      scanf("%d",&x);
      top++;
      stack[top]=x;
   }
}
void pop()
{
   if(top<=-1)
   {
      printf("\n\t Stack is under flow");
   }
   else
   {
      printf("\n\t The popped elements is %d",stack[top]);
      top--;
   }
}
void display()
{
   if(top>=0)
   {
```

```
        printf("\n The elements in STACK \n");
        for(i=top; i>=0; i--)
           printf("\n%d",stack[i]);
        printf("\n Press Next Choice");
     }
     else
     {
        printf("\n The STACK is empty");
     }

  }
```

2. Stack using Linked List:

We can represent a stack as a linked list. In a stack push and pop operations are performed at one end called top. We can perform similar operations at one end of list using top pointer. The linked stack looks as shown in figure.

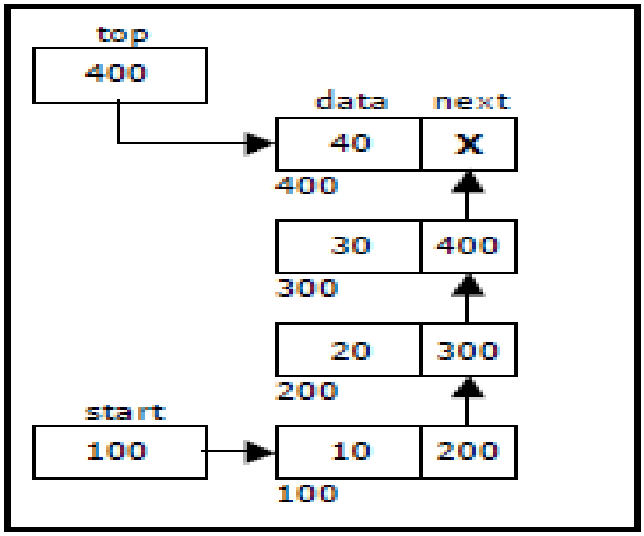


Figure Linked stack representation

Applications of stack:

1. Stack is used by compilers to check for balancing of parentheses, brackets and braces.
2. Stack is used to evaluate a postfix expression.
3. Stack is used to convert an infix expression into postfix/prefix form.
4. In recursion, all intermediate arguments and return values are stored on the processor's stack.
5. During a function call the return address and arguments are pushed onto a stack and on return they are popped off.

Converting and evaluating Algebraic expressions:

An **algebraic expression** is a legal combination of operators and operands. Operand is the quantity on which a mathematical operation is performed. Operand may be a variable like x, y, z or a constant like 5, 4, 6 etc. Operator is a symbol which signifies a mathematical or logical operation between the operands. Examples of familiar operators include +, -, *, /, ^ etc.

An algebraic expression can be represented using three different notations. They are infix, postfix and prefix notations:

Infix: It is the form of an arithmetic expression in which we fix (place) the arithmetic operator in between the two operands.

Example: A + B

Prefix: It is the form of an arithmetic notation in which we fix (place) the arithmetic operator before (pre) its two operands. The prefix notation is called as polish notation.

Example: + A B

Postfix: It is the form of an arithmetic expression in which we fix (place) the arithmetic operator after (post) its two operands. The postfix notation is called as *suffix notation* and is also referred to *reverse polish notation*.

Example: A B +

Conversion from infix to postfix:

Procedure to convert from infix expression to postfix expression is as follows:

1. Scan the infix expression from left to right.
2. a) If the scanned symbol is left parenthesis, push it onto the stack.
   b) If the scanned symbol is an operand, then place directly in the postfix expression (output).
   c) If the symbol scanned is a right parenthesis, then go on popping all the items from the stack and place them in the postfix expression till we get the matching left parenthesis.
   d) If the scanned symbol is an operator, then go on removing all the operators from the stack and place them in the postfix expression, if and only if the precedence of the operator which is on the top of the stack is greater than (*or greater than or equal*) to the precedence of the scanned operator and push the scanned operator onto the stack otherwise, push the scanned operator onto the stack.

The three important features of postfix expression are:

1. The operands maintain the same order as in the equivalent infix expression.
2. The parentheses are not needed to designate the expression unambiguously.
3. While evaluating the postfix expression the priority of the operators is no longer relevant.

We consider five binary operations: +, -, *, / and $ or ↑ (exponentiation). For these binary operations, the following in the order of precedence (highest to lowest):

OPERATOR	PRECEDENCE	VALUE
Exponentiation ($ or ↑ or ^)	Highest	3
*, /	Next highest	2
+, -	Lowest	1

Example 1:

Convert ((A – (B + C)) * D) ↑ (E + F) infix expression to postfix form:

SYMBOL	POSTFIX STRING	STACK	REMARKS
(		(	
(		((	
A	A	((	
-	A	((-	
(	A	((- (	
B	A B	((- (	
+	A B	((- (+	
C	A B C	((- (+	
)	A B C +	((-	
)	A B C + -	(	
*	A B C + -	(*	
D	A B C + - D	(*	
)	A B C + - D *		
↑	A B C + - D *	↑	
(	A B C + - D *	↑ (	
E	A B C + - D * E	↑ (	
+	A B C + - D * E	↑ (+	
F	A B C + - D * E F	↑ (+	
)	A B C + - D * E F +	↑	
End of string	A B C + - D * E F + ↑	The input is now empty. Pop the output symbols from the stack until it is empty.	

Example 2:

Convert the following infix expression A + B * C – D / E * H into its equivalent postfix expression.

SYMBOL	POSTFIX STRING	STACK	REMARKS
A	A		
+	A	+	
B	A B	+	
*	A B	+ *	
C	A B C	+ *	
-	A B C * +	-	
D	A B C * + D	-	
/	A B C * + D	- /	
E	A B C * + D E	- /	
*	A B C * + D E /	- *	
H	A B C * + D E / H	- *	
End of string	A B C * + D E / H * -	The input is now empty. Pop the output symbols from the stack until it is empty.	

Evaluation of postfix expression:

The postfix expression is evaluated easily by the use of a stack.

1. When a number is seen, it is pushed onto the stack;
2. When an operator is seen, the operator is applied to the two numbers that are popped from the stack and the result is pushed onto the stack.
3. When an expression is given in postfix notation, there is no need to know any precedence rules; this is our obvious advantage.

Example 1:

Evaluate the postfix expression: 6 5 2 3 + 8 * + 3 + *

SYMBOL	OPERAND 1	OPERAND 2	VALUE	STACK	REMARKS
6				6	
5				6, 5	
2				6, 5, 2	
3				6, 5, 2, 3	The first four symbols are placed on the stack.
+	2	3	5	6, 5, 5	Next a '+' is read, so 3 and 2 are popped from the stack and their sum 5, is pushed
8	2	3	5	6, 5, 5, 8	Next 8 is pushed
*	5	8	40	6, 5, 40	Now a '*' is seen, so 8 and 5 are popped as 8 * 5 = 40 is pushed
+	5	40	45	6, 45	Next, a '+' is seen, so 40 and 5 are popped and 40 + 5 = 45 is pushed
3	5	40	45	6, 45, 3	Now, 3 is pushed
+	45	3	48	6, 48	Next, '+' pops 3 and 45 and pushes 45 + 3 = 48 is pushed
*	6	48	288	288	Finally, a '*' is seen and 48 and 6 are popped, the result 6 * 48 = 288 is pushed

Example 2:

Evaluate the following postfix expression: 6 2 3 + - 3 8 2 / + * 2 ↑ 3 +

SYMBOL	OPERAND 1	OPERAND 2	VALUE	STACK
6				6
2				6, 2
3				6, 2, 3
+	2	3	5	6, 5
-	6	5	1	1
3	6	5	1	1, 3
8	6	5	1	1, 3, 8
2	6	5	1	1, 3, 8, 2
/	8	2	4	1, 3, 4
+	3	4	7	1, 7
*	1	7	7	7
2	1	7	7	7, 2
↑	7	2	49	49
3	7	2	49	49, 3
+	49	3	52	52

Advantages

- Maintains data in a LIFO manner
- The last element is readily available for use
- All operations are of O(1) complexity

Disadvantages

- Manipulation is restricted to the top of the stack
- Not much flexible

Applications

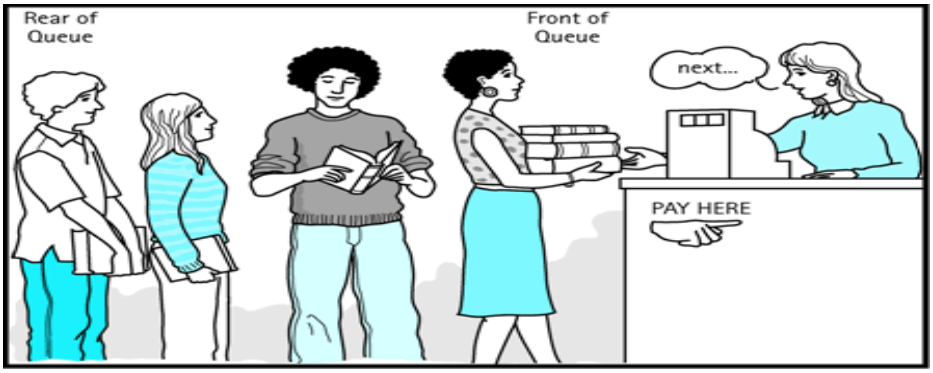
- Recursion
- Parsing
- Browser
- Editors

Queue

A queue is linear data structure and collection of elements. A queue is another special kind of list, where items are inserted at one end called the **rear** and deleted at the other end called the **front**. The principle of queue is a **"FIFO" or "First-in-first-out"**.
Queue is an abstract data structure. A queue is a useful data structure in programming. **It is similar to the ticket queue outside a cinema hall**, where the first person entering the queue is the first person who gets the ticket.

A real-world example of queue can be a single-lane one-way road, where the vehicle enters first, exits first.

More real-world examples can be seen as queues at the ticket windows and bus-stops and our college library.



The operations for a queue are analogues to those for a stack; the difference is that the

insertions go at the end of the list, rather than the beginning.

Operations on QUEUE:

A queue is an object or more specifically an abstract data structure (ADT) that allows the following operations:

- **Enqueue or insertion:** which inserts an element at the end of the queue.
- **Dequeue or deletion:** which deletes an element at the start of the queue.

Queue operations work as follows:

1. Two pointers called FRONT and REAR are used to keep track of the first and last elements in the queue.
2. When initializing the queue, we set the value of FRONT and REAR to 0.
3. On enqueing an element, we increase the value of REAR index and place the new element in the position pointed to by REAR.
4. On dequeueing an element, we return the value pointed to by FRONT and increase the FRONT index.
5. Before enqueing, we check if queue is already full.
6. Before dequeuing, we check if queue is already empty.
7. When enqueing the first element, we set the value of FRONT to 1.
8. When dequeing the last element, we reset the values of FRONT and REAR to 0.

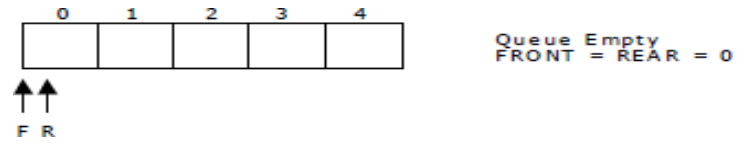
Representation of Queue (or) Implementation of Queue:

The queue can be represented in two ways:

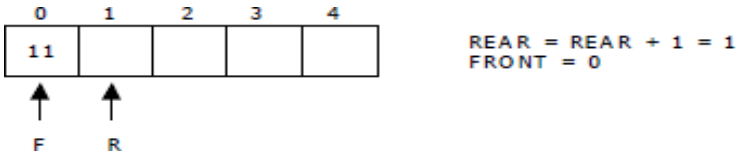
1. Queue using Array
2. Queue using Linked List

1. Queue using Array:

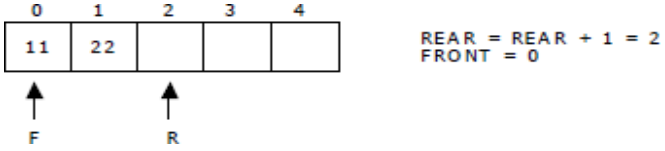
Let us consider a queue, which can hold maximum of five elements. Initially the queue is empty.



