



SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35



Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB212 – DIGITAL SIGNAL PROCESSING

II YEAR/ IV SEMESTER

UNIT 1 – DISCRETE FOURIER TRANSFORM

TOPIC – Introduction to DFT



EMPATHY



1

- Defects in signals is to identified

2

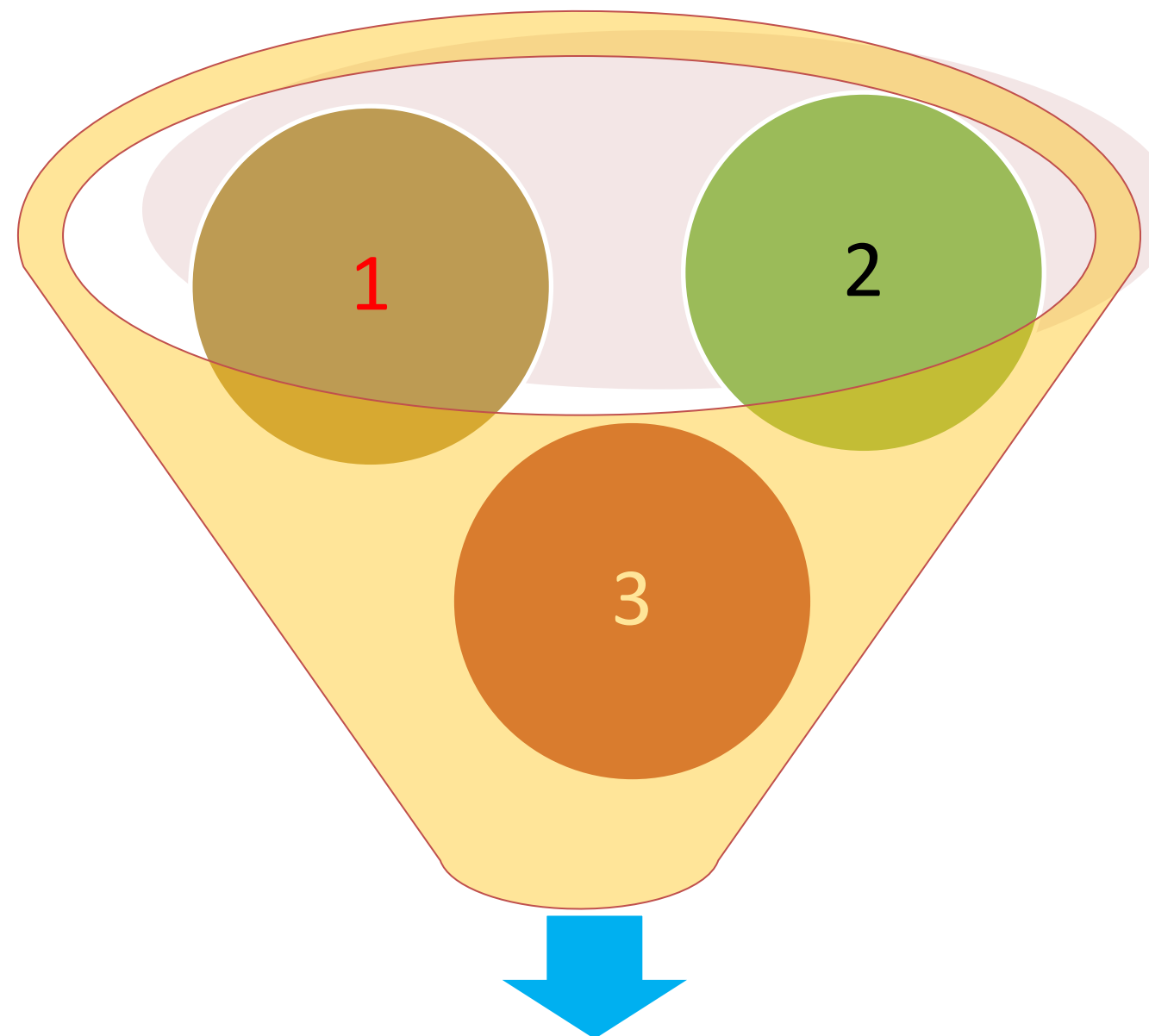
- Conversion from Time domain to frequency domain takes longer time

