



## DEPARTMENT OF MATHEMATICS

### UNIT - II DESIGN OF EXPERIMENTS

#### LATIN SQUARE :

2) An agriculturist wants to test the effects of four different fertilizers A, B, C and D on the yield of paddy. In order to eliminate sources of error due to variability in self-fertility he used the fertilizers in a Latin square arrangement as given below where the numbers indicate yields in quintals per unit area. Perform an analysis of variance to decide whether there is a difference between the fertilizers at 5% level of significance.

A	18	D	20	C	23	B	11
D	22	A	20	B	10	C	19
B	15	C	21	D	25	A	17
C	22	B	12	A	15	D	24

Soln: Let origin =  $n_{ij} - 18$  avg(min, max)

	$n_1$	$n_2$	$n_3$	$n_4$	total	$n_1^2$	$n_2^2$	$n_3^2$	$n_4^2$
$y_1$	0	3	5	-7	1	0	9	25	49
$y_2$	4	-2	-8	1	-1	16	4	64	1
$y_3$	-3	3	7	-1	6	9	9	49	1
$y_4$	4	-6	-3	6	1	16	36	9	36
total	5	2	1	-1	7	41	58	147	87
	$\sum n_1$	$\sum n_2$	$\sum n_3$	$\sum n_4$		$\sum n_1^2$	$\sum n_2^2$	$\sum n_3^2$	$\sum n_4^2$

Step 1: Formulate  $H_0$  &  $H_1$ :

$H_0$ : There is no difference between the fertilizers

$H_1$ : There is difference between the fertilizers.



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step 2: To find  $T$  &  $N$ :

$$T = \sum n_1 + \sum n_2 + \sum n_3 + \sum n_4$$

$$= 5 + 2 + 1 - 1 = 7$$

$$N = n_1 + n_2 + n_3 + n_4$$

$$= 4 + 4 + 4 + 4 = 16$$

step 3: To find correction factor, C.F.

$$C.F. = \frac{T^2}{N} = \frac{7^2}{16} = \frac{49}{16} = 3.0625$$

step 4: To find TSS;

$$TSS = \sum n_1^2 + \sum n_2^2 + \sum n_3^2 + \sum n_4^2 - C.F.$$

$$= 41 + 58 + 147 + 87 - 3.0625$$

$$= 333 - 3.0625 = 329.94$$

step 5: To find SSC, SSR, & SST

$$SSC = \frac{(\sum n_1)^2}{n_1} + \frac{(\sum n_2)^2}{n_2} + \frac{(\sum n_3)^2}{n_3} + \frac{(\sum n_4)^2}{n_4} - C.F.$$

$$= \frac{5^2}{4} + \frac{2^2}{4} + \frac{1^2}{4} + \frac{-1^2}{4} - 3.0625$$

$$= 4.6875$$

$$SSR = \frac{(\sum y_1)^2}{n_1'} + \frac{(\sum y_2)^2}{n_2'} + \frac{(\sum y_3)^2}{n_3'} + \frac{(\sum y_4)^2}{n_4'} - C.F.$$

$$= \frac{1^2}{4} + \frac{-1^2}{4} + \frac{6^2}{4} + \frac{1^2}{4} - 3.0625$$

$$= 6.6875$$



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To find SST :

A	0	2	-3	-1	-2 : $\Sigma z_1$
B	-3	-6	-8	-7	-24 : $\Sigma z_2$
C	4	3	5	1	13 : $\Sigma z_3$
D	4	3	7	6	20 : $\Sigma z_4$

$$\begin{aligned} SST &= \frac{(\Sigma z_1)^2}{4} + \frac{(\Sigma z_2)^2}{4} + \frac{(\Sigma z_3)^2}{4} + \frac{(\Sigma z_4)^2}{4} - C.F. \\ &= \frac{-2^2}{4} + \frac{-24^2}{4} + \frac{13^2}{4} + \frac{20^2}{4} - C.F. \\ &= 284.25 - 3.0625 = 284.1875 \end{aligned}$$

Step 6: to find SSE

$$\begin{aligned} SSE &= TSS - SSSC - SSR - SST \\ &= 329.94 - 4.6875 - 6.6875 - 284.1875 \\ &= 34.375 \end{aligned}$$



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Step 7: Anova table.

Source of variations	Sum of Squares	Degrees of Freedom	Mean Sum of Squares	F-Ratio
Column	SSC: 4.6875	$c-1 = 3$	MSC = $\frac{4.6875}{3}$ = 1.5625	$F_c = \frac{5.7291}{1.5625}$ = 3.6666 $F_{\alpha}(6,3) = 9.1$
Row	SSR: 6.6875	$r-1 = 3$	MSR = $\frac{6.6875}{3}$ = 2.2291	$F_r = \frac{5.7291}{2.2291}$ = 2.5701 $F_{\alpha}(6,3) = 9.1$
Treatment	SST: 284.1875	$T-1 = 3$	MST = $\frac{284.1875}{3}$ = 94.7291	$F_T = \frac{94.7291}{16.5347}$ = 5.7291 $F_{\alpha}(3,6) = 4.76$
Error	SSE: 34.375	$(n-1)(n-2)$ $3 \times 2 = 6$	MSE = $\frac{34.375}{6}$ = 5.7291	

Step 8: Conclusion:

$$F_c = 3.6666 < 9.94 = F_{\alpha}, \text{ } H_0 \text{ is accepted}$$

$$F_r = 2.5701 < 9.94 = F_{\alpha}, \text{ } H_0 \text{ is accepted}$$

$$F_T = 16.5347 > 4.76 = F_{\alpha}, \text{ } H_0 \text{ is rejected}$$

(a) There is difference between the fertilizers.



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2) Analyse the variance in the Latin square of yields (in quintals) of wheat where P, Q, R, S represent the different manures used.

S 222    P 221    R 223    Q 222

Q 224    R 223    P 222    S 225

P 220    Q 219    S 220    R 221

R 222    S 223    Q 221    P 222

Test whether the different manures used have given significantly different yields:

Soln:  $F_c = 1.34$  ;  $F_R = 12.31$  ,  $F_T = 2.12$  &  $F_\alpha = 4.76$