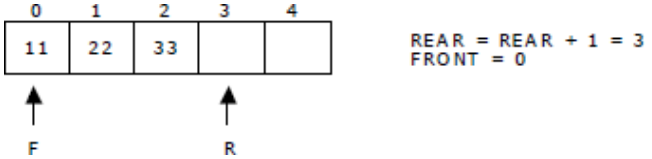
Now, insert 11 to the queue. Then queue status will be:



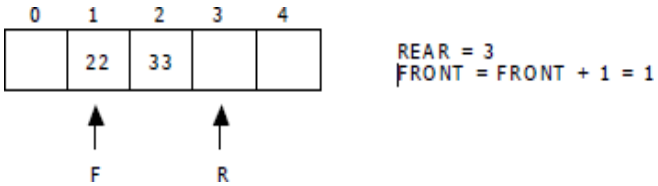
Next, insert 22 to the queue. Then the queue status is:



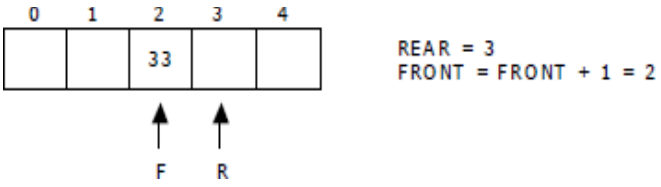
Again insert another element 33 to the queue. The status of the queue is:



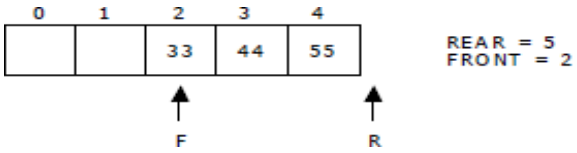
Now, delete an element. The element deleted is the element at the front of the queue.So the status of the queue is:



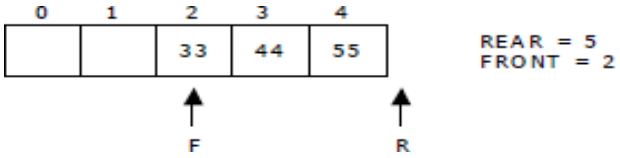
Again, delete an element. The element to be deleted is always pointed to by the FRONT pointer. So, 22 is deleted. The queue status is as follows:



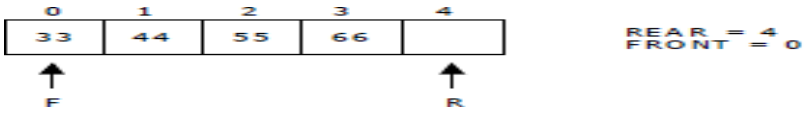
Now, insert new elements 44 and 55 into the queue. The queue status is



Next insert another element, say 66 to the queue. We cannot insert 66 to the queue as the rear crossed the maximum size of the queue (i.e., 5). There will be queue full signal. The queue status is as follows:



Now it is not possible to insert an element 66 even though there are two vacant positions in the linear queue. To overcome this problem the elements of the queue are to be shifted towards the beginning of the queue so that it creates vacant position at the rear end. Then the FRONT and REAR are to be adjusted properly. The element 66 can be inserted at the rear end. After this operation, the queue status is as follows:



This difficulty can overcome if we treat queue position with index 0 as a position that comes after position with index 4 i.e., we treat the queue as a **circular queue.**

Queue operations using array:

a.enqueue() or insertion():which inserts an element at the end of the queue.

void insertion()
{
if(rear==max)
printf("\n Queue is Full");
else
{
printf("\n Enter no %d:",j++);
scanf("%d",&queue[rear++]);
}
}
	Algorithm: Procedure for insertion():
	Step-1:START
	Step-2: if rear==max then
	Write 'Queue is full'
	Step-3: otherwise
	3.1: read element 'queue[rear]'
	Step-4:STOP

b.dequeue() or deletion(): which deletes an element at the start of the queue.

void deletion()
{
if(front==rear)
{
printf("\n Queue is empty");
}
else
{
printf("\n Deleted Element is
%d",queue[front++]);
x++;
} }
	Algorithm: procedure for deletion():
	Step-1:START
	Step-2: if front==rear then
	Write' Queue is empty'
	Step-3: otherwise
	3.1: print deleted element
	Step-4:STOP

c.dispaly(): which displays an elements in the queue.

void deletion()
{
if(front==rear)
{
printf("\n Queue is empty");
}
else
{
for(i=front; i<rear; i++)
{
printf("%d",queue[i]);
printf("\n");
}
}
}
	Algorithm: procedure for deletion():
	Step-1:START
	Step-2: if front==rear then
	Write' Queue is empty'
	Step-3: otherwise
	3.1: for i=front to rear then
	3.2: print 'queue[i]'
	Step-4:STOP

2. Queue using Linked list:

We can represent a queue as a linked list. In a queue data is deleted from the front end and inserted at the rear end. We can perform similar operations on the two ends of alist. We use two pointers *front* and *rear* for our linked queue implementation.

The linked queue looks as shown in figure:

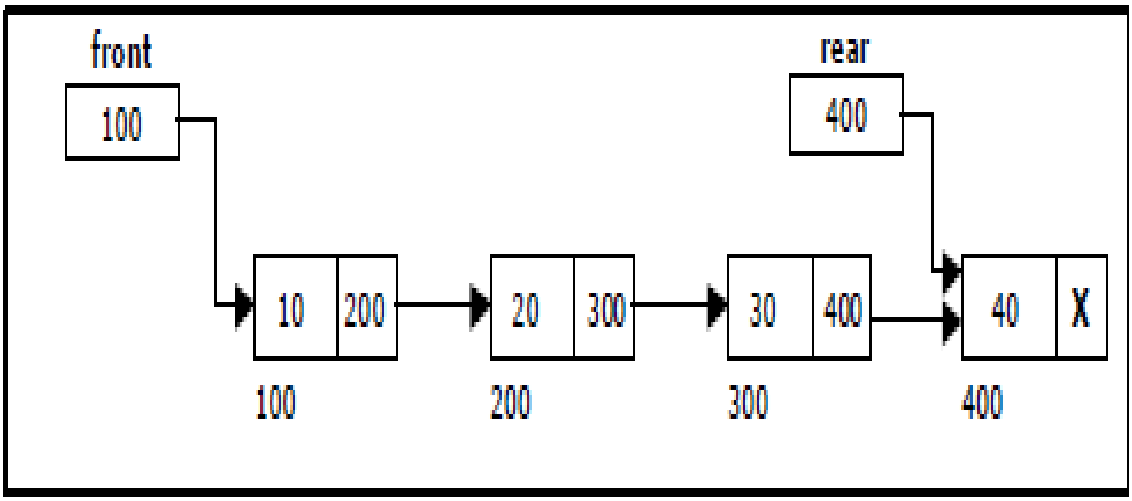


Figure ; Linked Queue representation

Applications of Queue:

1. It is used to schedule the jobs to be processed by the CPU.
2. When multiple users send print jobs to a printer, each printing job is kept in the printing queue. Then the printer prints those jobs according to first in first out (FIFO) basis.
3. Breadth first search uses a queue data structure to find an element from a graph.

Advantages

- Maintains data in FIFO manner
- Insertion from beginning and deletion from end takes O(1) time

Unit - II

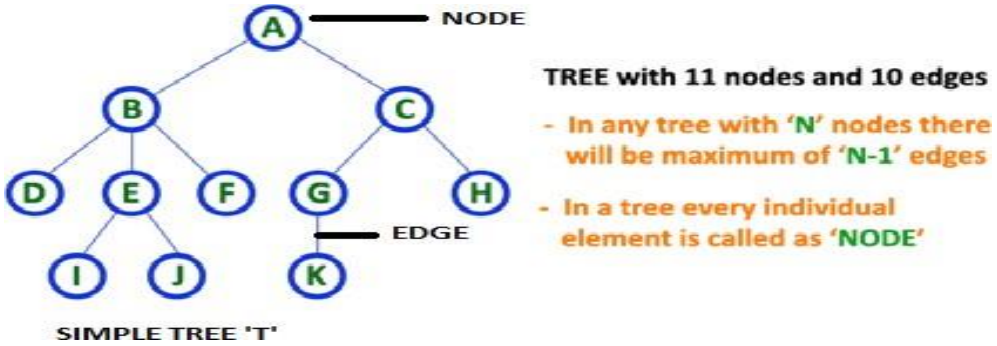
Tress

INTRODUCTION

In linear data structure data is organized in sequential order and in non-linear data structure data is organized in random order. A tree is a very popular non-linear data structure used in a wide range of applications. Tree is a non-linear data structure which organizes data in hierarchical structure and this is a recursive definition.

DEFINITION OF TREE:

Tree is collection of nodes (or) vertices and their edges (or) links. In tree data structure, every individual element is called as **Node**. Node in a tree data structure stores the actual data of that particular element and link to next element in hierarchical structure.



SIMPLE TREE 'T'

Note: 1. In a **Tree**, if we have **N** number of nodes then we can have a maximum of **N-1** number of links or edges.

2. **Tree** has no cycles.

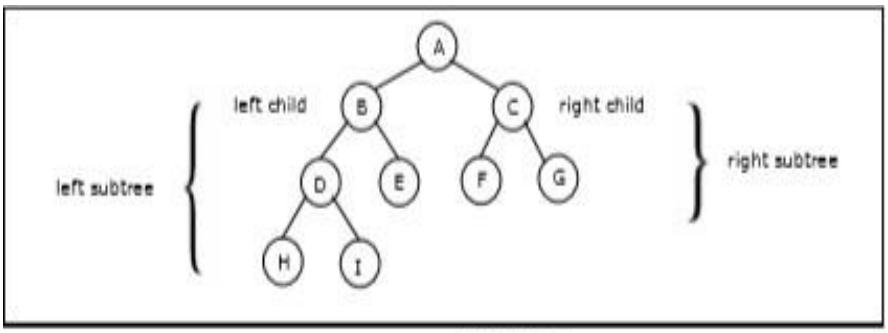
In a normal tree, every node can have any number of children. A binary tree is a special type of tree data structure in which every node can have a **maximum of 2 children**. One is known as a left child and the other is known as right child.

A tree in which every node can have a maximum of two children is called Binary Tree.

In a binary tree, every node can have either 0 children or 1 child or 2 children but not more than 2 children.

In general, tree nodes can have any number of children. In a binary tree, each node can have at most two children. A binary tree is either empty or consists of a node called the root together with two binary trees called the left subtree and the right subtree. A tree with no nodes is called as a null tree

Example:



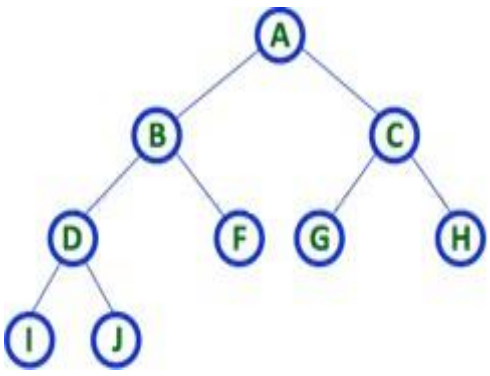
. Binary Tree

TYPES OF BINARY TREE:

1. Strictly Binary Tree:
In a binary tree, every node can have a maximum of two children. But in strictly binary tree, every node should have exactly two children or none. That means every internal node must have exactly two children. A strictly Binary Tree can be defined as follows...

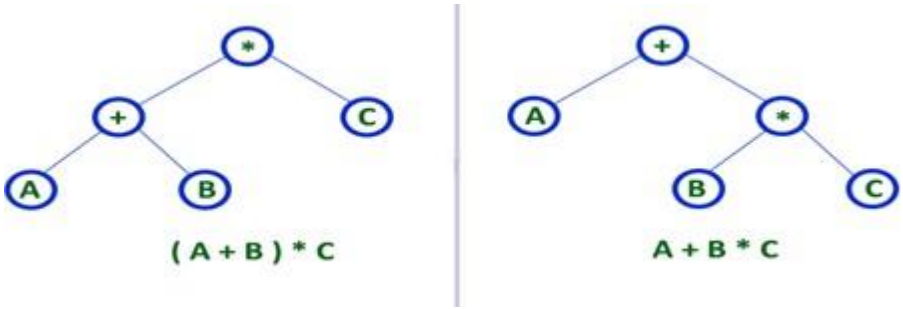
A binary tree in which every node has either two or zero number of children is called Strictly Binary Tree

Strictly binary tree is also called as **Full Binary Tree** or **Proper Binary Tree** or **2-Tree.**



Strictly binary tree data structure is used to represent mathematical expressions.

Example



2. Complete Binary Tree:

In a binary tree, every node can have a maximum of two children. But in strictly binary tree, every node should have exactly two children or none and in complete binary tree all the nodes must have exactly two children and at every level of complete binary tree there must be 2^{level} number of nodes. For example at level 2 there must be $2^2 = 4$ nodes and at level 3 there must be $2^3 = 8$ nodes.

A binary tree in which every internal node has exactly two children and all leaf nodes are at same level is called Complete Binary Tree.

Complete binary tree is also called as **Perfect Binary Tree.**

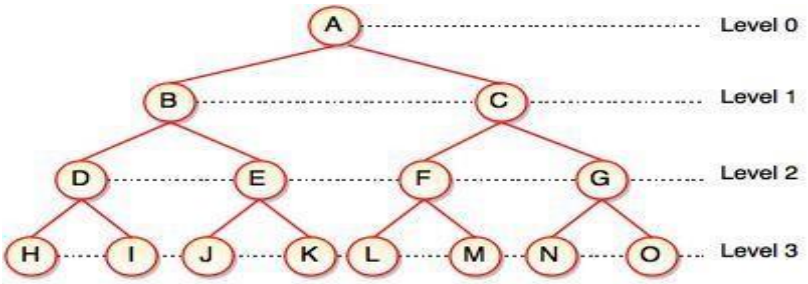
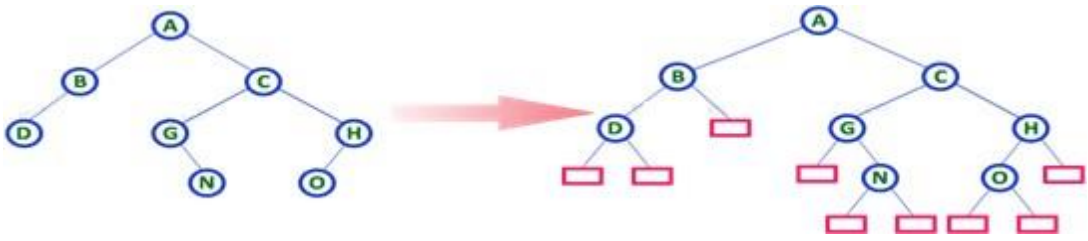


Fig. Complete Binary Tree

3. Extended Binary Tree:

A binary tree can be converted into Full Binary tree by adding dummy nodes to existing nodes wherever required.

