SNS COLLEGE OF TECHNOLOGY

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Coimbatore-35

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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB212 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 2 – IIR FILTER DESIGN

TOPIC – CHEBYSHEV FILTER

COMPARISON OF DIGITAL & ANALOG FILTERS

S.No.	Digital Filter	Analog Filter
1	Operates on digital samples of the signal	Operates on analog signals
2	It is governed by linear difference equation	It is governed by linear differential equation
3	It consists of adders, multipliers and delays implemented in digital logic	It consists of electrical components like resistors, capacitors and inductors
4	The filter coefficients are designed to satisfy the desired frequency response	The approximation problem is solved to satisfy the desired frequency response

DESIGN OF LOWPASS DIGITAL CHEBYSHEV FILTER

- For designing a Chebyshev IIR digital filter, analog filter is designed using the given specifications
- Then the analog filter transfer function is transformed to digital filter transfer function by using either Impulse Invariant or Bilinear Transformation
- The analog chebyshev filter is designed by approximating the ideal frequency response using an error function
- The approximation function is selected such that the error is minimized over a band of frequencies

TYPES OF CHEBYSHEV APPROXIMATION

- There are two types of Chebyshev approximation:

1. Type-1 Chebyshev Approximation
2. Type-2 Chebyshev Approximation

- In type - 1 approximation, the error function is selected such that magnitude response is equiripple in the passband and monotonic in the stopband
- In type - 2 approximation, the error function is selected such that magnitude response is monotonic in the passband and equiripple in the stopband
- The type – 2 magnitude response is also called Inverse chebyshev response

ANALOG CHEBYSHEV FILTER

- The magnitude response of Type-1 low pass filter is given by

$$\left|H(j\Omega)\right|^2 = \frac{1}{1+\epsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)}$$

- ϵ – Attenuation Constant

$C_N\left(\frac{\Omega}{\Omega_p}\right)$ = Chebyshev polynomial of order N

ANALOG CHEBYSHEV FILTER

- Attenuation Constant is given by

$$\epsilon = \left[\frac{1}{A_P^2} - 1 \right]^{1/2}$$

- $\mathbf{A_p}$ - is the gain or magnitude at pass band edge frequency $\mathbf{\Omega_p}$
- For small values of N the chebyshev polynomial is given by

$$C_N(x) = \begin{cases} \cos(N \cos^{-1} x) & ; \text{ for } |x| \le 1 \\ \cosh(N \cosh^{-1} x) & ; \text{ for } |x| > 1 \end{cases}$$

ANALOG CHEBYSHEV FILTER

- The transfer function of the analog system can be obtained from the magnitude response of Type – 1 low pass filter by substituting **Ω by s/j**

$$H(s)\,H(-s) = \frac{1}{1 + \epsilon^2 C_N^2\left(\frac{s/j}{\Omega_c}\right)}$$

- For the normalized transfer function, let us replace s/ **Ω_c by s_n**

$$H(s_n)\,H(-s_n) = \frac{1}{1 + \epsilon^2 C_N^2(-js_n)}$$

PROPERTIES OF CHEBYSHEV FILTERS (TYPE -1)

- The magnitude $|H(j\Omega)|$ oscillates between 1 and $1/\sqrt{1+\epsilon^2}$ within the pass band and so the filter is called equiripple I the pass band
- The normalized magnitude response has a value of $1/\sqrt{1+\epsilon^2}$ at cutoff frequency Ω_c
- The magnitude is monotonic outside the pass band
- The Chebyshev Type – 1 Filters are all pole designs
- With large values of N, the transition from pass band to stop band becomes more sharp and approaches ideal characteristics.