3

- Frequency domain Information must be extracted



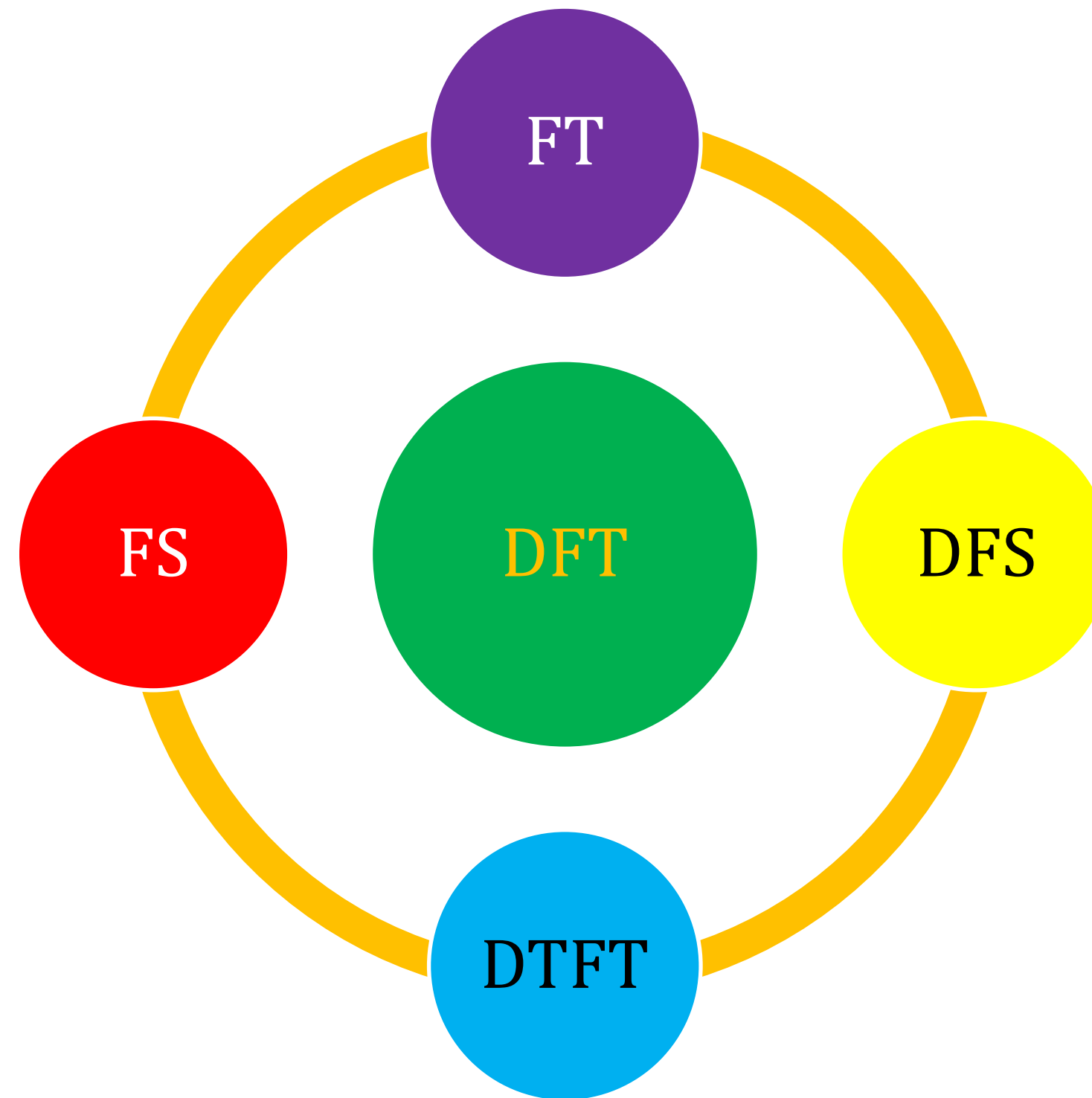
DEFINE



Time to frequency domain
conversion and process faster



IDEATE

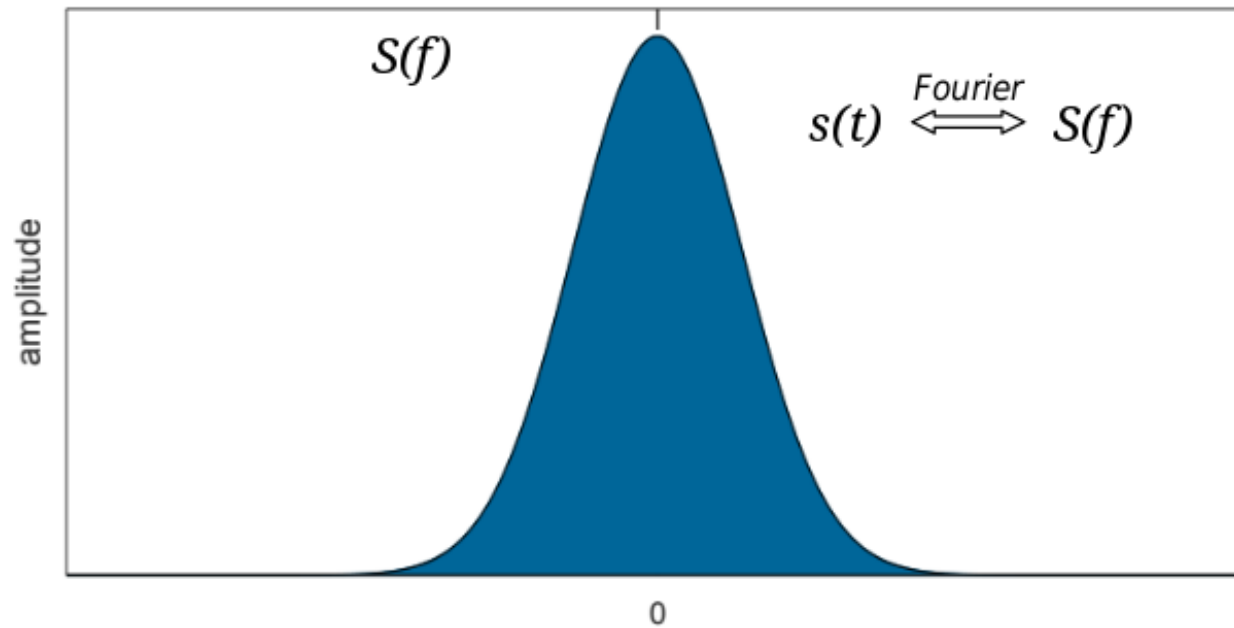




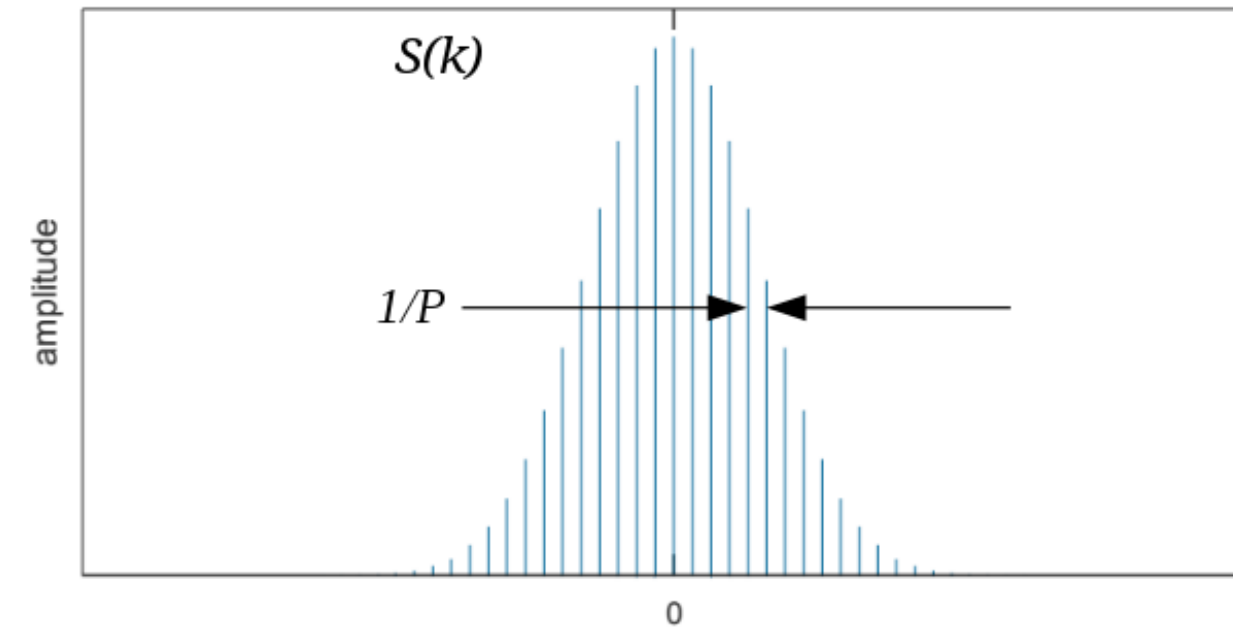
FOURIER COEFFICIENTS REPRESENTATION