The full binary tree obtained by adding dummy nodes to a binary tree is called as Extended Binary Tree.



In above figure, a normal binary tree is converted into full binary tree by adding dummy nodes.

4. Skewed Binary Tree:

If a tree which is dominated by left child node or right child node, is said to be a **Skewed Binary Tree**.

In a **skewed binary tree**, all nodes except one have only one child node. The remaining node has no child.

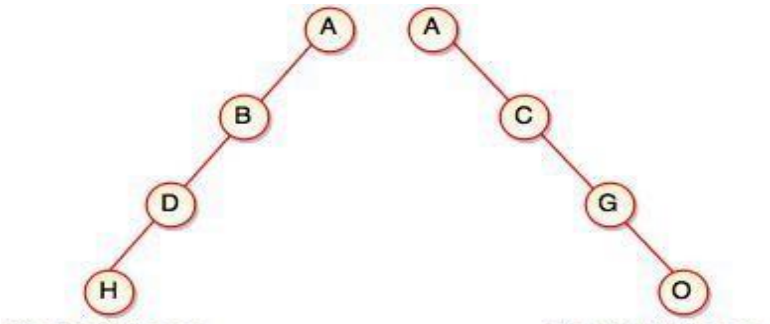


Fig. Left Skewed Binary Tree

Fig. Right Skewed Binary Tree

In a left skewed tree, most of the nodes have the left child without corresponding right child.

In a right skewed tree, most of the nodes have the right child without corresponding left child.

Properties of binary trees:

Some of the important properties of a binary tree are as follows:

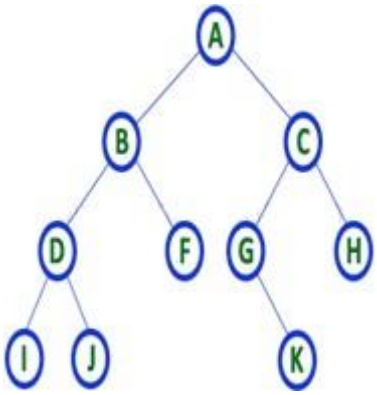
1. If h = height of a binary tree, then
    a. Maximum number of leaves = 2h
    b. Maximum number of nodes = 2h + 1 - 1
2. If a binary tree contains m nodes at level l, it contains at most 2m nodes at level l + 1.
3. Since a binary tree can contain at most one node at level 0 (the root), it can contain at most 2l node at level l.
4. The total number of edges in a full binary tree with n node is n –

BINARY TREE REPRESENTATIONS:

A binary tree data structure is represented using two methods. Those methods are as follows...

1. Array Representation
2. **Linked List Representation**

Consider the following binary tree...



1. Array Representation of Binary Tree

In array representation of a binary tree, we use one-dimensional array (1-D Array) to represent a binary tree.

Consider the above example of a binary tree and it is represented as follows...

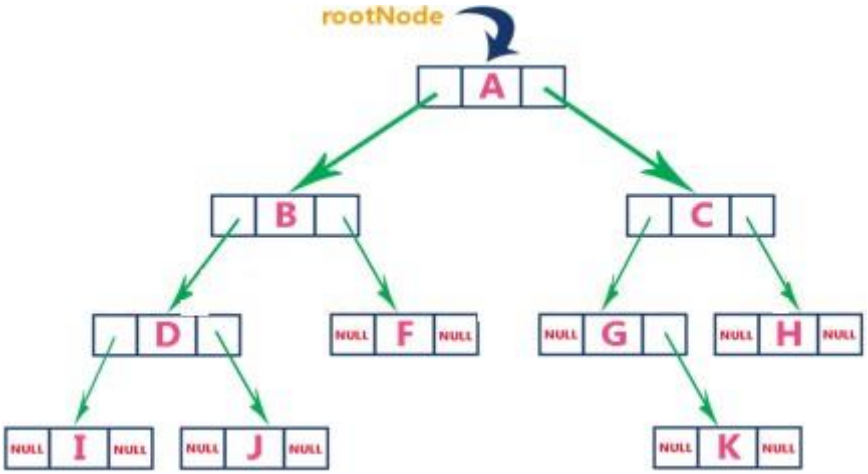
To represent a binary tree of depth **'n'** using array representation, we need one dimensional array with a maximum size of $\mathbf{2^{n+1}}$.

2. Linked List Representation of Binary Tree

We use a double linked list to represent a binary tree. In a double linked list, every node consists of three fields. First field for storing left child address, second for storing actual data and third for the right child address.

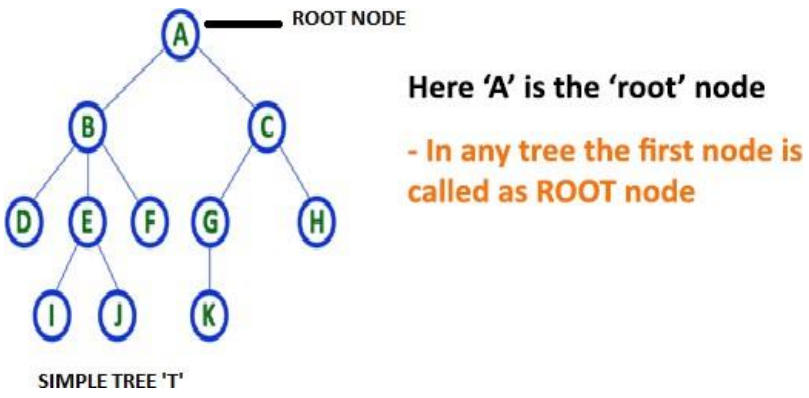
In this linked list representation, a node has the following structure...

The above example of the binary tree represented using Linked list representation is shown as follows...

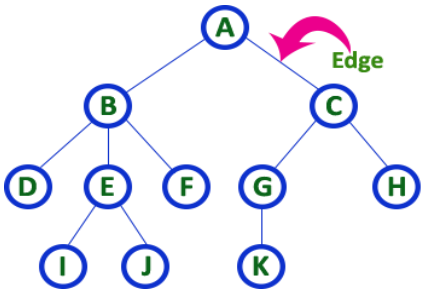


TREE TERMINOLOGIES:

1. Root Node: In a **Tree** data structure, the first node is called as **Root Node**. Every tree must have a root node. We can say that the root node is the origin of the tree data structure. In any tree, there must be only one root node. We never have multiple root nodes in a tree.

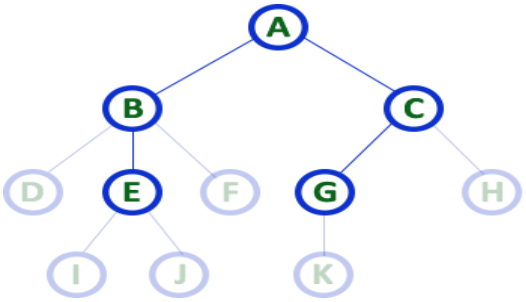


2. Edge: In a **Tree**, the connecting link between any two nodes is called as **EDGE**. In a tree with '**N**' number of nodes there will be a maximum of '**N-1**' number of edges.



- In any tree, 'Edge' is a connecting link between two nodes.

3. Parent Node: In a **Tree**, the node which is a predecessor of any node is called as **PARENT NODE**. In simple words, the node which has a branch from it to any other node is called a parent node. Parent node can also be defined as "**The node which has child / children**".

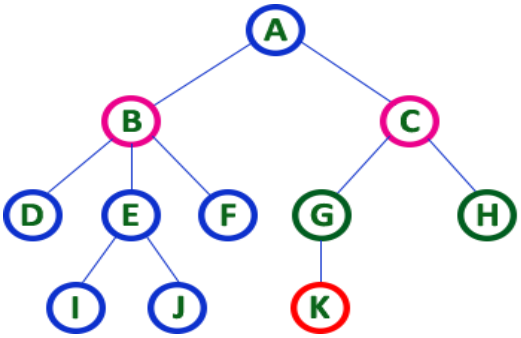


Here A, B, C, E & G are Parent nodes

- In any tree the node which has child / children is called 'Parent'
- A node which is predecessor of any other node is called 'Parent'

Here, A is parent of B&C. B is the parent of D,E&F and so on…

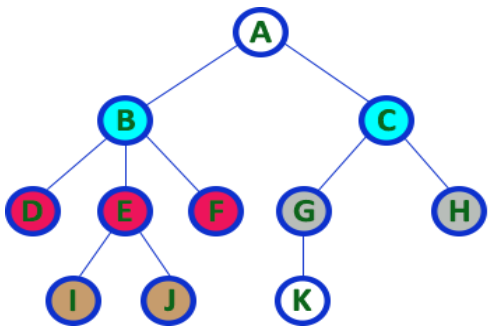
4. Child Node: In a **Tree** data structure, the node which is descendant of any node is called as **CHILD Node**. In simple words, the node which has a link from its parent node is called as child node. In a tree, any parent node can have any number of child nodes. In a tree, all the nodes except root are child nodes.



Here B & C are Children of A
Here G & H are Children of C
Here K is Child of G

- descendant of any node is called as CHILD Node

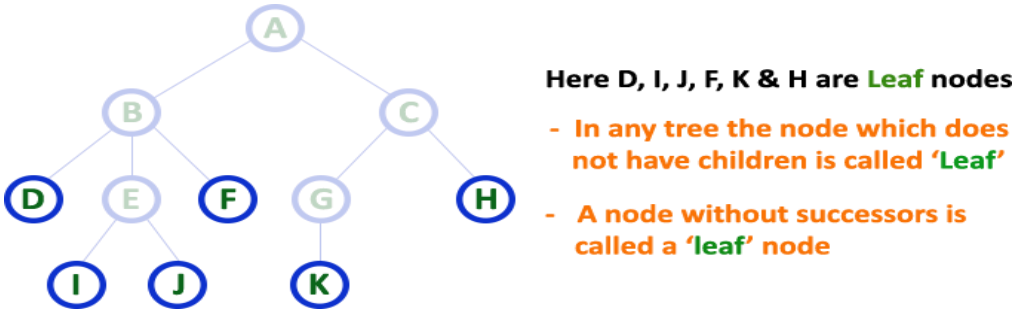
5. Siblings: In a **Tree** data structure, nodes which belong to same Parent are called as **SIBLINGS**. In simple words, the nodes with the same parent are called Sibling nodes.



Here B & C are Siblings
Here D E & F are Siblings
Here G & H are Siblings
Here I & J are Siblings

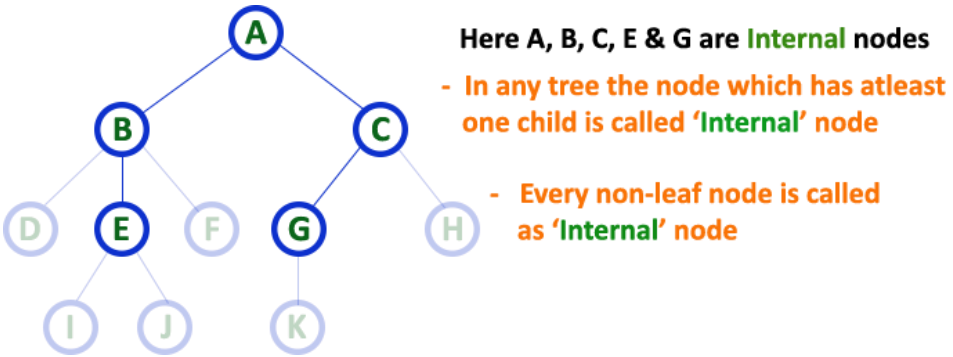
- In any tree the nodes which has same Parent are called 'Siblings'
- The children of a Parent are called 'Siblings'

6. Leaf Node: In a **Tree** data structure, the node which does not have a child is called as LEAF Node. In simple words, a leaf is a node with no child. In a tree data structure, the leaf nodes are also called as External Nodes. External node is also a node with no child. In a tree, leaf node is also called as 'Terminal' node.

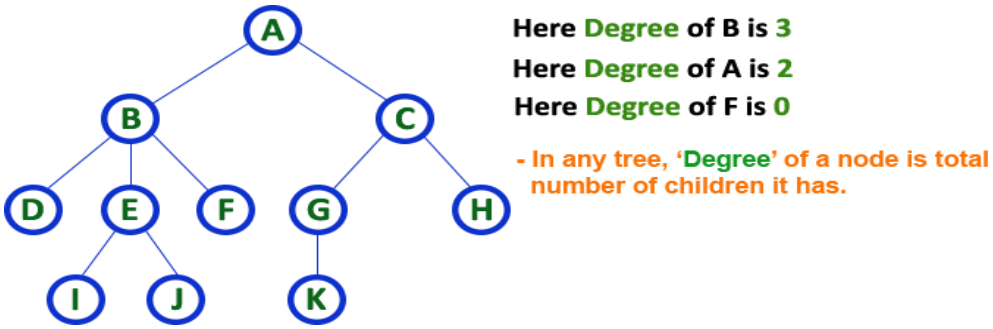


7. Internal Nodes: In a **Tree** data structure, the node which has atleast one child is called as INTERNAL Node. In simple words, an internal node is a node with atleast one child.