TRANSFER FUNCTION OF ANALOG CHEBYSHEV LOW PASS FILTER

- For a stable and causal filter the poles should lie on the left half of s –plane. Hence the desired filter transfer function is obtained by selecting N number of left half poles
- When N is even all the poles are complex and exist as conjugate pair
- When N is odd, one of the pole is real and all other poles are complex and exist as conjugate pair
- Therefore the transfer function of chebyshev filters will be a product f second order factors

NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

- N be the order of the filter
- $H(s_n)$ be the normalized Chebyshev lowpass filter function
- When N is even,

$$H(s_n) = \prod_{k=1}^{\frac{N}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}$$

- When N is odd,

$$H(s_n) = \frac{B_0}{s + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}$$

NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

$$\text{where, } b_k = 2\, y_N \sin\left(\frac{(2k-1)\pi}{2N}\right)$$

$$c_k = y_N^2 + \cos^2\left(\frac{(2k-1)\pi}{2N}\right)$$

$$c_0 = y_N$$

$$y_N = \frac{1}{2}\left\{\left[\left(\frac{1}{\epsilon^2}+1\right)^{\frac{1}{2}} + \frac{1}{\epsilon}\right]^{\frac{1}{N}} - \left[\left(\frac{1}{\epsilon^2}+1\right)^{\frac{1}{2}} + \frac{1}{\epsilon}\right]^{-\frac{1}{N}}\right\}$$

NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

- N be the order of the filter
- For even values of N parameter B_k are evaluated

$$H(s_n)\Big|_{s_n = 0} = \frac{1}{(1 + \epsilon^2)^{\frac{1}{2}}}$$

- For odd values of N the parameter B_k are evaluated

$$H(s_n)\Big|_{s_n = 0} = 1$$

- While evaluating B_k to take $B_0 = B_1 = B_2 = ... = B_k$

UNNORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

- The unnormalized transfer function is obtained by letting $s_n \rightarrow s/\Omega_c$ in the normalized transfer function, Where Ω_c is the cutoff frequency of the lowpass filter
- H(s) be the normalized Chebyshev low pass filter transfer function
- When N is even, H(s) is obtained by letting $s_n \rightarrow s/\Omega_c$ in normalized Chebyshev low pass filter function

$$\therefore H(s) = \prod_{k=1}^{\frac{N}{2}} \left. \frac{B_k}{s_n^2 + b_k s_n + c_k} \right|_{s_n = \frac{s}{\Omega_c}} = \prod_{k=1}^{\frac{N}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

UNNORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

- H(s) be the normalized Chebyshev low pass filter transfer function
- N be the order of the filter
- Ω_c is the cutoff frequency of the lowpass filter
- When N is odd, H(s) is obtained by letting $s_n \rightarrow s/\Omega_c$ in normalized Chebyshev low pass filter function

$$\therefore H(s) = \frac{B_0}{s_n + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k} \Bigg|_{s_n = \frac{s}{\Omega_c}} = \frac{B_0 \Omega_c}{s + c_0 \Omega_c} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

ORDER OF ANALOG LOWPASS CHEBYSHEV FILTER

- In Chebyshev filters the frequency response of the filter depends on the order N. The specifications of the filter are given in terms of gain at a passband and stopband frequency
- $\mathbf{A_p}$ - Gain or Magnitude at pass band edge frequency $\mathbf{\Omega_p}$
- $\mathbf{A_s}$ - Gain or Magnitude at Stop band edge frequency $\mathbf{\Omega_s}$

