Gauss Jordon Method:

This method is a modified form of Gaussian elimination method. In this method, the co-eff. matrix is reduced to a diagonal matrix or unit matrix rather than a triangular matrix. Here we get the soln. without using the back substitution method.

1) Using the Gauss-Jordan method solve the following equations:

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$x + y + 5z = 7$$

The system is equivalent to $Ax = B$.

$$\begin{pmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 13 \\ 7 \end{pmatrix}$$

Now Augmented matrix is $[A, B] = \begin{pmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{pmatrix}$

We've to reduce $[A, B]$ to diagonal matrix

Fix I row, change II, III row with row I

$$[A,B] = \begin{pmatrix} 10 & 1 & 1 & 12 \\ 0 & 9.8 & 0.8 & 10.6 \\ 0 & 0.9 & 4.9 & 5.8 \end{pmatrix} \begin{matrix} \\ R_2 \leftrightarrow R_2 - \frac{2}{10}R_1 \\ R_3 \leftrightarrow R_3 - \frac{1}{10}R_1 \end{matrix}$$

$$[A,B] \sim \begin{pmatrix} 10 & 1 & 1 & 12 \\ 0 & 9.8 & 0.8 & 10.6 \\ 0 & 0.9 & 4.9 & 5.8 \end{pmatrix}$$

Fix II, I row and change III row with row I

$$\sim \begin{pmatrix} 10 & 1 & 1 & 12 \\ 0 & 9.8 & 0.8 & 10.6 \\ 0 & 0 & 4.82 & 4.82 \end{pmatrix} \begin{matrix} R_1 \leftrightarrow R_1 - \frac{1}{9.8}R_2 \\ \\ R_3 \leftrightarrow R_3 - \left(\frac{0.9}{9.8}\right)R_2 \end{matrix}$$

Fix III row, change II, I row with row III

$$\sim \begin{pmatrix} 10 & 0 & 0 & 11 \\ 0 & 9.8 & 0 & 9.8 \\ 0 & 0 & 4.82 & 4.82 \end{pmatrix} \begin{matrix} R_1 \leftrightarrow R_1 - \frac{1}{4.82}R_3 \\ R_2 \leftrightarrow R_2 - \frac{0.8}{4.82}R_3 \\ \end{matrix}$$

Fix II, III row, change I row with row II

$$\sim \begin{pmatrix} 10 & 0 & 0 & 10 \\ 0 & 9.8 & 0 & 9.8 \\ 0 & 0 & 4.82 & 4.82 \end{pmatrix} \begin{matrix} R_1 \leftrightarrow R_1 - \frac{1}{9.8}R_2 \\ \\ \\ \end{matrix}$$

We get $10x_1 = 10 \Rightarrow x = 1$

$9.8\,y = 9.8 \Rightarrow y = 1$

$4.82\,z = 4.82 \Rightarrow z = 1$

(2) Solve the following equations using Gauss-Jordan method.

$$2x_1 + 2x_2 - x_3 + x_4 = 4$$
$$4x_1 + 3x_2 - x_3 + 2x_4 = 6$$
$$8x_1 + 5x_2 - 3x_3 + 4x_4 = 12$$
$$3x_1 + 3x_2 - 2x_3 + 2x_4 = 6$$

For the above, the augmented matrix $[A,B]$ is

$$[A,B] = \begin{pmatrix} 2 & 2 & -1 & 1 & 4 \\ 4 & 3 & -1 & 2 & 6 \\ 8 & 5 & -3 & 4 & 12 \\ 3 & 3 & -2 & 2 & 6 \end{pmatrix}$$

Fix I row, change II, III & IV row with row I

$$\sim \begin{pmatrix} 2 & 2 & -1 & 1 & 4 \\ 0 & -1 & 1 & 0 & -2 \\ 0 & -3 & 1 & 0 & -4 \\ 0 & 0 & -0.5 & 0.5 & 0 \end{pmatrix} \begin{matrix} \\ R_2 \leftrightarrow R_2 - \frac{4}{2}R_1 \\ R_3 \leftrightarrow R_3 - \frac{8}{2}R_1 \\ R_4 \leftrightarrow R_4 - \frac{3}{2}R_1 \end{matrix}$$

fix I, II, change III & IV row with row II

$$\sim \begin{pmatrix} 2 & 2 & -1 & 1 & 4 \\ 0 & -1 & 1 & 0 & -2 \\ 0 & 0 & -2 & 0 & 2 \\ 0 & 0 & -0.5 & 0.5 & 0 \end{pmatrix} \begin{matrix} \\ \\ R_3 \leftrightarrow R_3 - \left(\frac{-3}{-1}\right)R_2 \\ R_4 \leftrightarrow R_4 - \end{matrix}$$

Fix I, II, III row, change IV row with III.

$$\sim \begin{pmatrix} 2 & 2 & -1 & 1 & 4 \\ 0 & -1 & 1 & 0 & -2 \\ 0 & 0 & -2 & 0 & 2 \\ 0 & 0 & 0 & 0.5 & -0.5 \end{pmatrix} R_4 \leftrightarrow R_4 - \left(\frac{-0.5}{-2}\right) R_3$$

Fix IV, III, II row, change I row with row IV.

$$\sim \begin{pmatrix} 2 & 2 & -1 & 0 & 5 \\ 0 & -1 & 1 & 0 & -2 \\ 0 & 0 & -2 & 0 & 2 \\ 0 & 0 & 0 & 0.5 & -0.5 \end{pmatrix} R_1 \leftrightarrow R_1 - \frac{1}{0.5} R_4$$

Fix IV, III, row, change II, IV row with row III.

$$\sim \begin{pmatrix} 2 & 2 & 0 & 0 & 4 \\ 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & -2 & 0 & 2 \\ 0 & 0 & 0 & 0.5 & -0.5 \end{pmatrix} \begin{matrix} R_1 \leftrightarrow R_1 - \left(+\frac{1}{2}\right) R_3 \\ R_2 \leftrightarrow R_2 - \left(\frac{1}{-2}\right) R_3 \end{matrix}$$

$$\sim \begin{pmatrix} 2 & 0 & 0 & 0 & 2 \\ 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & -2 & 0 & 2 \\ 0 & 0 & 0 & 0.5 & -0.5 \end{pmatrix} R_1 \leftrightarrow R_1 - \left(\frac{2}{-1}\right) R_2$$

$\therefore$ We get $2x_1 = 2 \Rightarrow x_1 = 1$

$-1 x_2 = -1 \Rightarrow x_2 = 1$

$-2 x_3 = 2 \Rightarrow x_3 = -1$

$0.5 x_4 = -0.5 \Rightarrow x_4 = -1$