In a **Tree** data structure, nodes other than leaf nodes are called as Internal Nodes. The root node is also said to be Internal Node if the tree has more than one node. Internal nodes are also called as 'Non-Terminal' nodes.



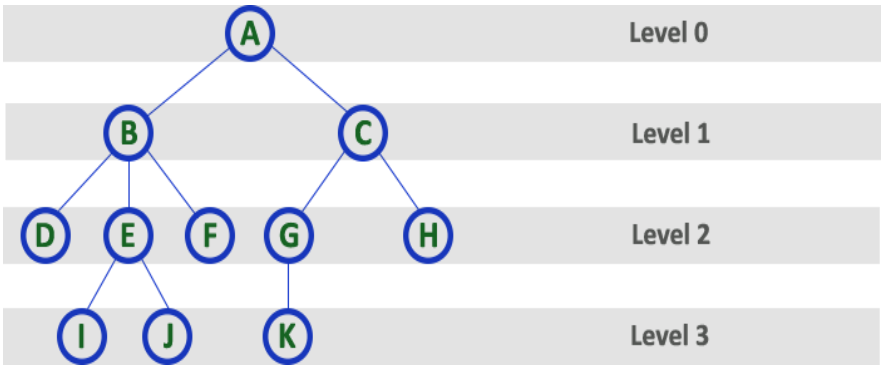
8. Degree: In a **Tree** data structure, the total number of children of a node is called as **DEGREE** of that Node. In simple words, the Degree of a node is total number of children it has. The highest degree of a node among all the nodes in a tree is called as '**Degree of Tree**'



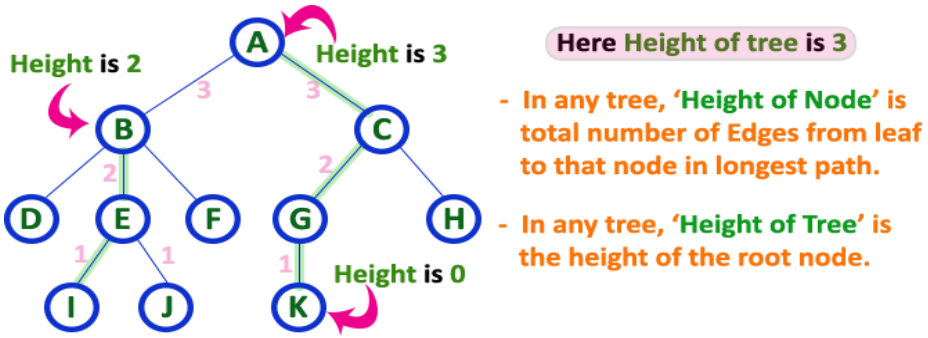
Degree of Tree is: 3

9. Level: In a **Tree** data structure, the root node is said to be at Level 0 and the children of root node are at Level 1 and the children of the nodes which are at Level 1 will be at Level 2

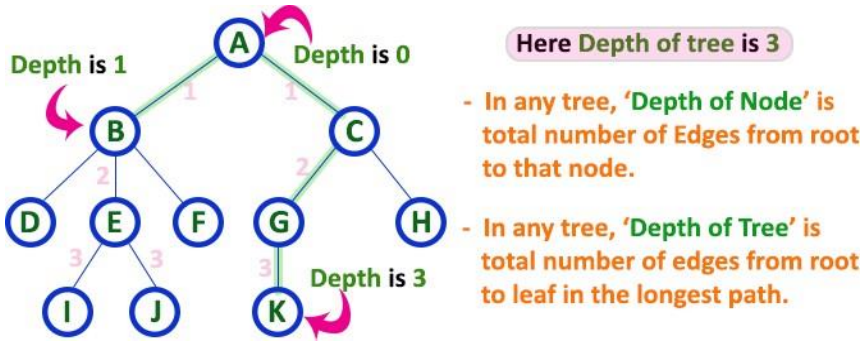
and so on... In simple words, in a tree each step from top to bottom is called as a Level and the Level count starts with '0' and incremented by one at each level (Step).



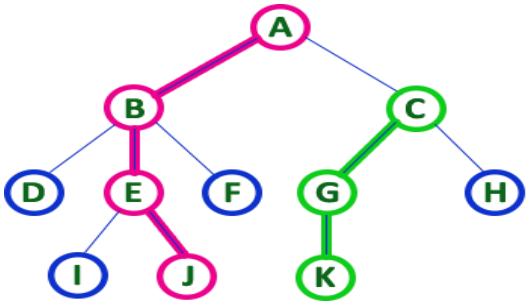
10. Height: In a **Tree** data structure, the total number of edges from leaf node to a particular node in the longest path is called as HEIGHT of that Node. In a tree, height of the root node is said to be height of the tree. In a tree, height of all leaf nodes is '0'.



11. Depth: In a **Tree** data structure, the total number of egdes from root node to a particular node is called as **DEPTH** of that Node. In a tree, the total number of edges from root node to a leaf node in the longest path is said to be **Depth of the tree**. In simple words, the highest depth of any leaf node in a tree is said to be depth of that tree. In a tree, **depth of the root node is '0'.**



12. Path: In a **Tree** data structure, the sequence of Nodes and Edges from one node to another node is called as **PATH** between that two Nodes. **Length of a Path** is total number of nodes in that path. In below example **the path A - B - E - J has length 4**.

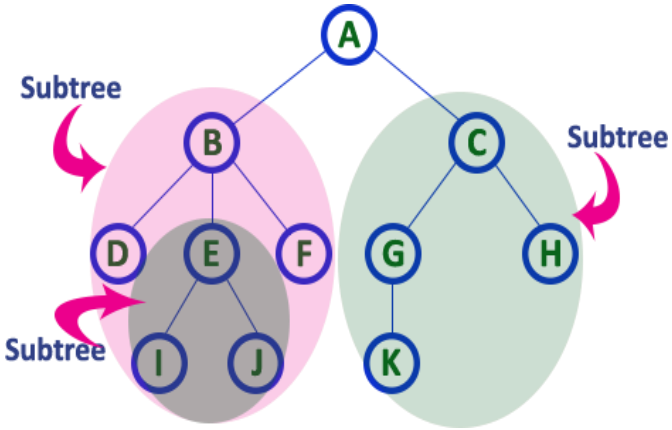


- In any tree, 'Path' is a sequence of nodes and edges between two nodes.

Here, 'Path' between A & J is
A - B - E - J

Here, 'Path' between C & K is
C - G - K

13. Sub Tree: In a **Tree** data structure, each child from a node forms a subtree recursively. Every child node will form a subtree on its parent node.

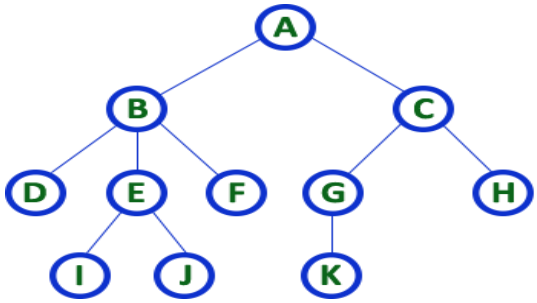


TREE REPRESENTATIONS:

A tree data structure can be represented in two methods. Those methods are as follows...

1. List Representation
2. **Left Child - Right Sibling Representation**

Consider the following tree...



TREE with 11 nodes and 10 edges

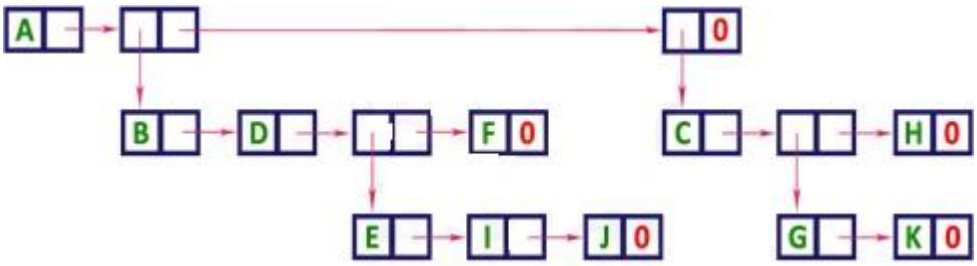
- In any tree with 'N' nodes there will be maximum of 'N-1' edges
- In a tree every individual element is called as 'NODE'

1. List Representation

In this representation, we use two types of nodes one for representing the node with data called 'data node' and another for representing only references called 'reference node'. We start with a 'data node' from the root node in the tree. Then it is linked to an internal node

through a 'reference node' which is further linked to any other node directly. This process repeats for all the nodes in the tree.

The above example tree can be represented using List representation as follows...

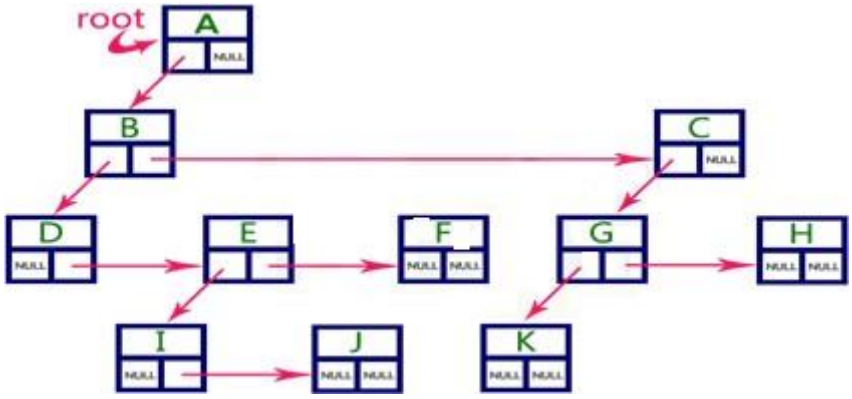


2. Left Child - Right Sibling Representation

In this representation, we use a list with one type of node which consists of three fields namely Data field, Left child reference field and Right sibling reference field. Data field stores the actual value of a node, left reference field stores the address of the left child and right reference field stores the address of the right sibling node. Graphical representation of that node is as follows...

In this representation, every node's data field stores the actual value of that node. If that node has left a child, then left reference field stores the address of that left child node otherwise stores NULL. If that node has the right sibling, then right reference field stores the address of right sibling node otherwise stores NULL.

The above example tree can be represented using Left Child - Right Sibling representation as follows...



BINARY TREE TRAVERSALS:

Unlike linear data structures (Array, Linked List, Queues, Stacks, etc) which have only one logical way to traverse them, binary trees can be traversed in different ways. Following are the generally used ways for traversing binary trees.

When we wanted to display a binary tree, we need to follow some order in which all the nodes of that binary tree must be displayed. In any binary tree, displaying order of nodes depends on the traversal method.

Displaying (or) visiting order of nodes in a binary tree is called as Binary Tree Traversal.

There are three types of binary tree traversals.

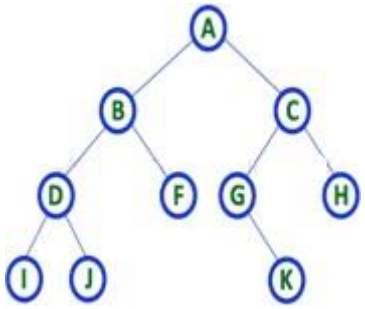
1. In - Order Traversal
2. **Pre - Order Traversal**
3. Post - Order Traversal

1. In - Order Traversal (left Child - root - right Child):

In In-Order traversal, the root node is visited between the left child and right child. In this traversal, the left child node is visited first, then the root node is visited and later we go for visiting the right child node. This in-order traversal is applicable for every root node of all sub trees in the tree. This is performed recursively for all nodes in the tree.

Algorithm:

Step-1: Visit the left subtree, using inorder.
Step-2: Visit the root.
Step-3: Visit the right subtree, using inorder.



In the above example of a binary tree, first we try to visit left child of root node 'A', but A's left child 'B' is a root node for left subtree. so we try to visit its (B's) left child 'D' and again D is a root for subtree with nodes D, I and J. So we try to visit its left child 'I' and it is the leftmost child. So first we visit **'I'** then go for its root node **'D'** and later we visit D's right child **'J'**. With this we have completed the left part of node B. Then visit **'B'** and next B's right child **'F'** is visited. With this we have completed left part of node A. Then visit root node **'A'**. With this we have completed left and root parts of node A. Then we go for the right part of the node A. In right of A again there is a subtree with root C. So go for left child of C and again it is a subtree with root G. But G does not have left part so we visit **'G'** and then visit G's right child K. With this we have completed the left part of node C. Then visit root node **'C'** and next visit C's right child **'H'** which is the rightmost child in the tree. So we stop the process.

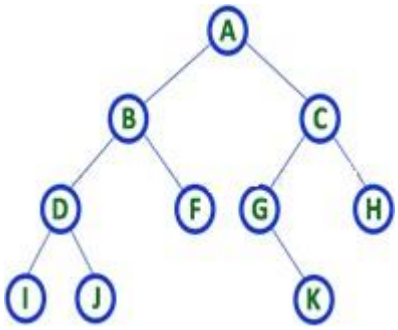
That means here we have visited in the order of **I - D - J - B - F - A - G - K - C - H** using In-Order Traversal.

2. Pre - Order Traversal (root - leftChild - rightChild):

In Pre-Order traversal, the root node is visited before the left child and right child nodes. In this traversal, the root node is visited first, then its left child and later its right child. This pre-order traversal is applicable for every root node of all subtrees in the tree. Preorder search is also called backtracking.

Algorithm:

Step-1: Visit the root.