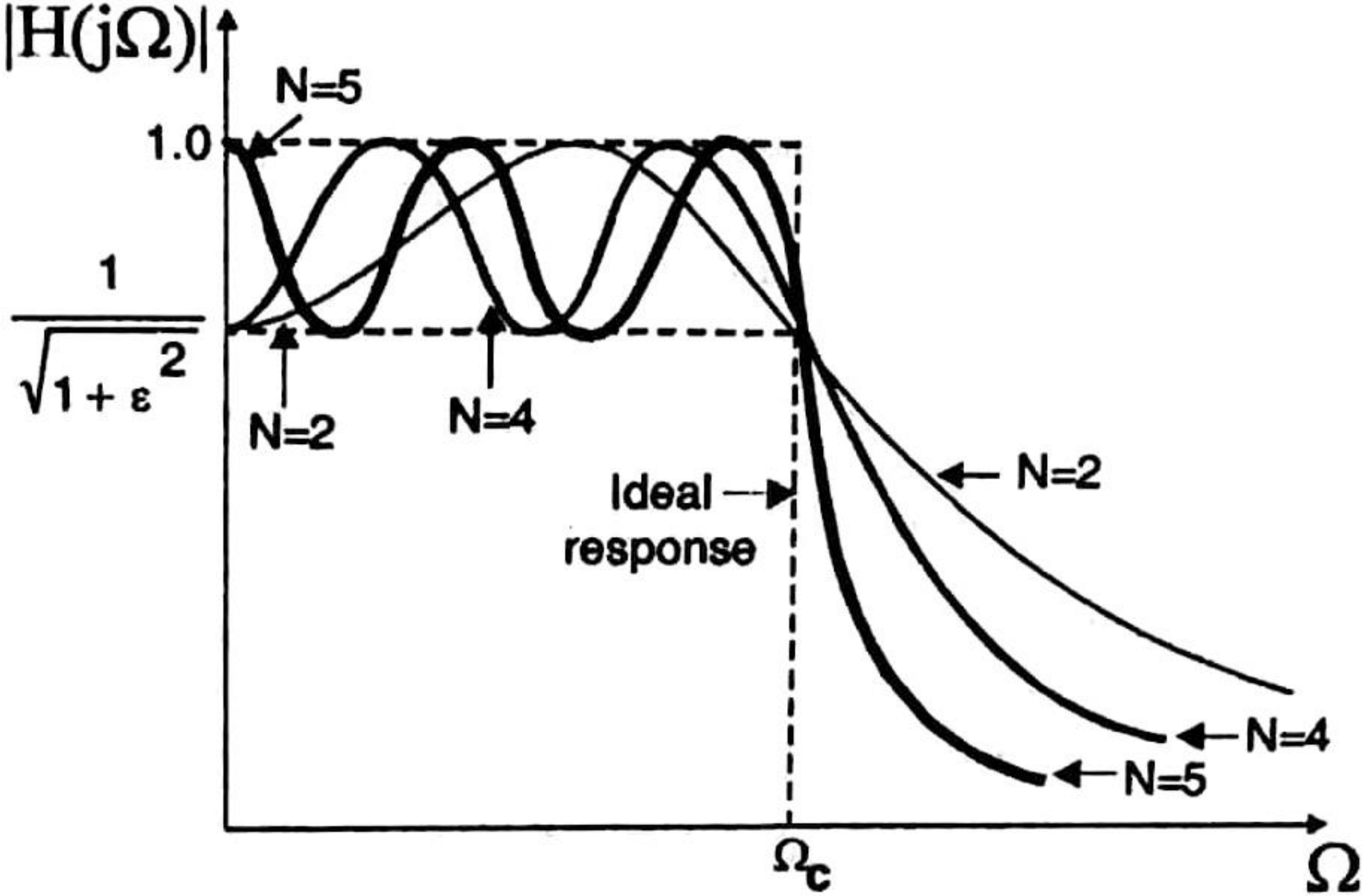
$$N_1 = \frac{\cosh^{-1}\left[\left(\frac{(1/A_s^2) - 1}{(1/A_p^2) - 1}\right)^{\frac{1}{2}}\right]}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

ORDER OF THE LOWPASS CHEBYSHEV FILTER

- The specifications of the filter are given in terms of dB attenuation at a passband and stopband frequency
- $\alpha_{p,\,dB}$ - dB attenuation at pass band edge frequency Ω_p
- $\alpha_{s,\,dB}$ - dB attenuation at Stop band edge frequency Ω_s