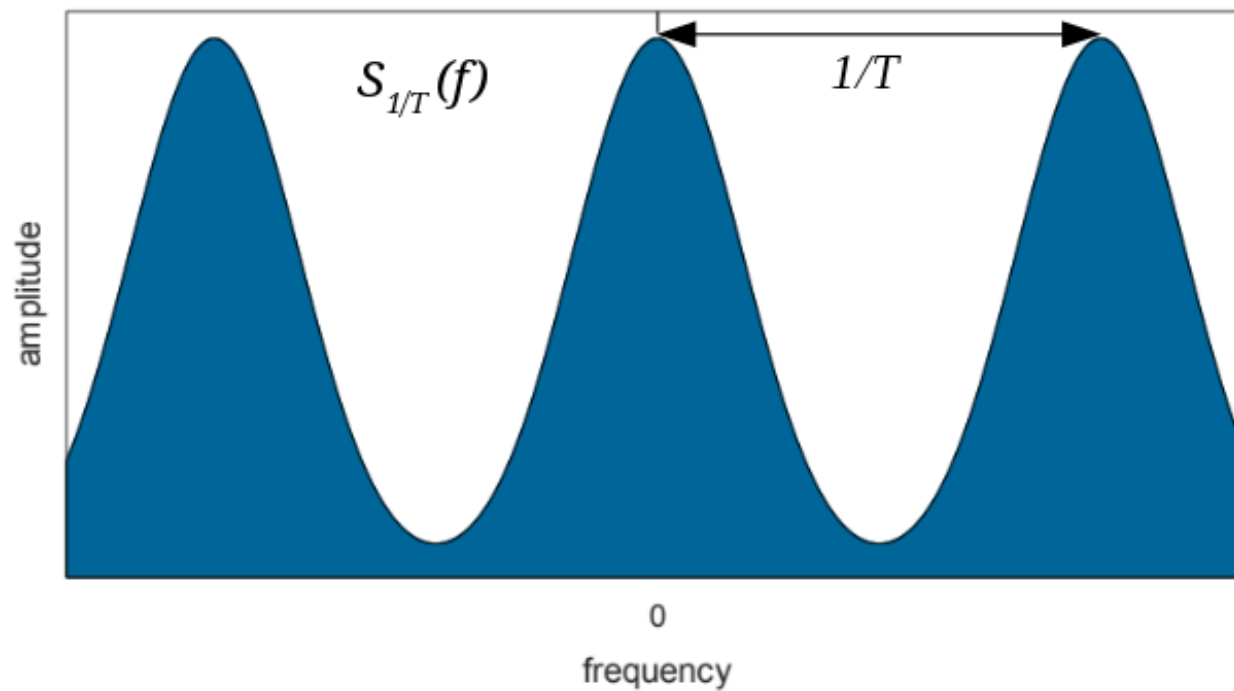
Fourier transform of a function $s(t)$



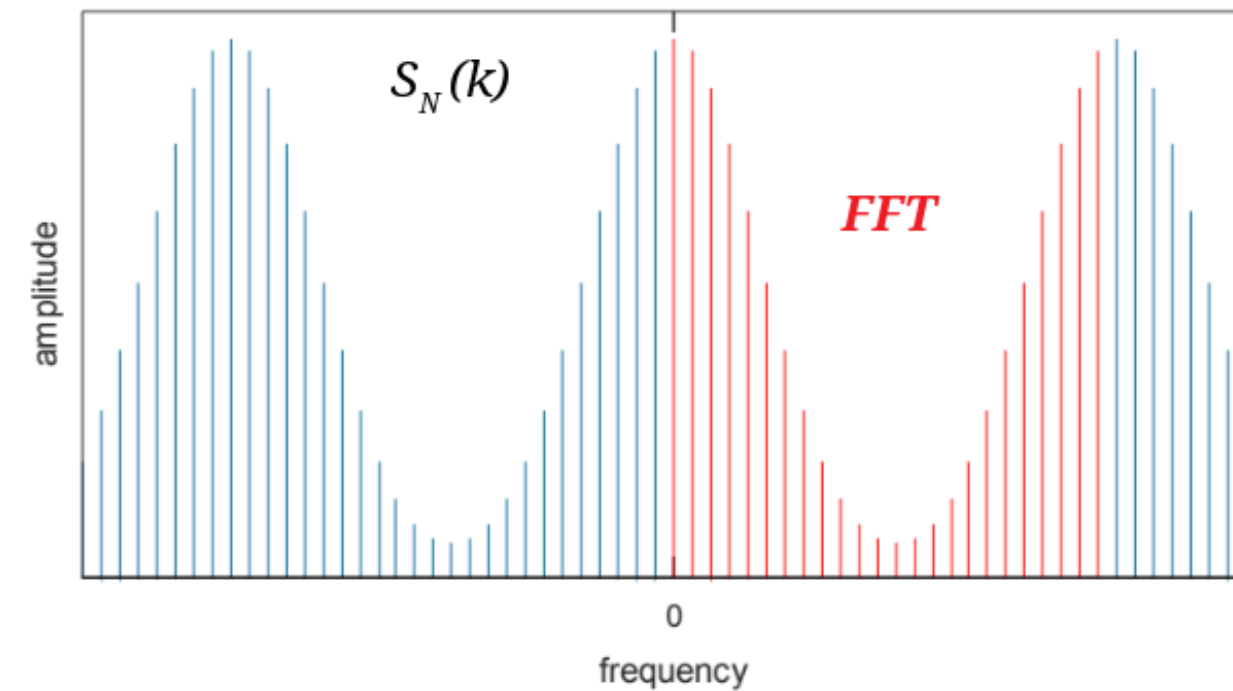
Transform of the periodic summation of $s(t)$
"Fourier series coefficients"



Transform of periodically sampled $s(t)$
"Discrete-time Fourier transform"



Transform of both periodic sampling and periodic summation
"Discrete Fourier transform"





DISCRETE FOURIER TRANSFORM



For a discrete time sequence we define two classes of Fourier Transforms:

- The *DTFT (Discrete Time FT)* for sequences having ***infinite*** duration,
- The *DFT (Discrete FT)* for sequences having ***finite*** duration.