Step-2: Visit the left subtree, using preorder.
Step-3: Visit the right subtree, using preorder.



In the above example of binary tree, first we visit root node **'A'** then visit its left child **'B'** which is a root for D and F. So we visit B's left child **'D'** and again D is a root for I and J. So we visit D's left child **'I'** which is the leftmost child. So next we go for visiting D's right child **'J'**. With this we have completed root, left and right parts of node D and root, left parts of node B. Next visit B's right child **'F'**. With this we have completed root and left parts of node A. So we go for A's right child **'C'** which is a root node for G and H. After visiting C, we go for its left child **'G'** which is a root for node K. So next we visit left of G, but it does not have left child so we go for G's right child **'K'**. With this, we have completed node C's root and left parts. Next visit C's right child **'H'** which is the rightmost child in the tree. So we stop the process.

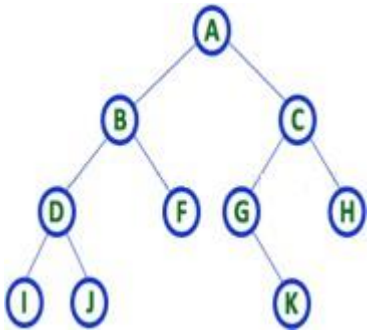
That means here we have visited in the order of **A-B-D-I-J-F-C-G-K-H** using Pre-Order Traversal.

3. Post - Order Traversal (leftChild - rightChild - root):

In Post-Order traversal, the root node is visited after left child and right child. In this traversal, left child node is visited first, then its right child and then its root node. This is recursively performed until the right most nodes are visited.

Algorithm:

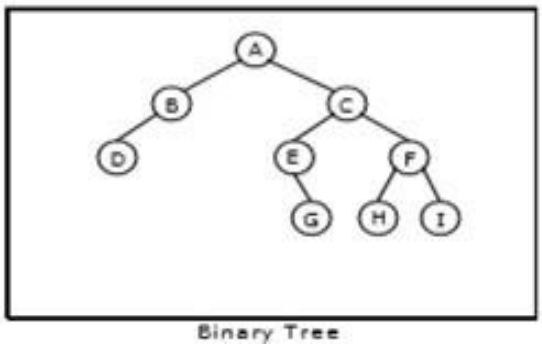
Step-1: Visit the left subtree, using postorder.
Step-2: Visit the right subtree, using postorder
Step-3: Visit the root.



Here we have visited in the order of **I - J - D - F - B - K - G - H - C - A** using Post-Order Traversal.

Example 1:

Traverse the following binary tree in pre, post, inorder and level order.



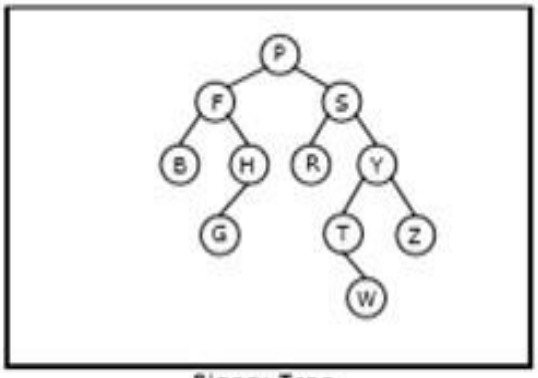
Binary Tree

- Preorder traversal yields:
  A, B, D, C, E, G, F, H, I
- Postorder traversal yields:
  D, B, G, E, H, I, F, C, A
- Inorder traversal yields:
  D, B, A, E, G, C, H, F, I

Pre, Post, Inorder and level order Traversing

Example 2:

Traverse the following binary tree in pre, post, inorder and level order.



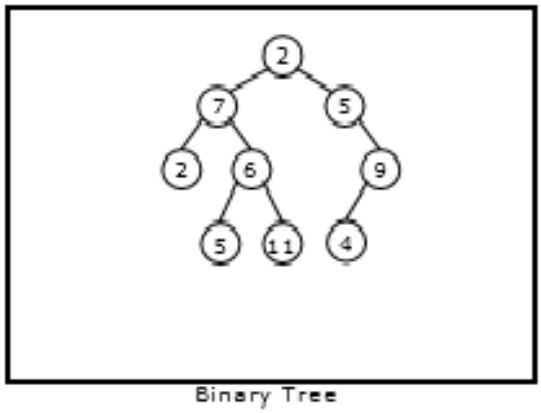
Binary Tree

- Preorder traversal yields:
  P, F, B, H, G, S, R, Y, T, W, Z
- Postorder traversal yields:
  B, G, H, F, R, W, T, Z, Y, S, P
- Inorder traversal yields:
  B, F, G, H, P, R, S, T, W, Y, Z

Pre, Post, Inorder and level order Traversing

Example 3:

Traverse the following binary tree in pre, post, inorder and level order.



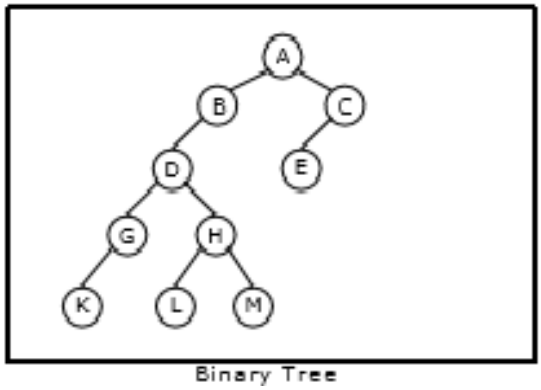
Binary Tree

- Preorder traversal yields:
  2, 7, 2, 6, 5, 11, 5, 9, 4
- Postorder travarsal yields:
  2, 5, 11, 6, 7, 4, 9, 5, 2
- Inorder travarsal yields:
  2, 7, 5, 6, 11, 2, 5, 4, 9

Pre, Post, Inorder and level order Traversing

Example 4:

Traverse the following binary tree in pre, post, inorder and level order.



Binary Tree

- Preorder traversal yields:
  A, B, D, G, K, H, L, M, C, E
- Postorder travarsal yields:
  K, G, L, M, H, D, B, E, C, A
- Inorder travarsal yields:
  K, G, D, L, H, M, B, A, E, C

Pre, Post, Inorder and level order Traversing

Unit – III

Sorting and Searching

Bubble sort Algorithm

In this article, we will discuss the Bubble sort Algorithm. The working procedure of bubble sort is simplest. This article will be very helpful and interesting to students as they might face bubble sort as a question in their examinations. So, it is important to discuss the topic.

Bubble sort works on the repeatedly swapping of adjacent elements until they are not in the intended order. It is called bubble sort because the movement of array elements is just like the movement of air bubbles in the water. Bubbles in water rise up to the surface; similarly, the array elements in bubble sort move to the end in each iteration.

Although it is simple to use, it is primarily used as an educational tool because the performance of bubble sort is poor in the real world. It is not suitable for large data sets. The average and worst-case complexity of Bubble sort is **$O(n^2)$,** where **n** is a number of items.

Bubble short is majorly used where –

- complexity does not matter
- simple and shortcode is preferred

Algorithm

In the algorithm given below, suppose **arr** is an array of **n** elements. The assumed **swap** function in the algorithm will swap the values of given array elements.

```
1. begin BubbleSort(arr)
2.   for all array elements
3.     if arr[i] > arr[i+1]
4.       swap(arr[i], arr[i+1])
5.     end if
6.   end for
7.   return arr
8. end BubbleSort
```

Insertion Sort Algorithm

In this article, we will discuss the Insertion sort Algorithm. The working procedure of insertion sort is also simple. This article will be very helpful and interesting to students as they might face insertion sort as a question in their examinations. So, it is important to discuss the topic.

Insertion sort works similar to the sorting of playing cards in hands. It is assumed that the first card is already sorted in the card game, and then we select an unsorted card. If the selected unsorted card is greater than the first card, it will be placed at the right side; otherwise, it will be placed at the left side. Similarly, all unsorted cards are taken and put in their exact place.

The same approach is applied in insertion sort. The idea behind the insertion sort is that first take one element, iterate it through the sorted array. Although it is simple to use, it is not appropriate for large data sets as the time complexity of insertion sort in the average case and worst case is $\mathbf{O(n^2)}$, where n is the number of items. Insertion sort is less efficient than the other sorting algorithms like heap sort, quick sort, merge sort, etc.

Insertion sort has various advantages such as -

- Simple implementation
- Efficient for small data sets
- Adaptive, i.e., it is appropriate for data sets that are already substantially sorted.

Now, let's see the algorithm of insertion sort.

Algorithm

The simple steps of achieving the insertion sort are listed as follows -

Step 1 - If the element is the first element, assume that it is already sorted. Return 1.

Step2 - Pick the next element, and store it separately in a **key.**

Step3 - Now, compare the **key** with all elements in the sorted array.

Step 4 - If the element in the sorted array is smaller than the current element, then move to the next element. Else, shift greater elements in the array towards the right.

Step 5 - Insert the value.

Step 6 - Repeat until the array is sorted.

Merge Sort Algorithm

In this article, we will discuss the merge sort Algorithm. Merge sort is the sorting technique that follows the divide and conquer approach. This article will be very helpful and interesting to students as they might face merge sort as a question in their examinations. In coding or technical interviews for software engineers, sorting algorithms are widely asked. So, it is important to discuss the topic.

Merge sort is similar to the quick sort algorithm as it uses the divide and conquer approach to sort the elements. It is one of the most popular and efficient sorting algorithm. It divides the

given list into two equal halves, calls itself for the two halves and then merges the two sorted halves. We have to define the **merge()** function to perform the merging.

The sub-lists are divided again and again into halves until the list cannot be divided further. Then we combine the pair of one element lists into two-element lists, sorting them in the process. The sorted two-element pairs is merged into the four-element lists, and so on until we get the sorted list.

Now, let's see the algorithm of merge sort.

Algorithm

In the following algorithm, **arr** is the given array, **beg** is the starting element, and **end** is the last element of the array.

```
1. MERGE_SORT(arr, beg, end)
2.
3. if beg < end
4. set mid = (beg + end)/2
5. MERGE_SORT(arr, beg, mid)
6. MERGE_SORT(arr, mid + 1, end)
7. MERGE (arr, beg, mid, end)
8. end of if
```

Quick Sort Algorithm

In this article, we will discuss the Quicksort Algorithm. The working procedure of Quicksort is also simple. This article will be very helpful and interesting to students as they might face quicksort as a question in their examinations. So, it is important to discuss the topic.

Sorting is a way of arranging items in a systematic manner. Quicksort is the widely used sorting algorithm that makes **n log n** comparisons in average case for sorting an array of n elements. It is a faster and highly efficient sorting algorithm. This algorithm follows the divide and conquer approach. Divide and conquer is a technique of breaking down the algorithms into subproblems, then solving the subproblems, and combining the results back together to solve the original problem.

Divide: In Divide, first pick a pivot element. After that, partition or rearrange the array into two sub-arrays such that each element in the left sub-array is less than or equal to the pivot element and each element in the right sub-array is larger than the pivot element.

Conquer: Recursively, sort two subarrays with Quicksort.

Combine: Combine the already sorted array.

Quicksort picks an element as pivot, and then it partitions the given array around the picked pivot element. In quick sort, a large array is divided into two arrays in which one holds values

that are smaller than the specified value (Pivot), and another array holds the values that are greater than the pivot.

After that, left and right sub-arrays are also partitioned using the same approach. It will continue until the single element remains in the sub-array.

Radix Sort Algorithm

In this article, we will discuss the Radix sort Algorithm. Radix sort is the linear sorting algorithm that is used for integers. In Radix sort, there is digit by digit sorting is performed that is started from the least significant digit to the most significant digit.

The process of radix sort works similar to the sorting of students names, according to the alphabetical order. In this case, there are 26 radix formed due to the 26 alphabets in English. In the first pass, the names of students are grouped according to the ascending order of the first letter of their names. After that, in the second pass, their names are grouped according to the ascending order of the second letter of their name. And the process continues until we find the sorted list.

Now, let's see the algorithm of Radix sort.

Algorithm

1. radixSort(arr)
2. max = largest element in the given array
3. d = number of digits in the largest element (or, max)
4. Now, create d buckets of size 0 - 9
5. **for** i -> 0 to d
6. sort the array elements using counting sort (or any stable sort) according to the digits at
7. the ith place

Selection Sort Algorithm

In this article, we will discuss the Selection sort Algorithm. The working procedure of selection sort is also simple. This article will be very helpful and interesting to students as they might face selection sort as a question in their examinations. So, it is important to discuss the topic.

In selection sort, the smallest value among the unsorted elements of the array is selected in every pass and inserted to its appropriate position into the array. It is also the simplest algorithm. It is an in-place comparison sorting algorithm. In this algorithm, the array is divided into two parts, first is sorted part, and another one is the unsorted part. Initially, the sorted part of the array is empty, and unsorted part is the given array. Sorted part is placed at the left, while the unsorted part is placed at the right.

In selection sort, the first smallest element is selected from the unsorted array and placed at the first position. After that second smallest element is selected and placed in the second position. The process continues until the array is entirely sorted.

The average and worst-case complexity of selection sort is **O(n^2)**, where **n** is the number of items. Due to this, it is not suitable for large data sets.

Selection sort is generally used when –

- A small array is to be sorted
- Swapping cost doesn't matter
- It is compulsory to check all elements

Now, let's see the algorithm of selection sort.

