GAUSS - SEIDEL ITERATIVE METHOD:

Let the system of simultaneous equations be

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

check: $|a_1| > |b_1| + |c_1|$

$|b_2| > |a_2| + |c_2|$

$|c_3| > |a_3| + |b_3|$

The diagonal elts should be dominant, so that, the iteration process can be applied.

The gn. system can be written as.

$$x = \frac{1}{a_1}(d_1 - b_1 y - c_1 z)$$
$$y = \frac{1}{b_2}(d_2 - a_2 x - c_2 z)$$
$$z = \frac{1}{c_3}(d_3 - a_3 x - b_3 y)$$

Let the I approximation be y_0 and $z_0 = 0$

$$x_1 = \frac{1}{a_1}(d_1 - b_1 y_0 - c_1 z_0)$$
$$y_1 = \frac{1}{b_2}(d_2 - a_2 x_1 - c_2 z_0)$$
$$z_1 = \frac{1}{c_3}(d_3 - a_3 x_1 - b_3 y_1)$$

II iteration:

$$x_2 = \frac{1}{a_1}(d_1 - b_1 y_1 - c_1 z_1)$$

$$y_2 = \frac{1}{b_2}(d_2 - a_2 x_2 - c_2 z_1)$$

$$z_2 = \frac{1}{c_3}(d_3 - a_3 x_2 - b_3 y_2)$$

III iteration:

$$x_3 = \frac{1}{a_1}(d_1 - b_1 y_2 - c_1 z_2)$$

$$y_3 = \frac{1}{b_2}(d_2 - a_2 x_3 - c_2 z_2)$$

$$z_3 = \frac{1}{c_3}(d_3 - a_3 x_3 - b_3 y_3)$$

The process is repeated until we get difference btwn. two consecutive approx. is negligible.

(1) Solve the system of eqns: [Form the Table]

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$

using Gauss-Seidel iteration method.

Soln: The given system is,

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$

Here the diagonal elts. does not dominant. So we are interchanging the system as

$$27x+6y-z=85$$
$$6x+15y+2z=72$$
$$x+y+54z=110$$

Since diagonal elts. are dominant, the iteration process is applied here. The above system can be written as

$$x=\frac{1}{27}(85-6y+z)$$
$$y=\frac{1}{15}(72-6x-2z)$$
$$z=\frac{1}{54}(110-x-y)$$

I iteration:

$$x_1=\frac{1}{27}(85-6y_0+z_0)$$
$$y_1=\frac{1}{15}(72-6x_1-2z_0)$$
$$z_1=\frac{1}{54}(110-x_1-y_1)$$

Let the initial values, $y_0=z_0=0$.

$$x_1=\frac{1}{27}\times 85 = 3.1481$$
$$y_1=\frac{1}{15}(72-6\times 3.1481-2\times 0)=3.5407$$
$$z_1=\frac{1}{54}(110-3.1481-3.5407)=1.9131$$

II iteration:

$$x_2=\frac{1}{27}(85-6\times 3.5407+1.9131)=2.4321$$
$$y_2=\frac{1}{15}(72-6\times 2.4321-2\times 1.9131)=3.5720$$
$$z_2=\frac{1}{54}(110-2.4321-3.5720)=1.9258$$

III iteration:

$x_3 = 2.4256$

$y_3 = 3.5729$

$z_3 = 1.9259$

IV iteration:

$x_4 = 2.4255$

$y_4 = 3.5730$

$z_4 = 1.9259$

V iteration:

$x_5 = 2.4254$

$y_5 = 3.5730$

$z_5 = 1.9259$

VI iteration:

$x_6 = 2.4254$

$y_6 = 3.5730$

$z_6 = 1.9259$

From V, VI iteration we get the solutions as

$x = 2.4254$; $y = 3.5730$; $z = 1.9259$.