



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore – 35



DEPARTMENT OF MATHEMATICS

UNIT – II DESIGN OF EXPERIMENTS

FACTORIAL EXPERIMENT

Introduction:

In the experimental designs either in C.R.D or R.B.D or L.S.D we were primarily concerned with the comparison and estimation of the effects of a single set of treatments like varieties of wheat, manure or different methods of cultivation etc., such experiments which deal with one factor only, called as simple experiments

In factorial experiments, the effects of several factors of variation are studied and investigate simultaneously, the treatments being all the combinations of different factors under study. in these experiments, an attempt is made to estimate the effects of each of the factors and also the interaction effects .ie., variation in the effect of one factor as a result to different levels of others factors.

Uses advantages of factorial experiments

1. Factorial design are widely used in experiments involving several factors where it is necessary to study the effect of the factors on a response
2. Factorial designs allow effects of a factor to be estimated at several levels of the others, giving conclusions that are valid over a range of experimental conditions.
3. The factorial designs are more efficient than one –factor-at a time experiments
4. In factorial designs, individual factorial effect is estimated with precision, as whole of the experiment is devoted to it.
5. Factorial designs from the basis of other designs of considerable practical value.
6. Factorial designs are widely used in research work. These design are used to apply the results over a wide range of conditions

2² FACTORIAL EXPERIMENT

Suppose there are 2 factors with 2 levels each which may effect the characteristic in which we are interested. to study their effects .we investigate the 4 possible combinations of the levels of these factors in each complete trial or in the replicate



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of the experiments. This experiment is called a 2^2 factorial experiment and can be performed in the form of CRD, RBD in the form of LSD or in other design as well.

Definition:

A factorial design with two factors, each at two levels is called a 2^2 factorial experiment

Yate's method of computing factorial effect totals

For calculation of various factorial effect totals for 2^2 factorial experiments, the following table provides a special computational rule for the totals of the main effects or the interactions corresponding to the treatment combinations

Problem:

Find out the main effects and interaction in the following 2^2 factorial experiments

And write down the analysis of variance table

Block	Treatments			
I	64(1)	6(kp)	25(k)	30(p)
II	14(k)	75(1)	33(kp)	50(p)
III	17(kp)	41(p)	12(k)	76(1)
IV	25(p)	33(k)	75(1)	10(kp)

Solution:

Null hypothesis H_0 : There is no significant difference between treatments (rows) and between blocks (columns)

We re arrange the given data in new table given below for computations of the SS due to treatments and blocks.



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Treatment combination	Blocks			
	I	II	III	IV
(1)	64	25	30	6
(k)	75	14	50	33
(p)	76	12	41	17
(kp)	75	33	25	10

Code the data by subtracting 37 from each value.

Treatment combination	Blocks				Row Total R_i	R_i^2
	I	II	III	IV		
(1)	27	38	39	38	142	20164
(k)	-12	-23	-25	-4	-64	4096
(p)	-7	13	4	-12	-2	4
(kp)	-31	-4	-20	-27	-82	6724
Row Total C_j	-23	24	-2	-5	-6	$\sum R_i^2 = 309882$
C_j^2	529	576	4	25	$\sum C_j^2 = 1134$	

$$N = 4 \times 4 = 16$$

Step 1:

Grand total $T = -6$

Step 2:

$$\text{Correction Factor C.F} = \frac{T^2}{N}$$



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$$C.F = \frac{(-6)^2}{16}$$

$$C.F = 2.25$$

Step 3:

$$\begin{aligned}SSC &= \text{Sum of squares of individual observations} \\ &= 8939\end{aligned}$$

Step 4:

$$\begin{aligned}TSS &= \text{Sum of squares of individual observation} - C.F \\ &= 8936 - 2.25 \\ &= 8933.75\end{aligned}$$

Step 5:

SSR = Sum Of Squares Between Rows

$$\begin{aligned}SSR &= \frac{(\sum R_i)^2}{n_1} - C.F \\ &= 7744.75\end{aligned}$$

Step 6:

SSC = Sum Of Squares Between columns

$$\begin{aligned}SSC &= \frac{(\sum c_j)^2}{n_2} - C.F \\ &= 283.5 - 2.25 \\ &= 281.25\end{aligned}$$

Step 7:

$$\text{Residual SSE} = TSS - (SSR + SSC + SST)$$



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$$=8933.75-(7744+281.25)$$

$$=907.75$$

Step 8:

$$[K] = [KP] - [P] + [K] - [1]$$

$$=-82-(-2)+(-64)-142$$

$$=-286$$

Step 9:

$$[P] = [KP] + [P] - [K] - [1]$$

$$=-82+(-2)-(-64)-142$$

$$=-162$$

Step 10:

$$[KP] = [KP] - [K] + [P] + [1]$$

$$=-82-(-64)+(-2)+142$$

$$= 126$$

Step 11:

$$S_K = \frac{[K]^2}{4r}$$

$$= \frac{(-286)^2}{4 \times 4}$$

$$=5112.25$$

Step 12:



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$$\begin{aligned} S_P &= \frac{[P]^2}{4r} \\ &= \frac{(-162)^2}{4 \times 4} \\ &= 1640.25 \end{aligned}$$

Step 13:

$$\begin{aligned} S_{KP} &= \frac{[KP]^2}{4r} \\ &= \frac{(-126)^2}{4 \times 4} \\ &= 992.25 \end{aligned}$$

Step 13:

ANOVA TABLE

Source of variation	Degrees of freedom	Sum of squares	Mean squares	F-ratio	Table value
k	1	5112.25	$MS_K=5112.25$	$F_K = 50.69$	6.99
p	1	1640.25	$MS_P=1640.25$	$F_P = 16.26$	6.99
kp	1	992.25	$MS_{KP}=992.25$	$F_{KP} = 50.699.84$	6.99
Residual	9	9076.75	$MSE=100.86$	-	-