Algorithm

1. SELECTION SORT(arr, n)
2.
3. Step 1: Repeat Steps 2 **and** 3 **for** i = 0 to n-1
4. Step 2: CALL SMALLEST(arr, i, n, pos)
5. Step 3: SWAP arr[i] with arr[pos]
6. [END OF LOOP]
7. Step 4: EXIT
8.
9. SMALLEST (arr, i, n, pos)
10. Step 1: [INITIALIZE] SET SMALL = arr[i]
11. Step 2: [INITIALIZE] SET pos = i
12. Step 3: Repeat **for** j = i+1 to n
13. **if** (SMALL > arr[j])
14. SET SMALL = arr[j]
15. SET pos = j
16. [END OF **if**]
17. [END OF LOOP]
18. Step 4: RETURN pos

Linear Search Algorithm

In this article, we will discuss the Linear Search Algorithm. Searching is the process of finding some particular element in the list. If the element is present in the list, then the process is called successful, and the process returns the location of that element; otherwise, the search is called unsuccessful.

Two popular search methods are Linear Search and Binary Search. So, here we will discuss the popular searching technique, i.e., Linear Search Algorithm.

Linear search is also called as **sequential search algorithm.** It is the simplest searching algorithm. In Linear search, we simply traverse the list completely and match each element of the list with the item whose location is to be found. If the match is found, then the location of the item is returned; otherwise, the algorithm returns NULL.

It is widely used to search an element from the unordered list, i.e., the list in which items are not sorted. The worst-case time complexity of linear search is **O(n).**

The steps used in the implementation of Linear Search are listed as follows –

- First, we have to traverse the array elements using a **for** loop.
- In each iteration of **for loop,** compare the search element with the current array element, and -
  - If the element matches, then return the index of the corresponding array element.
  - If the element does not match, then move to the next element.
- If there is no match or the search element is not present in the given array, return **-1.**

Algorithm

1. Linear_Search(a, n, val) // 'a' is the given array, 'n' is the size of given array, 'val' is the value t o search
2. Step 1: set pos = -1
3. Step 2: set i = 1
4. Step 3: repeat step 4 while i <= n
5. Step 4: if a[i] == val
6. set pos = i
7. print pos
8. go to step 6
9. [end of if]
10. set ii = i + 1
11. [end of loop]
12. Step 5: if pos = -1
13. print "value is not present in the array "
14. [end of if]
15. Step 6: exit

Binary Search Algorithm

In this article, we will discuss the Binary Search Algorithm. Searching is the process of finding some particular element in the list. If the element is present in the list, then the process is called successful, and the process returns the location of that element. Otherwise, the search is called unsuccessful.

Linear Search and Binary Search are the two popular searching techniques. Here we will discuss the Binary Search Algorithm.

Binary search is the search technique that works efficiently on sorted lists. Hence, to search an element into some list using the binary search technique, we must ensure that the list is sorted.

Binary search follows the divide and conquer approach in which the list is divided into two halves, and the item is compared with the middle element of the list. If the match is found then, the location of the middle element is returned. Otherwise, we search into either of the halves depending upon the result produced through the match.

Algorithm

1. Binary_Search(a, lower_bound, upper_bound, val) // 'a' is the given array, 'lower_bound' is th e index of the first array element, 'upper_bound' is the index of the last array element, 'val' is t he value to search
2. Step 1: set beg = lower_bound, end = upper_bound, pos = - 1
3. Step 2: repeat steps 3 and 4 while beg <=end
4. Step 3: set mid = (beg + end)/2
5. Step 4: if a[mid] = val
6. set pos = mid
7. print pos
8. go to step 6
9. else if a[mid] > val
10. set end = mid - 1
11. else
12. set beg = mid + 1
13. [end of if]
14. [end of loop]
15. Step 5: if pos = -1
16. print "value is not present in the array"
17. [end of if]
18. Step 6: exit

Unit – IV

Greedy and Backtracking

What is an Algorithm?

An algorithm is a process or a set of rules required to perform calculations or some other problem-solving operations especially by a computer. The formal definition of an algorithm is that it contains the finite set of instructions which are being carried in a specific order to perform the specific task. It is not the complete program or code; it is just a solution (logic) of a problem, which can be represented either as an informal description using a Flowchart or Pseudocode.

Characteristics of an Algorithm

The following are the characteristics of an algorithm:

- **Input:** An algorithm has some input values. We can pass 0 or some input value to an algorithm.
- **Output:** We will get 1 or more output at the end of an algorithm.
- **Unambiguity:** An algorithm should be unambiguous which means that the instructions in an algorithm should be clear and simple.
- **Finiteness:** An algorithm should have finiteness. Here, finiteness means that the algorithm should contain a limited number of instructions, i.e., the instructions should be countable.
- **Effectiveness:** An algorithm should be effective as each instruction in an algorithm affects the overall process.
- **Language independent:** An algorithm must be language-independent so that the instructions in an algorithm can be implemented in any of the languages with the same output.

Dataflow of an Algorithm

- **Problem:** A problem can be a real-world problem or any instance from the real-world problem for which we need to create a program or the set of instructions. The set of instructions is known as an algorithm.
- **Algorithm:** An algorithm will be designed for a problem which is a step by step procedure.

- **Input:** After designing an algorithm, the required and the desired inputs are provided to the algorithm.
- **Processing unit:** The input will be given to the processing unit, and the processing unit will produce the desired output.
- **Output:** The output is the outcome or the result of the program.

Why do we need Algorithms?

We need algorithms because of the following reasons:

- **Scalability:** It helps us to understand the scalability. When we have a big real-world problem, we need to scale it down into small-small steps to easily analyze the problem.
- **Performance:** The real-world is not easily broken down into smaller steps. If the problem can be easily broken into smaller steps means that the problem is feasible.

Asymptotic Analysis

As we know that data structure is a way of organizing the data efficiently and that efficiency is measured either in terms of time or space. So, the ideal data structure is a structure that occupies the least possible time to perform all its operation and the memory space. Our focus would be on finding the time complexity rather than space complexity, and by finding the time complexity, we can decide which data structure is the best for an algorithm.

The main question arises in our mind that on what basis should we compare the time complexity of data structures?. The time complexity can be compared based on operations performed on them. Let's consider a simple example.

Suppose we have an array of 100 elements, and we want to insert a new element at the beginning of the array. This becomes a very tedious task as we first need to shift the elements towards the right, and we will add new element at the starting of the array.

Suppose we consider the linked list as a data structure to add the element at the beginning. The linked list contains two parts, i.e., data and address of the next node. We simply add the address of the first node in the new node, and head pointer will now point to the newly added node. Therefore, we conclude that adding the data at the beginning of the linked list is faster than the arrays. In this way, we can compare the data structures and select the best possible data structure for performing the operations.

Example: Running time of one operation is x(n) and for another operation, it is calculated as f(n2). It refers to running time will increase linearly with an increase in 'n' for the first operation, and running time will increase exponentially for the second operation. Similarly, the running time of both operations will be the same if n is significantly small.

Usually, the time required by an algorithm comes under three types:

Worst case: It defines the input for which the algorithm takes a huge time.

Average case: It takes average time for the program execution.

Best case: It defines the input for which the algorithm takes the lowest time

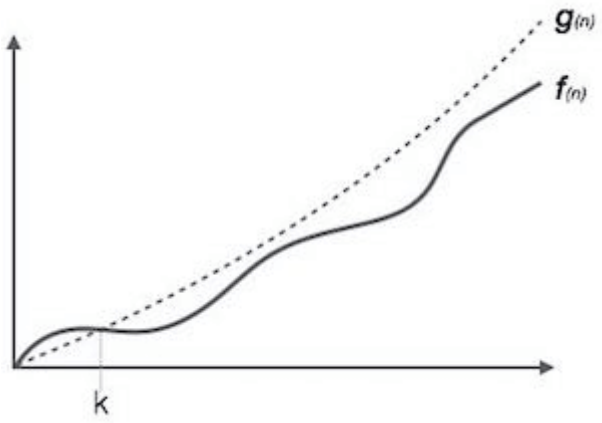
Asymptotic Notations

The commonly used asymptotic notations used for calculating the running time complexity of an algorithm is given below:

- Big oh Notation (?)
- Omega Notation (Ω)
- Theta Notation (θ)

Big oh Notation (O)

- Big O notation is an asymptotic notation that measures the performance of an algorithm by simply providing the order of growth of the function.
- This notation provides an upper bound on a function which ensures that the function never grows faster than the upper bound. So, it gives the least upper bound on a function so that the function never grows faster than this upper bound.



Omega Notation (Ω)

- It basically describes the best-case scenario which is opposite to the big o notation.
- It is the formal way to represent the lower bound of an algorithm's running time. It measures the best amount of time an algorithm can possibly take to complete or the best-case time complexity.
- It determines what is the fastest time that an algorithm can run.

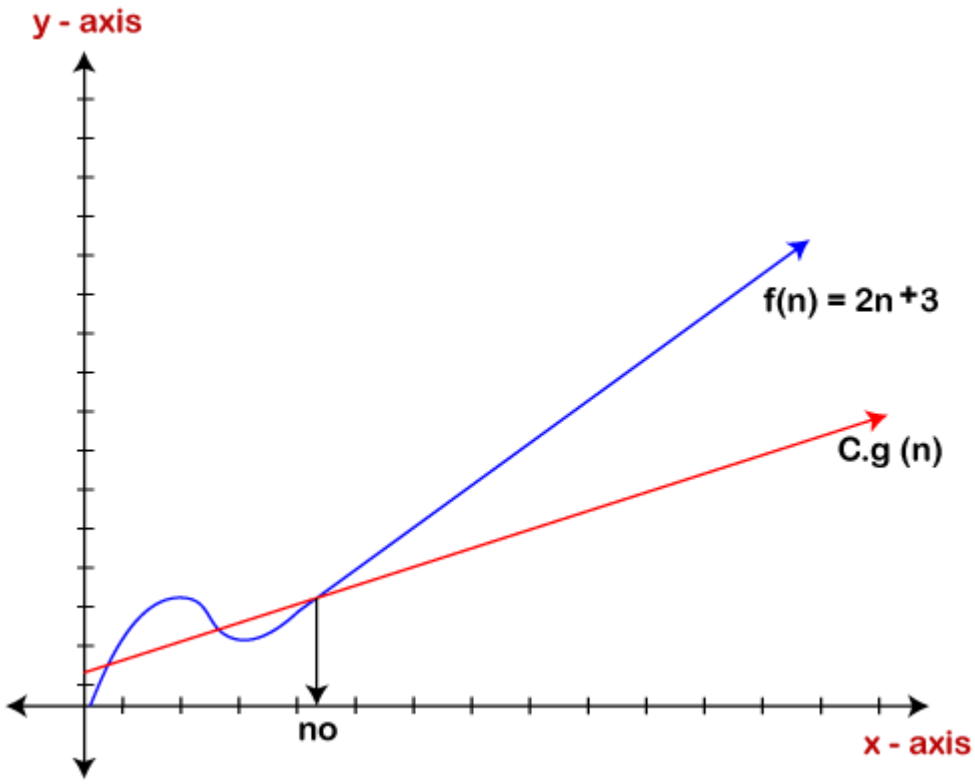
If we required that an algorithm takes at least certain amount of time without using an upper bound, we use big- Ω notation i.e. the Greek letter "omega". It is used to bound the growth of running time for large input size.

If **f(n)** and **g(n)** are the two functions defined for positive integers,

then **f(n) = Ω (g(n))** as **f(n) is Omega of g(n)** or f(n) is on the order of g(n)) if there exists constants c and no such that:

f(n)>=c.g(n) for all n≥no and c>0

It is the formal way to express the upper boundary of an algorithm running time. It measures the worst case of time complexity or the algorithm's longest amount of time to complete its operation. It is represented as shown below:



Theta Notation (θ)

- The theta notation mainly describes the average case scenarios.
- It represents the realistic time complexity of an algorithm. Every time, an algorithm does not perform worst or best, in real-world problems, algorithms mainly fluctuate between the worst-case and best-case, and this gives us the average case of the algorithm.
- Big theta is mainly used when the value of worst-case and the best-case is same.

- It is the formal way to express both the upper bound and lower bound of an algorithm running time.

Let's understand the big theta notation mathematically:

Let f(n) and g(n) be the functions of n where n is the steps required to execute the program then:

f(n)= θg(n)

The above condition is satisfied only if when

c1.g(n)<=f(n)<=c2.g(n)

where the function is bounded by two limits, i.e., upper and lower limit, and f(n) comes in between. The condition **f(n)= θg(n)** will be true if and only if c1.g(n) is less than or equal to f(n) and c2.g(n) is greater than or equal to f(n). The graphical representation of theta notation is given below:

Greedy Algorithm

The greedy method is one of the strategies like Divide and conquer used to solve the problems. This method is used for solving optimization problems. An optimization problem is a problem that demands either maximum or minimum results. Let's understand through some terms.

The Greedy method is the simplest and straightforward approach. It is not an algorithm, but it is a technique. The main function of this approach is that the decision is taken on the basis of the currently available information. Whatever the current information is present, the decision is made without worrying about the effect of the current decision in future.

This technique is basically used to determine the feasible solution that may or may not be optimal. The feasible solution is a subset that satisfies the given criteria. The optimal solution is the solution which is the best and the most favorable solution in the subset. In the case of feasible, if more than one solution satisfies the given criteria then those solutions will be considered as the feasible, whereas the optimal solution is the best solution among all the solutions.