DTFT AND INVERSE DTFT



DTFT

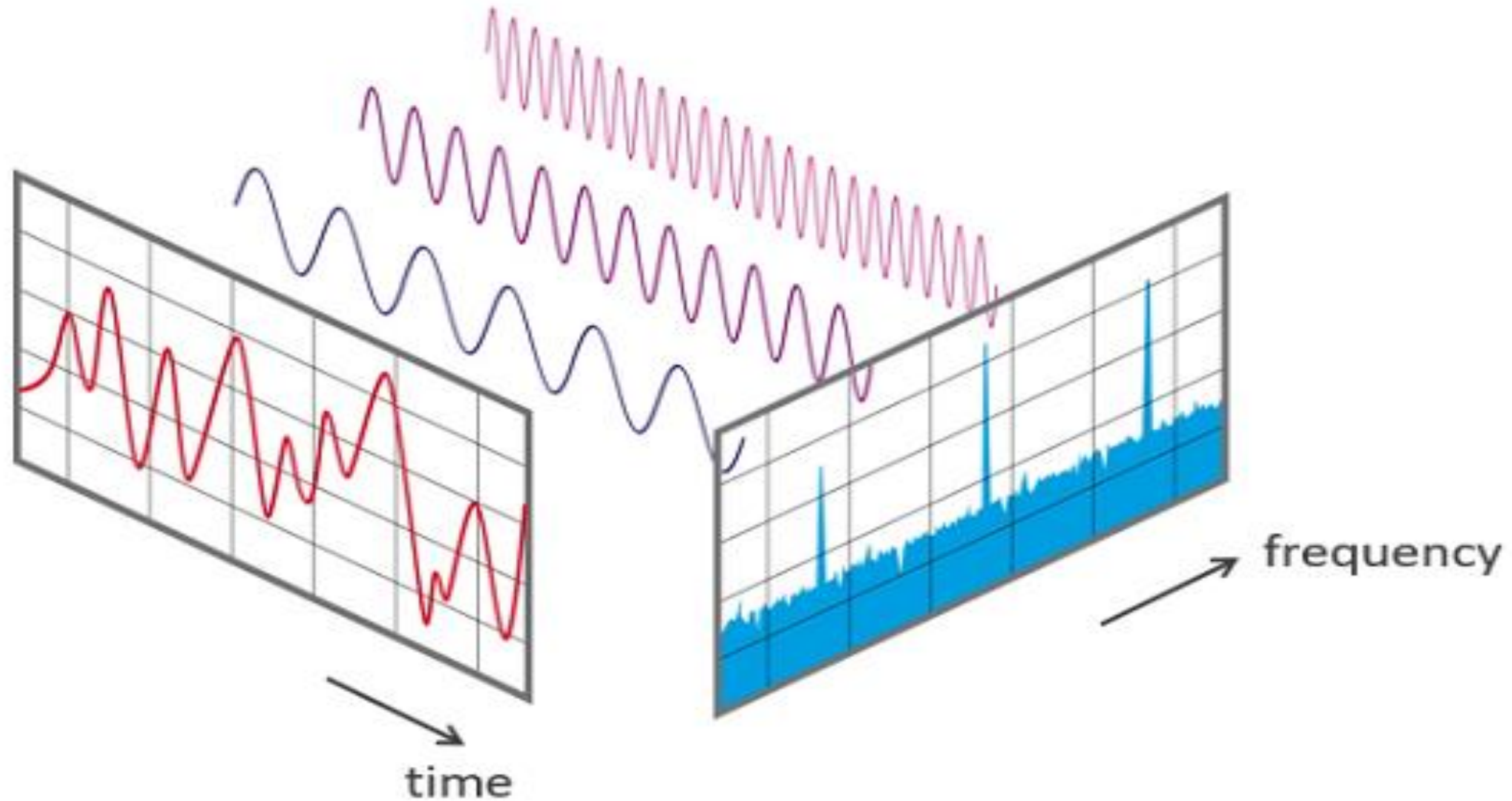
$$X(\omega) = DTFT\{x(n)\} = \sum_{n=-\infty}^{+\infty} x(n)e^{-j\omega n}$$

Inverse DFT

$$x(n) = IDTFT\{X(\omega)\} = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(\omega)e^{j\omega n} d\omega$$