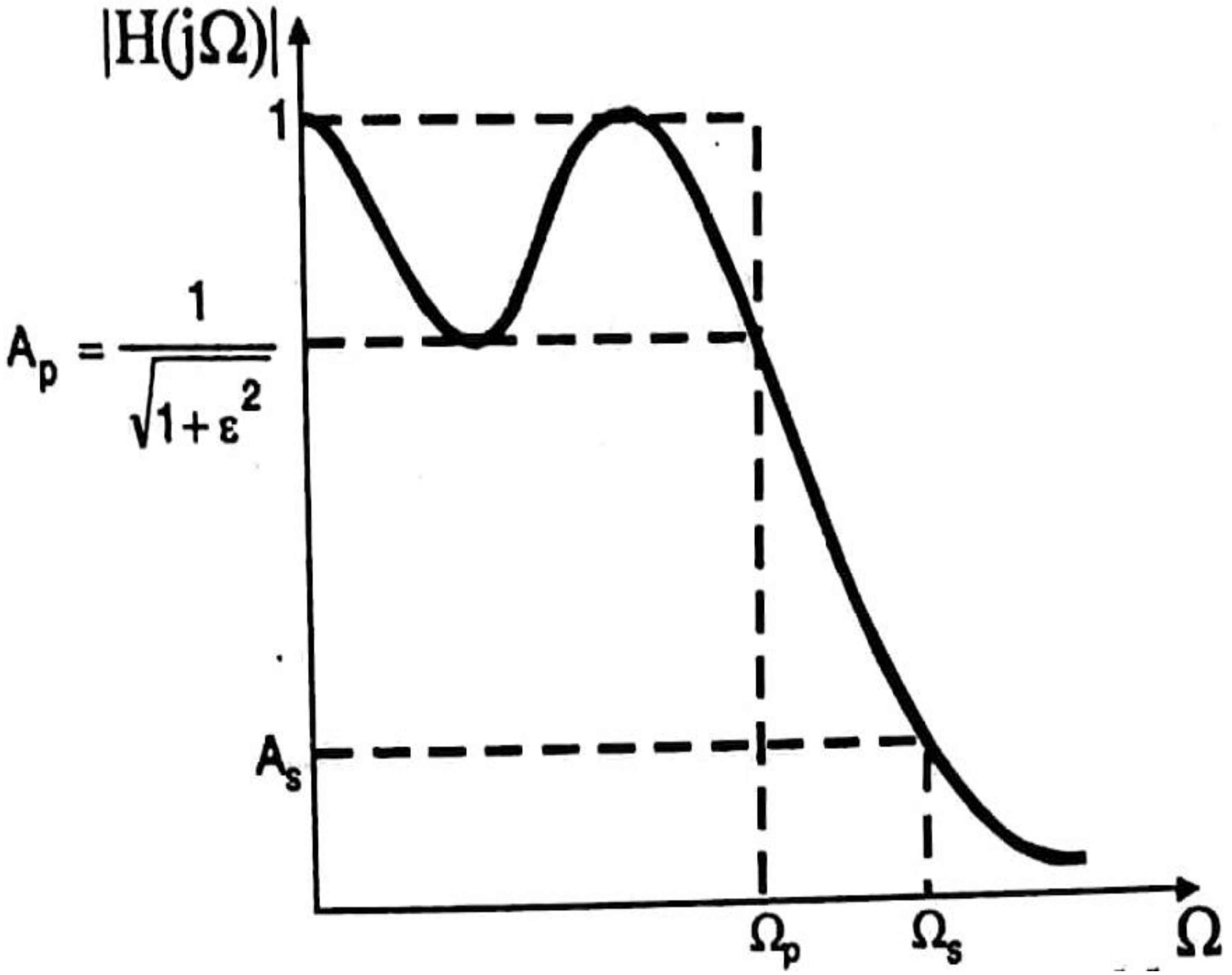
$$N_1 = \frac{\cosh^{-1}\left[\left(\frac{10^{0.1\alpha_{s,dB}}-1}{10^{0.1\alpha_{p,dB}}-1}\right)^{\frac{1}{2}}\right]}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