Characteristics of Greedy method

The following are the characteristics of a greedy method:

- To construct the solution in an optimal way, this algorithm creates two sets where one set contains all the chosen items, and another set contains the rejected items.
- A Greedy algorithm makes good local choices in the hope that the solution should be either feasible or optimal.

Components of Greedy Algorithm

The components that can be used in the greedy algorithm are:

- **Candidate set:** A solution that is created from the set is known as a candidate set.
- **Selection function:** This function is used to choose the candidate or subset which can be added in the solution.
- **Feasibility function:** A function that is used to determine whether the candidate or subset can be used to contribute to the solution or not.
- **Objective function:** A function is used to assign the value to the solution or the partial solution.
- **Solution function:** This function is used to intimate whether the complete function has been reached or not.

Applications of Greedy Algorithm

- It is used in finding the shortest path.
- It is used to find the minimum spanning tree using the prim's algorithm or the Kruskal's algorithm.
- It is used in a job sequencing with a deadline.
- This algorithm is also used to solve the fractional knapsack problem.

Prim's Algorithm

In this article, we will discuss the prim's algorithm. Along with the algorithm, we will also see the complexity, working, example, and implementation of prim's algorithm.

Before starting the main topic, we should discuss the basic and important terms such as spanning tree and minimum spanning tree.

Spanning tree - A spanning tree is the subgraph of an undirected connected graph.

Minimum Spanning tree - Minimum spanning tree can be defined as the spanning tree in which the sum of the weights of the edge is minimum. The weight of the spanning tree is the sum of the weights given to the edges of the spanning tree.

Now, let's start the main topic.

Prim's Algorithm is a greedy algorithm that is used to find the minimum spanning tree from a graph. Prim's algorithm finds the subset of edges that includes every vertex of the graph such that the sum of the weights of the edges can be minimized.

Prim's algorithm starts with the single node and explores all the adjacent nodes with all the connecting edges at every step. The edges with the minimal weights causing no cycles in the graph got selected.

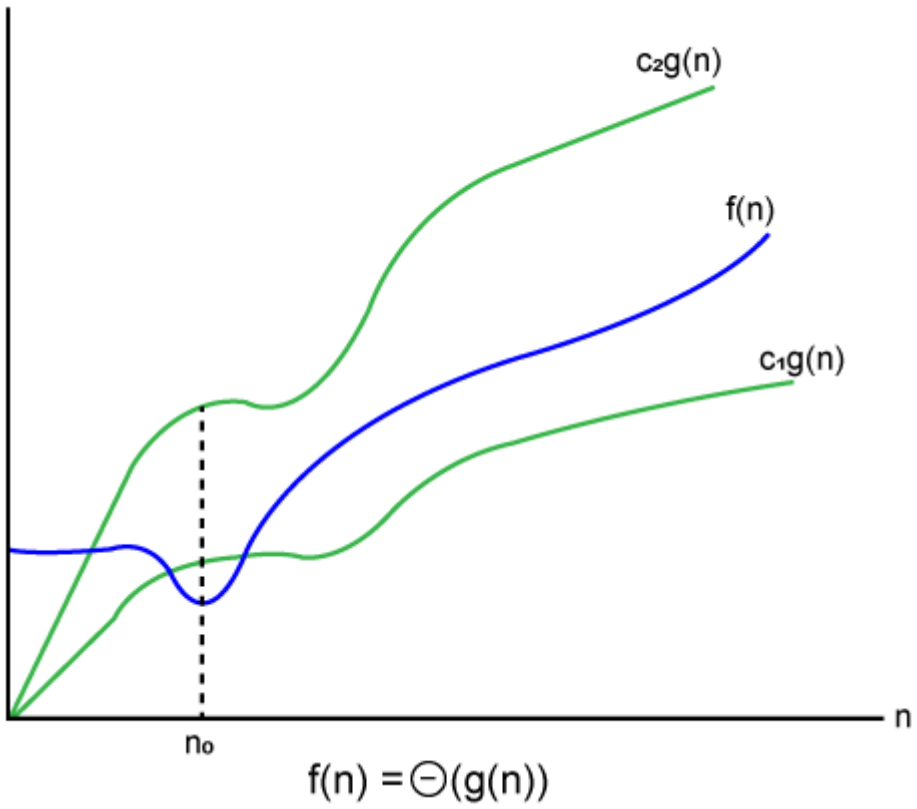
How does the prim's algorithm work?

Prim's algorithm is a greedy algorithm that starts from one vertex and continue to add the edges with the smallest weight until the goal is reached. The steps to implement the prim's algorithm are given as follows -

- First, we have to initialize an MST with the randomly chosen vertex.
- Now, we have to find all the edges that connect the tree in the above step with the new vertices. From the edges found, select the minimum edge and add it to the tree.
- Repeat step 2 until the minimum spanning tree is formed.

The applications of prim's algorithm are -

- Prim's algorithm can be used in network designing.
- It can be used to make network cycles.
- It can also be used to lay down electrical wiring cables.



Kruskal's Algorithm

In this article, we will discuss Kruskal's algorithm. Here, we will also see the complexity, working, example, and implementation of the Kruskal's algorithm.

But before moving directly towards the algorithm, we should first understand the basic terms such as spanning tree and minimum spanning tree.

Spanning tree - A spanning tree is the subgraph of an undirected connected graph.

Minimum Spanning tree - Minimum spanning tree can be defined as the spanning tree in which the sum of the weights of the edge is minimum. The weight of the spanning tree is the sum of the weights given to the edges of the spanning tree.

Now, let's start with the main topic.

Kruskal's Algorithm is used to find the minimum spanning tree for a connected weighted graph. The main target of the algorithm is to find the subset of edges by using which we can traverse every vertex of the graph. It follows the greedy approach that finds an optimum solution at every stage instead of focusing on a global optimum.

How does Kruskal's algorithm work?

In Kruskal's algorithm, we start from edges with the lowest weight and keep adding the edges until the goal is reached. The steps to implement Kruskal's algorithm are listed as follows -

- First, sort all the edges from low weight to high.
- Now, take the edge with the lowest weight and add it to the spanning tree. If the edge to be added creates a cycle, then reject the edge.
- Continue to add the edges until we reach all vertices, and a minimum spanning tree is created.

The applications of Kruskal's algorithm are -

- Kruskal's algorithm can be used to layout electrical wiring among cities.
- It can be used to lay down LAN connections.

An Introduction to Dijkstra's Algorithm

Now that we know some basic Graphs concepts let's dive into understanding the concept of Dijkstra's Algorithm.

Ever wondered how does Google Maps finds the shortest and fastest route between two places?

Well, the answer is **Dijkstra's Algorithm**. **Dijkstra's Algorithm** is a Graph algorithm **that finds the shortest path** from a source vertex to all other vertices in the Graph (single source shortest path). It is a type of Greedy Algorithm that only works on Weighted Graphs having positive weights. The time complexity of Dijkstra's Algorithm is $\mathbf{O(V^2)}$ with the help of the adjacency matrix representation of the graph. This time complexity can be reduced to **O((V + E) log V)** with the help of an adjacency list representation of the graph, where **V** is the number of vertices and **E** is the number of edges in the graph.

History of Dijkstra's Algorithm

Dijkstra's Algorithm was designed and published by **Dr. Edsger W. Dijkstra**, a Dutch Computer Scientist, Software Engineer, Programmer, Science Essayist, and Systems Scientist.

During an Interview with Philip L. Frana for the Communications of the ACM journal in the year 2001, Dr. Edsger W. Dijkstra revealed:

"What is the shortest way to travel from Rotterdam to Groningen, in general: from given city to given city? It is the algorithm for the shortest path, which I designed in about twenty minutes. One morning I was shopping in Amsterdam with my young fiancée, and tired, we sat down on the café terrace to drink a cup of coffee and I was just thinking about whether I could do this, and I then designed the algorithm for the shortest path. As I said, it was a twenty-minute invention. In fact, it was published in '59, three years later. The publication is still readable, it is, in fact, quite nice. One of the reasons that it is so nice was that I designed it without pencil and paper. I learned later that one of the advantages of designing without pencil and paper is that you are almost forced to avoid all avoidable complexities. Eventually, that algorithm became to my great amazement, one of the cornerstones of my fame."

Dijkstra thought about the shortest path problem while working as a programmer at the Mathematical Centre in Amsterdam in 1956 to illustrate the capabilities of a new computer known as ARMAC. His goal was to select both a problem and a solution (produced by the computer) that people with no computer background could comprehend. He developed the shortest path algorithm and later executed it for ARMAC for a vaguely shortened transportation map of 64 cities in the Netherlands (64 cities, so 6 bits would be sufficient to encode the city number). A year later, he came across another issue from hardware engineers operating the next computer of the institute: Minimize the amount of wire required to connect the pins on the machine's back panel. As a solution, he re-discovered the algorithm called Prim's minimal spanning tree algorithm and published it in the year 1959.

Fundamentals of Dijkstra's Algorithm

The following are the basic concepts of Dijkstra's Algorithm:

1. Dijkstra's Algorithm begins at the node we select (the source node), and it examines the graph to find the shortest path between that node and all the other nodes in the graph.
2. The Algorithm keeps records of the presently acknowledged shortest distance from each node to the source node, and it updates these values if it finds any shorter path.

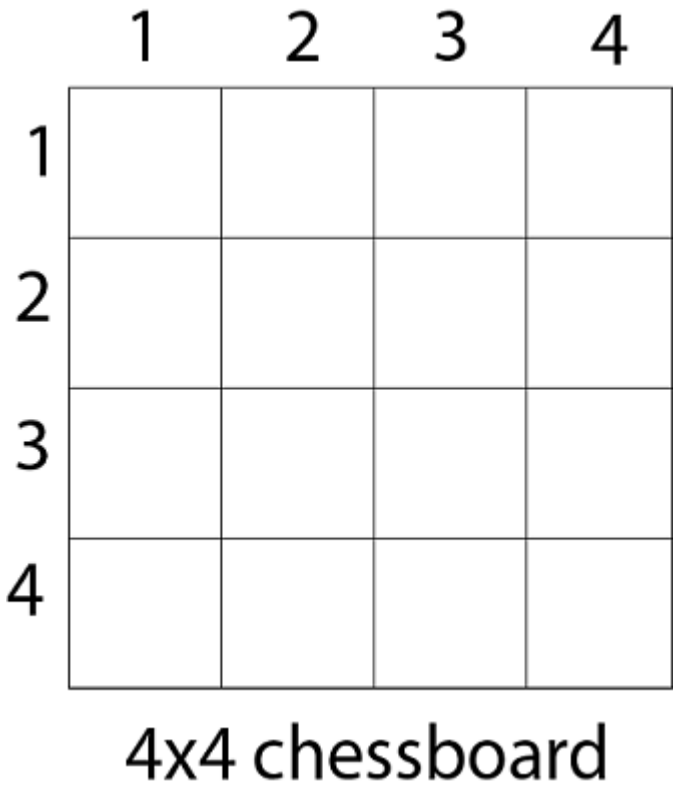
3. Once the Algorithm has retrieved the shortest path between the source and another node, that node is marked as 'visited' and included in the path.
4. The procedure continues until all the nodes in the graph have been included in the path. In this manner, we have a path connecting the source node to all other nodes, following the shortest possible path to reach each node.

N-Queens Problem

N - Queens problem is to place n - queens in such a manner on an n x n chessboard that no queens attack each other by being in the same row, column or diagonal.

It can be seen that for n =1, the problem has a trivial solution, and no solution exists for n =2 and n =3. So first we will consider the 4 queens problem and then generate it to n - queens problem.

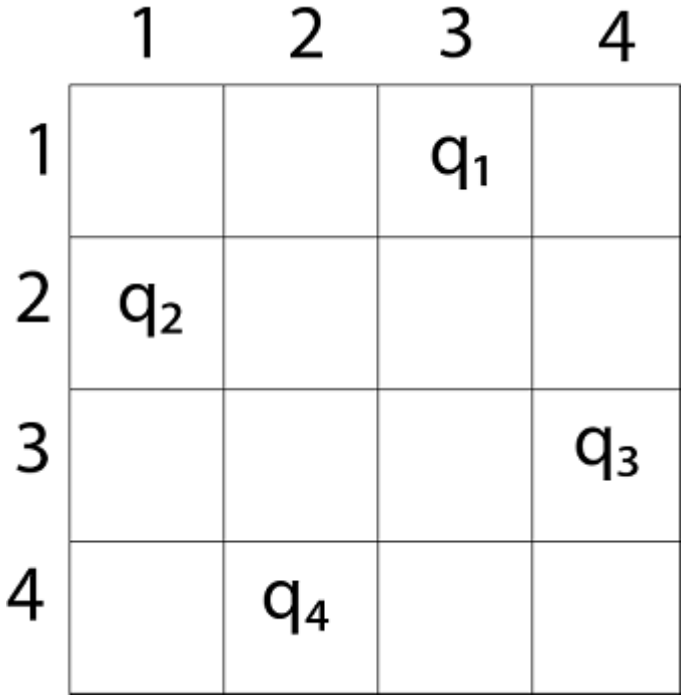
Given a 4 x 4 chessboard and number the rows and column of the chessboard 1 through 4.



Since, we have to place 4 queens such as q_1 q_2 q_3 and q_4 on the chessboard, such that no two queens attack each other. In such a conditional each queen must be placed on a different row, i.e., we put queen "i" on row "i."