DISCRETE FOURIER TRANSFORM

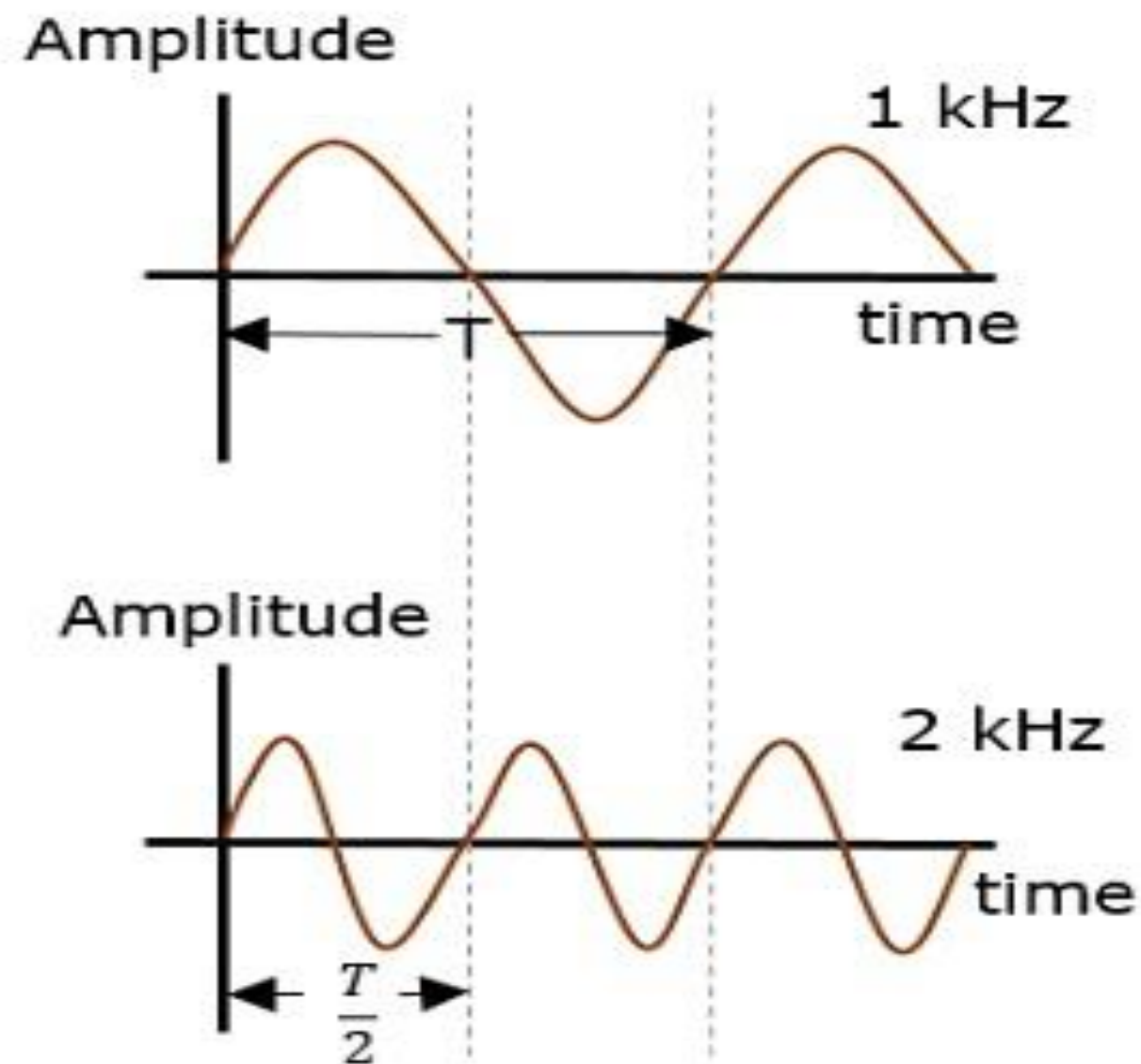




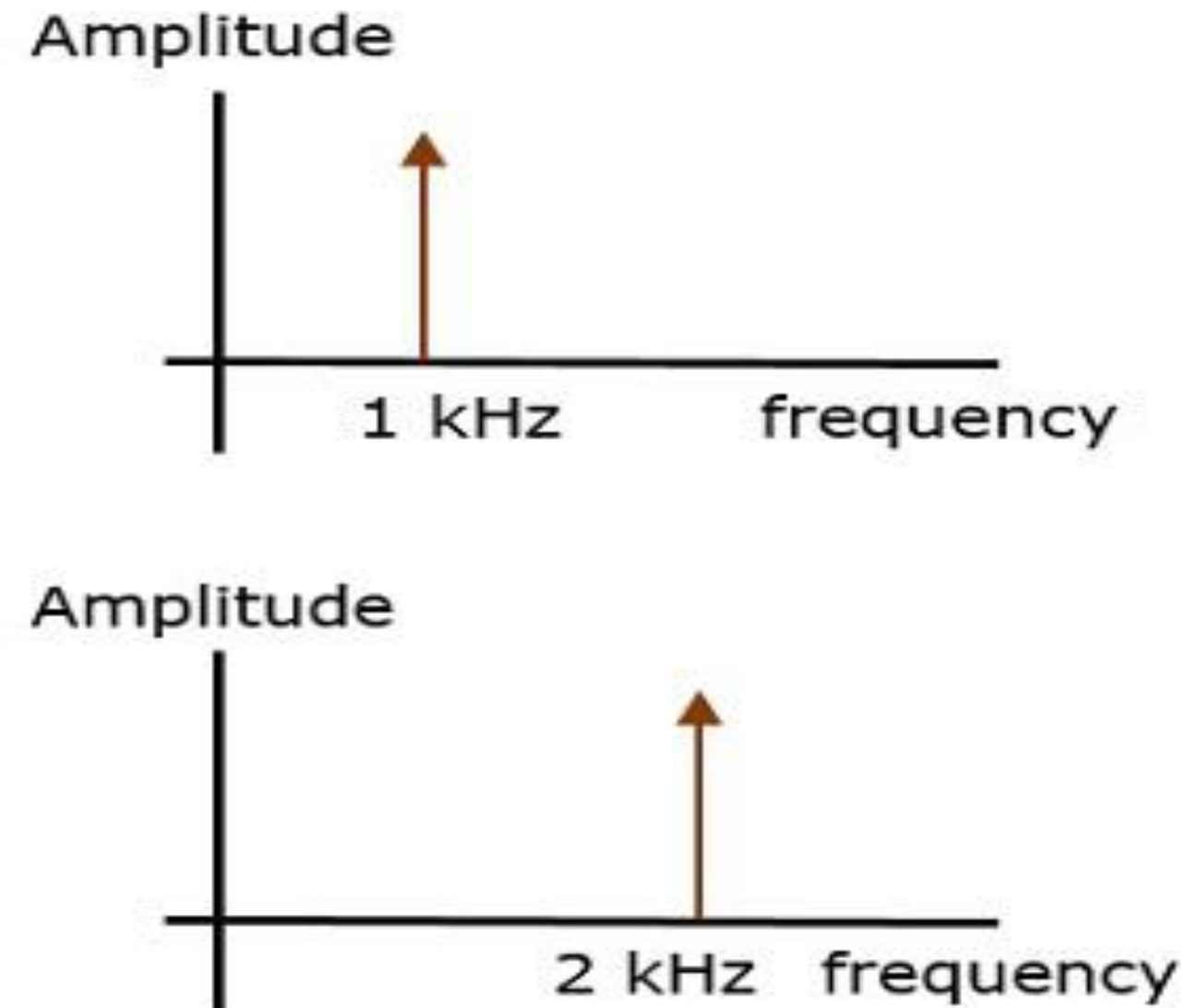
REPRESENTATION OF SIGNALS



Time Domain Representation



Frequency Domain Representation





DISCRETE FOURIER TRANSFORM

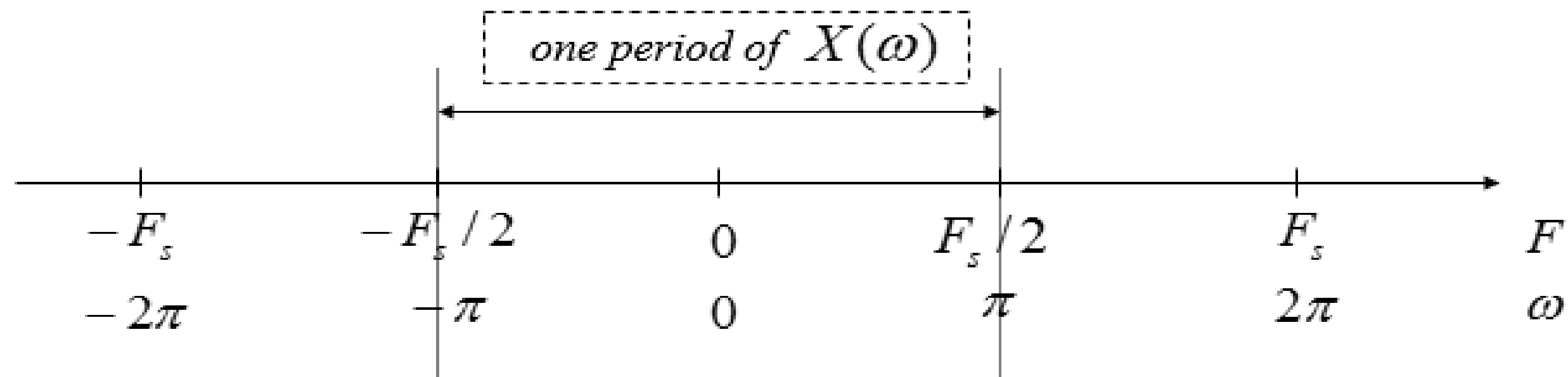


- The DTFT $X(\omega)$ is periodic with period 2π
- The frequency ω is the digital frequency and therefore it is limited to the interval

$$-\pi < \omega < +\pi$$

- The digital frequency ω is a normalized frequency relative to the sampling frequency, defined as

