



Using the moment distribution method, determine the end moments and the reactions at the supports of the beam shown in Figure 12.6a. Draw the shearing force and the bending moment diagrams.  $EI = \text{constant}$ .

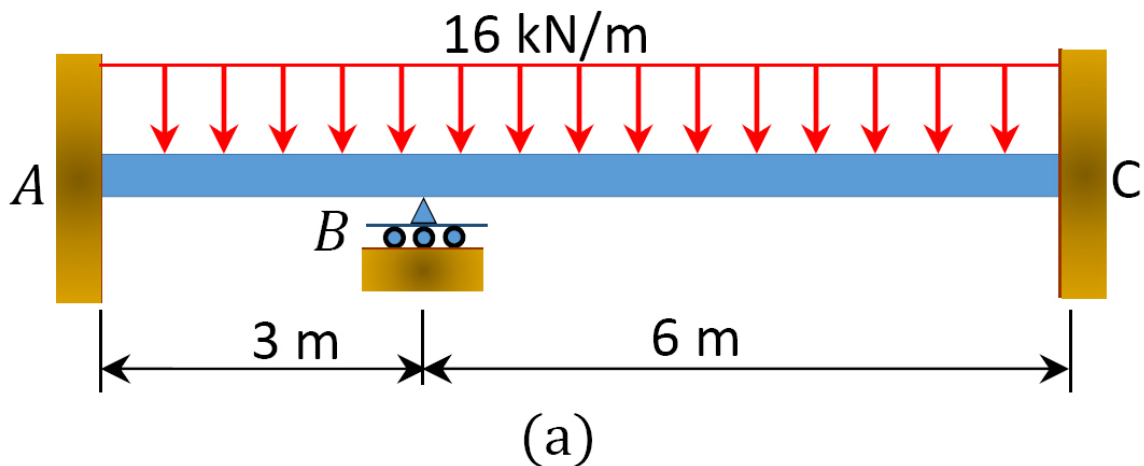


Fig. 12.6. Beam.

Solution

Fixed end moment.

$$(FEM)_{AB} = -\frac{wL^2}{12} = -\frac{16 \times 3^2}{12} = -12 \text{ kN.m}$$

$$(FEM)_{BA} = \frac{wL^2}{12} = 12 \text{ kN.m}$$

$$(FEM)_{BC} = -\frac{16 \times 6^2}{12} = -48 \text{ kN.m}$$

$$(FEM)_{CB} = 48 \text{ kN.m}$$

Stiffness factor.

$$K_{AB} = K_{BA} = \frac{1}{3} = 0.333I$$

$$K_{BC} = K_{CB} = \frac{1}{6} = 0.167I$$

Distribution factor.

$$(DF)_{AB} = \frac{K_{AB}}{\sum K} = \frac{K_{AB}}{K_{AB} + \infty} = \frac{0.333I}{0.333I + \infty} = 0$$

$$(DF)_{BA} = \frac{K_{BA}}{\sum K} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{0.333I}{0.333I + 0.167I} = 0.67$$

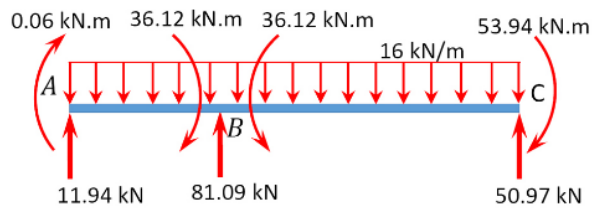
$$(DF)_{BC} = \frac{K_{BC}}{\sum K} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{0.167I}{0.333I + 0.167I} = 0.33$$

$$(DF)_{CB} = \frac{K_{CB}}{\sum K} = \frac{K_{CB}}{K_{AB} + \infty} = \frac{0.167I}{0.167I + \infty} = 0$$

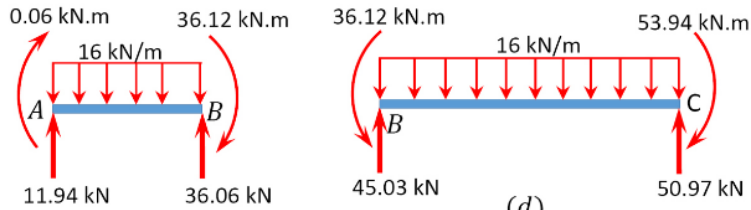
Table 12.1. Distribution table.

| Joint  | A      | B      |        | C      |
|--------|--------|--------|--------|--------|
| Member | AB     | BA     | BC     | CB     |
| DF     | 0      | 0.33   | 0.67   | 0      |
| FEM    | -12    | +12    | -48    | +48    |
| Bal    |        | +24.12 | +11.88 |        |
| CO     | +12.06 |        |        | +5.94  |
| Total  | +0.06  | +36.12 | -36.12 | +53.94 |

Shear force and bending moment diagrams.

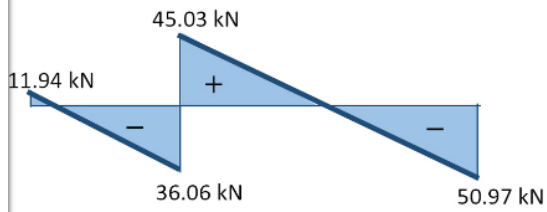


(b)

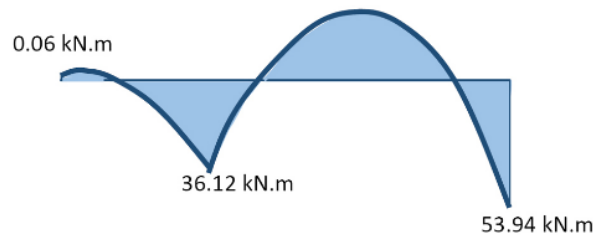


(c)

(d)



(e) Shearing force diagram for the indeterminate beam



(f) Bending moment diagram for the indeterminate beam