MAGNITUDE RESPONSE OF ANALOG CHEBYSHEV TYPE – 1 LOW PASS FILTER

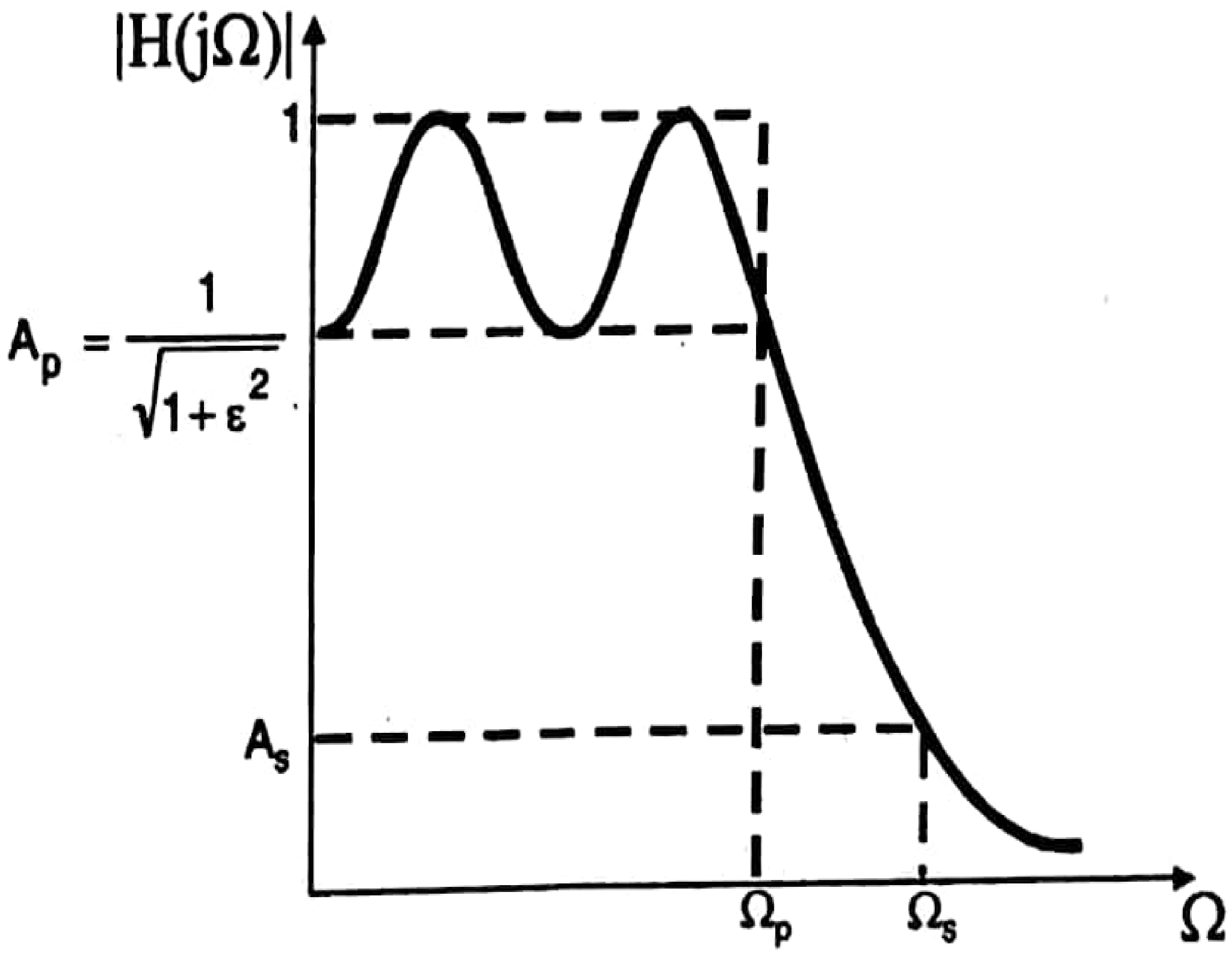


MAGNITUDE RESPONSE OF ANALOG CHEBYSHEV TYPE – 1 FILTERS

N is Odd

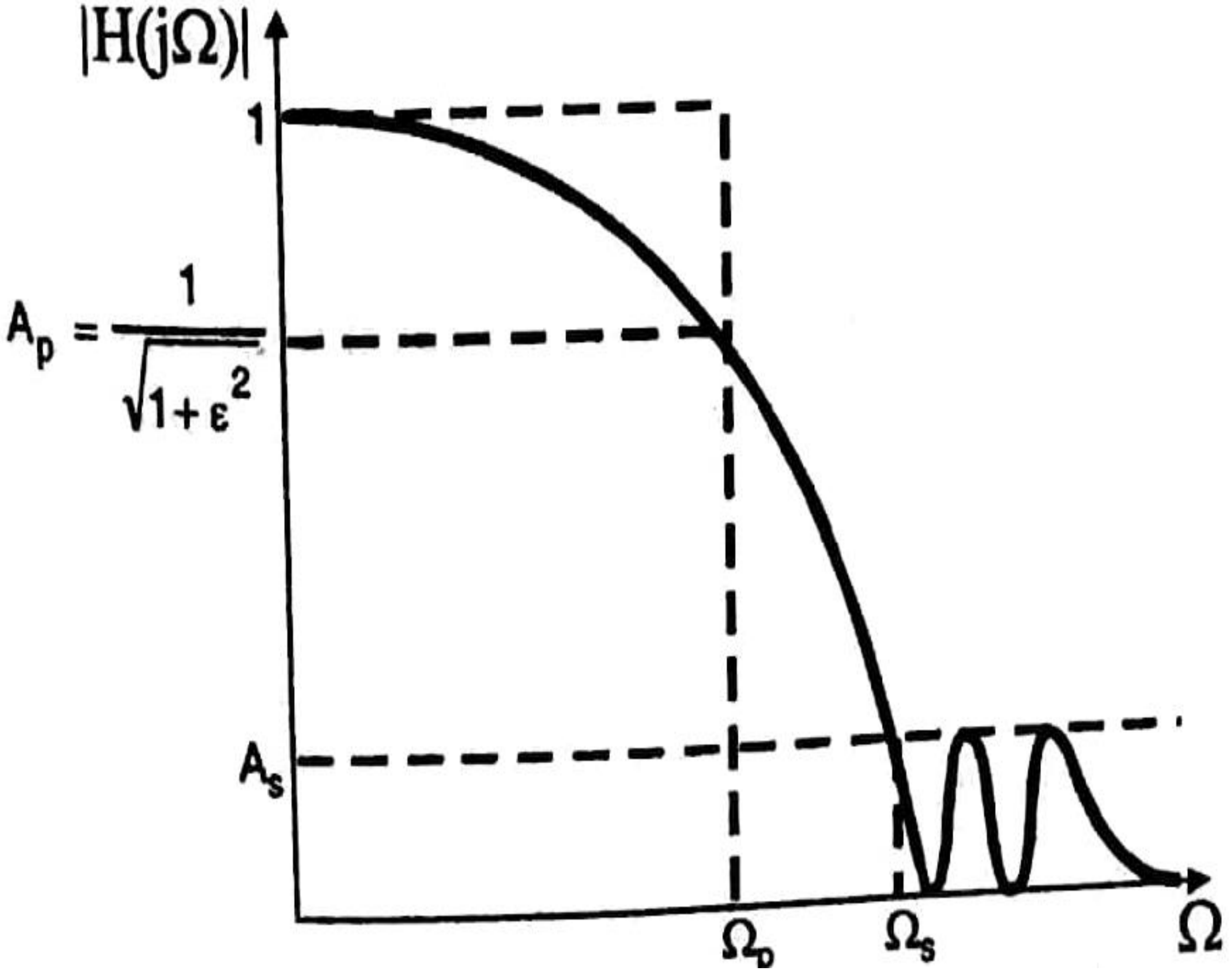


N is Even

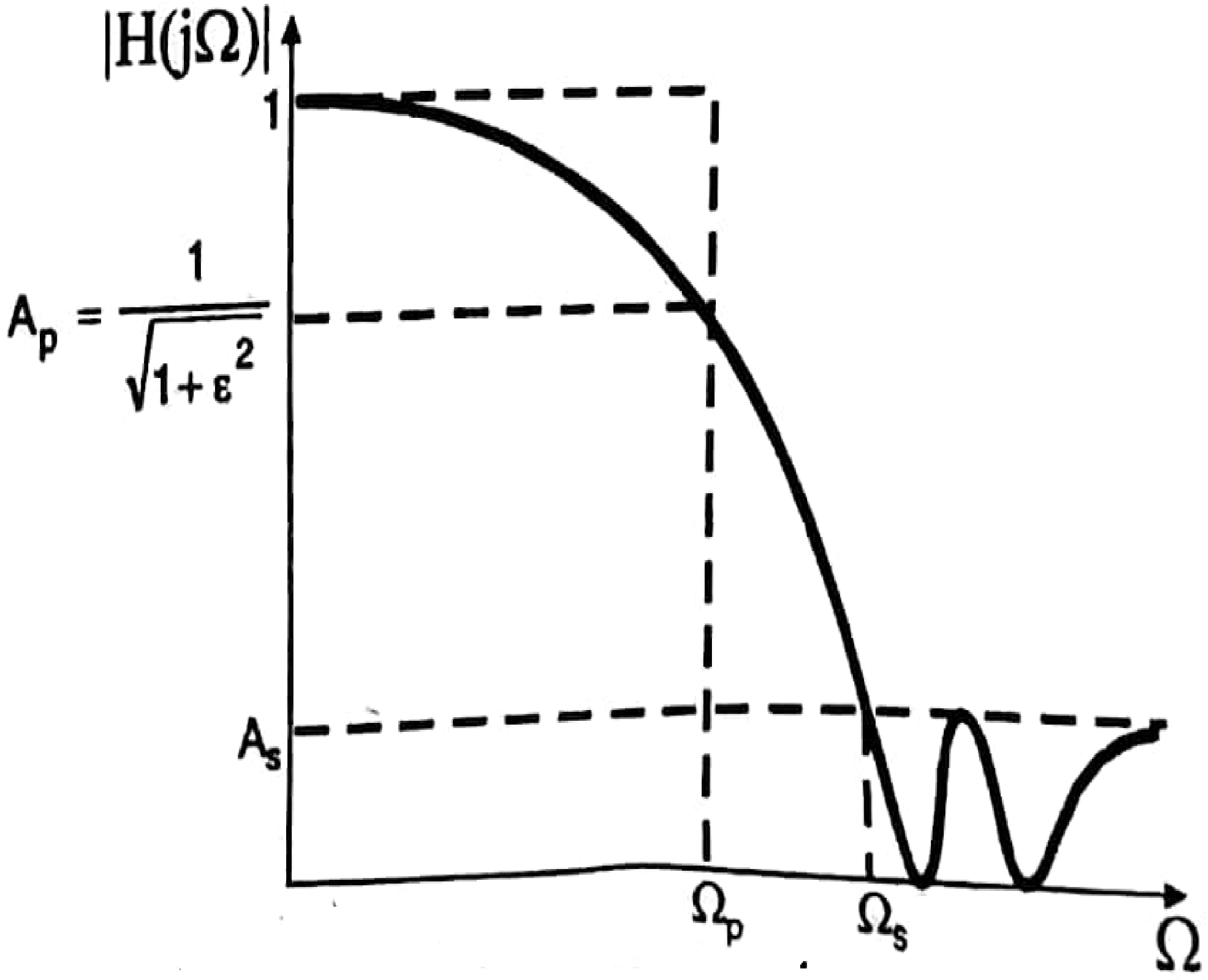


MAGNITUDE RESPONSE OF ANALOG CHEBYSHEV TYPE – 2 FILTERS

N is Odd



N is Even



CUTOFF FREQUENCY OF ANALOG LOWPASS CHEBYSHEV FILTER

- The IIR filters are designed to satisfy a prescribed gain or attenuation at a pass band or stop band frequency. But Practically the cutoff frequency $\boldsymbol{\Omega_c}$ is used to decide the useful frequency range of the filter
- In chebyshev filter design the passband and stopband specifications are used to estimate the order, N of the filter and $\mathbf{N^{th}}$ order normalized Chebyshev lowpass filter is designed. Then the normalized LPF is unnormalized using the cutoff frequency
- In Chebyshev filters the passband edge frequency, Ω_p is considered as cutoff frequency Ω_c and this cutoff is not equal to 3 dB cutoff frequency Ω_{3dB}

$$\Omega_{3dB} = \Omega_c \cosh\left(\frac{1}{N} \cosh^{-1} \frac{1}{\epsilon}\right)$$

DESIGN PROCEDURE FOR LOWPASS DIGITAL CHEBYSHEV IIR FILTER

- ω_p - Pass band edge digital frequency in rad /sample
- ω_s - Stop band edge digital frequency in rad/sample
- A_p - Gain at pass band edge frequency ω_p
- A_s - Gain at Stop band edge frequency ω_s
- $T = 1/F_s$ - Sampling time in sec.
- Where F_s = Sampling frequency in Hz
- Ω_p - Pass band edge analog frequency corresponding to ω_p
- Ω_s - Stop band edge analog frequency corresponding to ω_s

