UNIT-IV PART C

Level 1 Questions

1. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ $prove$ $\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$
2. If $u = \cos^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot u$
3. If x = r sin θ cos ϕ,y = r sin θ sin ϕ, z = r cos θ find J = $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$.
4. If $u = e^{xy}, show\ that \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{u}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]$.
5. If F is a function of x and y and if x = e^u sin v , y = e^u cos v, prove that $\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = e^{-2u}\left[\frac{\partial^2 F}{\partial u^2} + \frac{\partial^2 F}{\partial v^2}\right]$

Level 2 Questions

6. The temperature T at any point (x, y, z) in space is T = $Kxyz^2$ where K is a constant. Find the highest temperature on the surface of the sphere $x^2 + y^2 + z^2 = a^2$.
7. A rectangular box open at the top is to have a volume of 32 c.c. find the dimensions of the box that requires the least material for its construction.
8. Find the volume of the greatest rectangular parallelepiped inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
9. If $x^2 + y^2 + z^2 = r^2$, show that the maximum value of yz + zx + xy is r^2 and the minimum value is $\frac{-r^2}{2}$.
10. If x + y + z =u , y + z = uz, z = uvw prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$.
11. If u= $x^2 + y^2 + z^2$ and $x = e^{2t}, y = e^{2t}\cos 3t, z = e^{2t}\sin 3t$, find $\frac{du}{dt}$.

Level 3 Questions

12. If u = f(x,y) where x = r cos θ ,y = r sin θ, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$.
13. Find the Taylor's series expansion of $e^x \sin y$ at the point (-1, π/4) up to third degree terms.
14. Find the Taylor's series expansion of $e^x \cos y$ in the neighborhood of the point (1, π/4) up to third degree terms.
15. Use Taylor's series expansion of $x^2y^2 + 2x^2y + 3xy^2$ in the powers of (x - 1) and (y - 2).
16. Expand $x^2y - 2 + 3y$ in powers of (x – 1) and (y + 2) upto third degree terms.
17. Find the extreme values of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.
18. Find the maximum and minimum values of $x^2 - xy - 2x + y^2 + y$.