Now, we place queen q_1 in the very first acceptable position (1, 1). Next, we put queen q_2 so that both these queens do not attack each other. We find that if we place q_2 in column 1 and 2, then the dead end is encountered. Thus the first acceptable position for q_2 in column 3, i.e. (2,

3) but then no position is left for placing queen 'q_3' safely. So we backtrack one step and place the queen 'q_2' in (2, 4), the next best possible solution. Then we obtain the position for placing 'q_3' which is (3, 2). But later this position also leads to a dead end, and no place is found where 'q_4' can be placed safely. Then we have to backtrack till 'q_1' and place it to (1, 2) and then all other queens are placed safely by moving q_2 to (2, 4), q_3 to (3, 1) and q_4 to (4, 3). That is, we get the solution (2, 4, 1, 3). This is one possible solution for the 4-queens problem. For another possible solution, the whole method is repeated for all partial solutions. The other solutions for 4 - queens problems is (3, 1, 4, 2) i.e.



The implicit tree for 4 - queen problem for a solution (2, 4, 1, 3) is as follows:

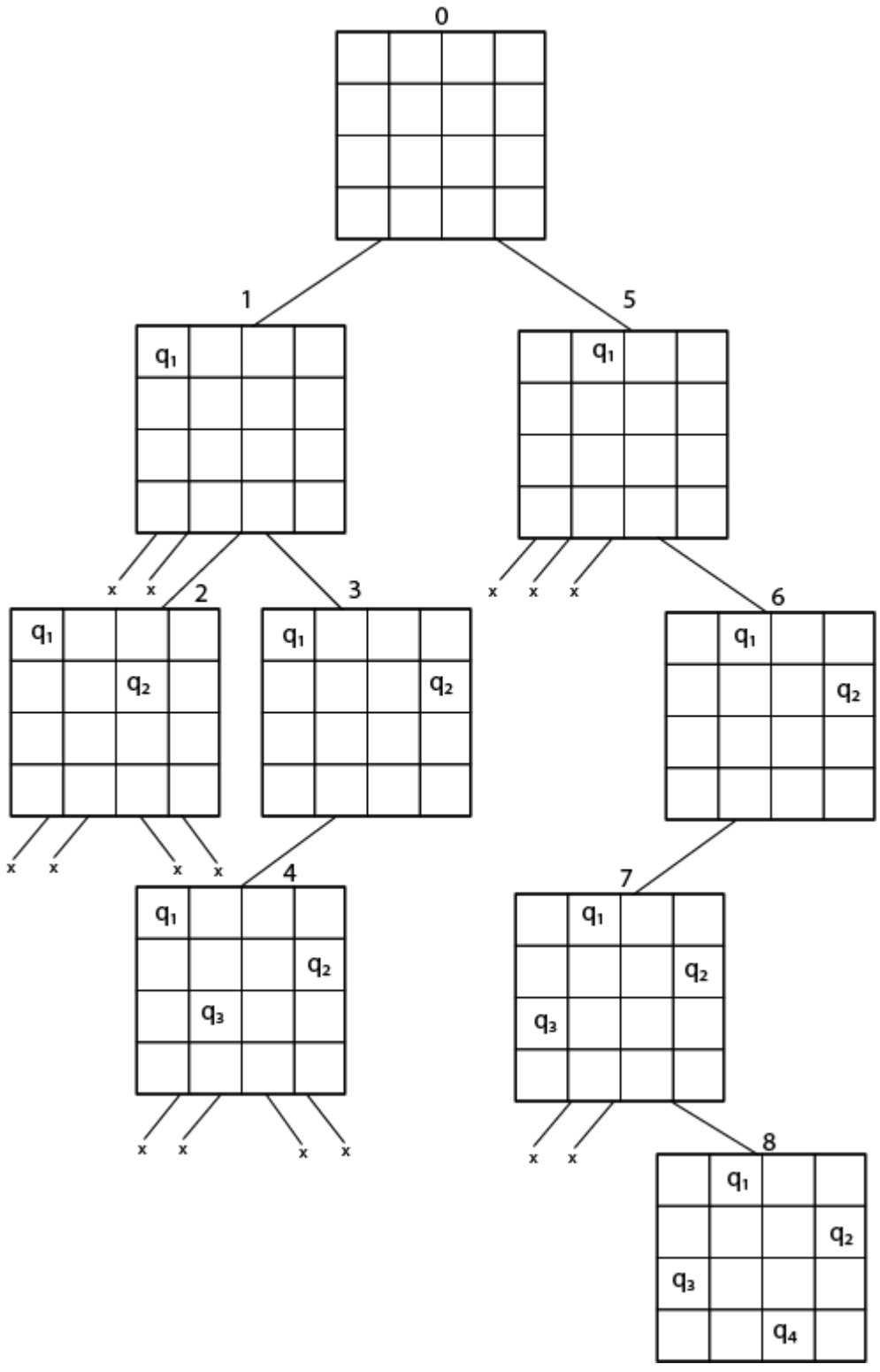
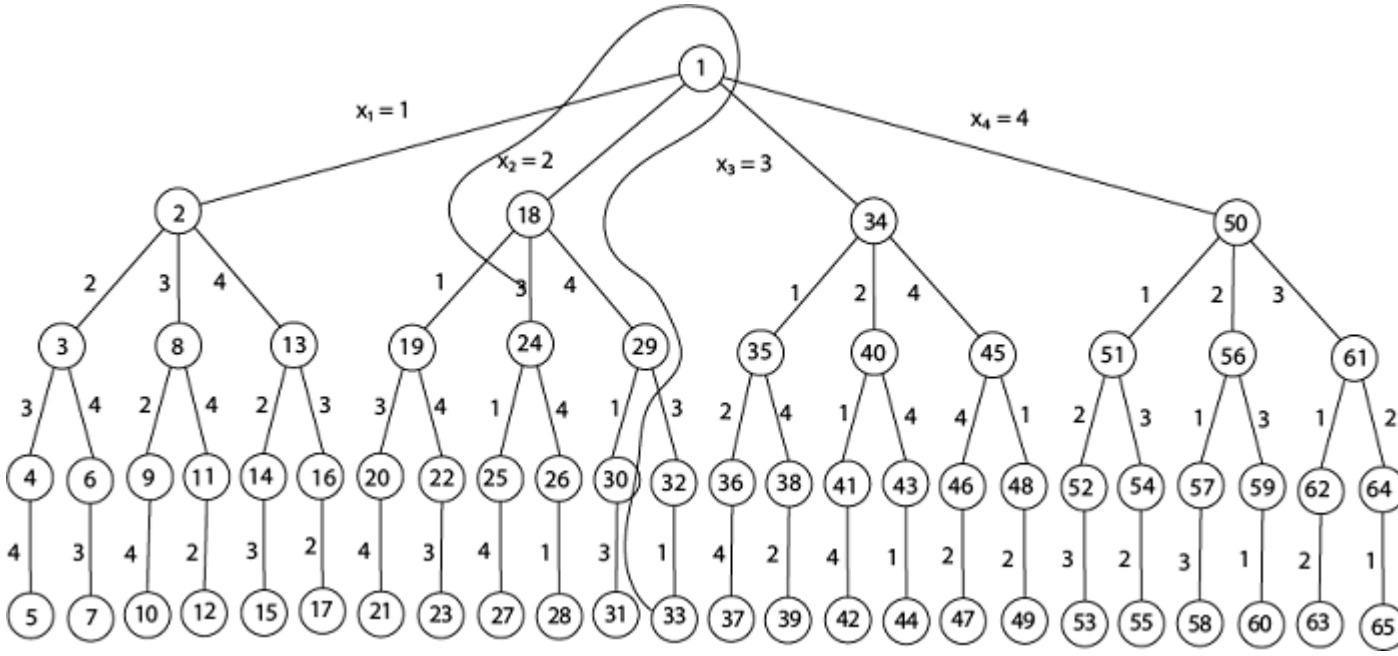


Fig shows the complete state space for 4 - queens problem. But we can use backtracking method to generate the necessary node and stop if the next node violates the rule, i.e., if two queens are attacking.



4 - Queens solution space with nodes numbered in DFS

It can be seen that all the solutions to the 4 queens problem can be represented as 4 - tuples (x_1, x_2, x_3, x_4) where x_i represents the column on which queen "q_i" is placed.

One possible solution for 8 queens problem is shown in fig:

	1	2	3	4	5	6	7	8
1				q_1				
2						q_2		
3								q_3
4		q_4						
5							q_5	
6	q_6							
7			q_7					
8					q_8			

Unit – V

NP-Hard and NP-Complete Problems

Complexity Classes

Definition of NP class Problem: - The set of all decision-based problems came into the division of NP Problems who can't be solved or produced an output within polynomial time but verified in the **polynomial time**. NP class contains P class as a subset. NP problems being hard to solve.

Definition of P class Problem: - The set of decision-based problems come into the division of P Problems who can be solved or produced an output within polynomial time. P problems being easy to solve

Definition of Polynomial time: - If we produce an output according to the given input within a specific amount of time such as within a minute, hours. This is known as Polynomial time.

Definition of Non-Polynomial time: - If we produce an output according to the given input but there are no time constraints is known as Non-Polynomial time. But yes output will produce but time is not fixed yet.

Definition of Decision Based Problem: - A problem is called a decision problem if its output is a simple "yes" or "no" (or you may need this of this as true/false, 0/1, accept/reject.) We will phrase many optimization problems as decision problems. For example, Greedy method, D.P., given a graph G= (V, E) if there exists any Hamiltonian cycle.

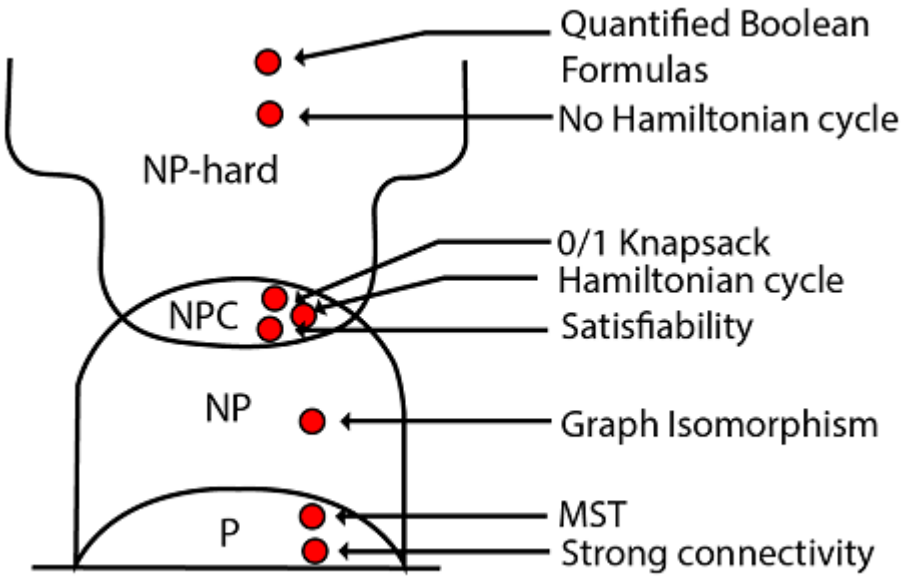
Definition of NP-hard class: - Here you to satisfy the following points to come into the division of NP-hard

1. If we can solve this problem in polynomial time, then we can solve all NP problems in polynomial time
2. If you convert the issue into one form to another form within the polynomial time

Definition of NP-complete class: - A problem is in NP-complete, if

1. It is in NP
2. It is NP-hard

Pictorial representation of all NP classes which includes NP, NP-hard, and NP-complete



Relation of P and NP classes

1. P contains in NP
2. P=NP

1. Observe that P contains in NP. In other words, if we can solve a problem in polynomial time, we can indeed verify the solution in polynomial time. More formally, we do not need to see a certificate (there is no need to specify the vertex/intermediate of the specific path) to solve the problem; we can explain it in polynomial time anyway.
2. However, it is not known whether P = NP. It seems you can verify and produce an output of the set of decision-based problems in NP classes in a polynomial time which is impossible because according to the definition of NP classes you can verify the solution within the polynomial time. So this relation can never be held.

NP-Completeness

A decision problem L is NP-Hard if

L' ≤p L for all L' ϵ NP.

Definition: L is NP-complete if

1. L ϵ NP and

2. L' ≤ p L for some known NP-complete problem L.' Given this formal definition, the complexity classes are:

P: is the set of decision problems that are solvable in polynomial time.

NP: is the set of decision problems that can be verified in polynomial time.

NP-Hard: L is NP-hard if for all L' ϵ NP, L' ≤p L. Thus if we can solve L in polynomial time, we can solve all NP problems in polynomial time.

NP-Complete L is NP-complete if

1. L ϵ NP and
2. L is NP-hard

If any NP-complete problem is solvable in polynomial time, then every NP-Complete problem is also solvable in polynomial time. Conversely, if we can prove that any NP-Complete problem cannot be solved in polynomial time, every NP-Complete problem cannot be solvable in polynomial time.

Example of NP-Complete problem

NP problem: - Suppose a DECISION-BASED problem is provided in which a set of inputs/high inputs you can get high output.

Criteria to come either in NP-hard or NP-complete.

1. The point to be noted here, the output is already given, and you can verify the output/solution within the polynomial time but can't produce an output/solution in polynomial time.
2. Here we need the concept of reduction because when you can't produce an output of the problem according to the given input then in case you have to use an emphasis on the concept of reduction in which you can convert one problem into another problem.