1. Choose either Bilinear or Impulse Invariant transformation and determine the specifications of equivalent analog filter

- The gain or attenuation of analog filter is same as digital filter
- **Bilinear Transformation:**

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} \qquad \Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2}$$

- **Impulse Invariant Transformation:**

$$\Omega_p = \frac{\omega_p}{T} \qquad \Omega_s = \frac{\omega_s}{T}$$

ORDER OF THE LOWPASS DIGITAL CHEBYSHEV FILTER

2. Decide the order N of the filter. In order to estimate the order N, Calculate the Parameter N_1 using the following equation:

$$N_1 = \frac{\cosh^{-1}\left[\left(\frac{(1/A_s^2) - 1}{(1/A_p^2) - 1}\right)^{\frac{1}{2}}\right]}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

- Choose N such that, $N \geq N_1$, Usually N is chosen as nearest integer just greater than N_1

NORMALIZED BUTTERWORTH LPF TRANSFER FUNCTION

3. Determine the normalized transfer function $H(s_n)$ of the analog lowpass filter function

- When N is even,

$$H(s_n) = \prod_{k=1}^{\frac{N}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}$$

- When N is odd,

$$H(s_n) = \frac{B_0}{s + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}$$

NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

where, $b_k = 2\, y_N \sin\left(\frac{(2k-1)\pi}{2N}\right)$

$$\epsilon = \left[\frac{1}{A_P^2} - 1\right]^{1/2}$$

$$c_k = y_N^2 + \cos^2\left(\frac{(2k-1)\pi}{2N}\right)$$

$$c_0 = y_N$$

$$y_N = \frac{1}{2}\left\{\left[\left(\frac{1}{\epsilon^2}+1\right)^{\frac{1}{2}} + \frac{1}{\epsilon}\right]^{\frac{1}{N}} - \left[\left(\frac{1}{\epsilon^2}+1\right)^{\frac{1}{2}} + \frac{1}{\epsilon}\right]^{-\frac{1}{N}}\right\}$$

NORMALIZED CHEBYSHEV LPF TRANSFER FUNCTION

- N be the order of the filter
- For even values of N parameter B_k are evaluated

$$H(s_n)\big|_{s_n = 0} = \frac{1}{(1 + \epsilon^2)^{\frac{1}{2}}}$$

- For odd values of N the parameter B_k are evaluated

$$H(s_n)\big|_{s_n = 0} = 1$$

- While evaluating B_k to take $B_0 = B_1 = B_2 = ... = B_k$

UNNORMALIZED ANALOG TRANSFER FUNCTION

4. Determine the unnormalized analog transfer function H (s) is obtained by replacing s_n by s/Ω_c in the normalized transfer function of the low pass filter function

- When N is even,

$$\therefore H(s) = \prod_{k=1}^{\frac{N}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}\Bigg|_{s_n = \frac{s}{\Omega_c}} = \prod_{k=1}^{\frac{N}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

- When N is odd,

$$\therefore H(s) = \frac{B_0}{s_n + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}\Bigg|_{s_n = \frac{s}{\Omega_c}} = \frac{B_0 \Omega_c}{s + c_0 \Omega_c} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

5. Determine the transfer function of digital filter H(z). Using the suitable transformation to transform H(s) to H(z). When the Impulse invariant transformation is employed, if T<1, then multiply H(z) by T to normalize the magnitude.

6. Realize the digital filter transfer function H(z) by a suitable structure

7. Verify the design by sketching the frequency response $H(e^{j\omega})$

$$H(e^{j\omega}) = H(z) / z = e^{j\omega}$$

ASSESSMENT

1. What is Chebyshev approximation?
2. How will you choose the order N for a Chebyshev Filter?
3. List the Properties of Chebyshev Filter.
4. Compare Analog filter and Digital filter?
5. Analog filter transfer function is converted to a digital filter transfer function by using either -------------------- (or) -------------------------
6. Define Sampling Time.
7. Attenuation Constant is given by ----------------
8. List the types of Chebyshev filters.

THANK YOU