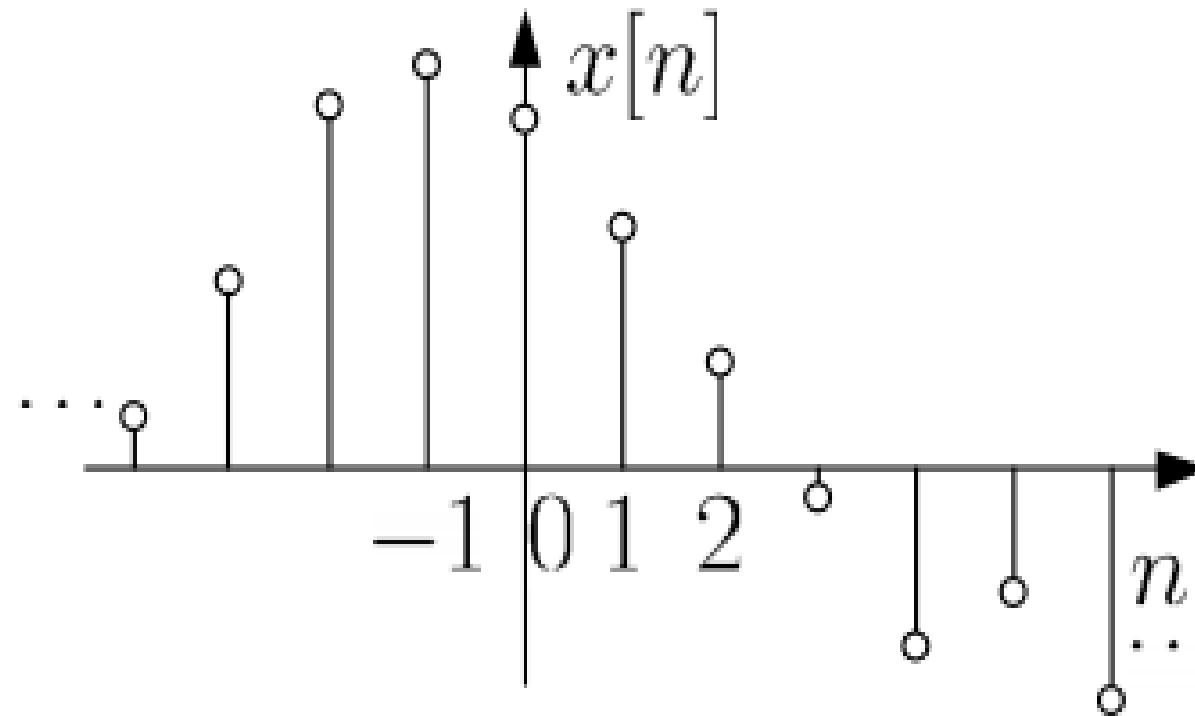
$$\omega = 2\pi \frac{F}{F_s}$$



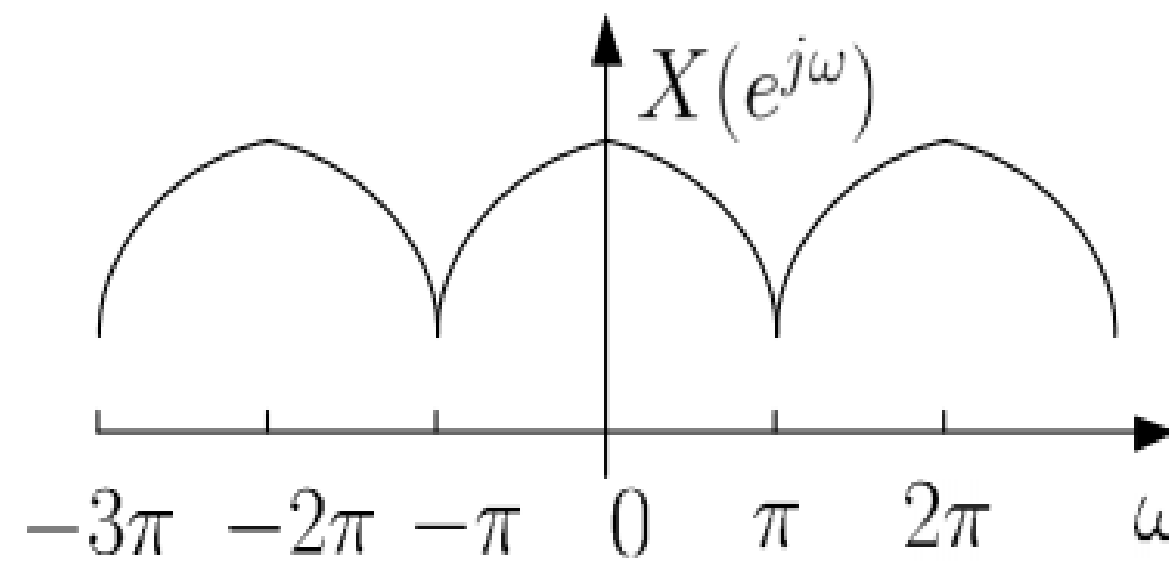


DFT REPRESENTATION

Time Domain



Freq Domain





DISCRETE FOURIER TRANSFORM



In Discrete Fourier Transform, Given a finite sequence

$$x = [x(0), x(1), \dots, x(N - 1)]$$

its Discrete Fourier Transform (DFT) is a finite sequence

$$X = DFT(x) = [X(0), X(1), \dots, X(N - 1)]$$

Where

$$X(k) = \sum_{n=0}^{N-1} x(n) w_N^{kn}, \quad w_N = e^{-j2\pi/N}$$





INVERSE DISCRETE FOURIER TRANSFORM



In Inverse Discrete Fourier Transform, Given a sequence

$$X = [X(0), X(1), \dots, X(N-1)]$$

its Inverse Discrete Fourier Transform (IDFT) is a finite sequence

$$x = IDFT(X) = [x(0), x(1), \dots, x(N-1)]$$

Where

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) w_N^{-kn}, \quad w_N = e^{-j2\pi/N}$$

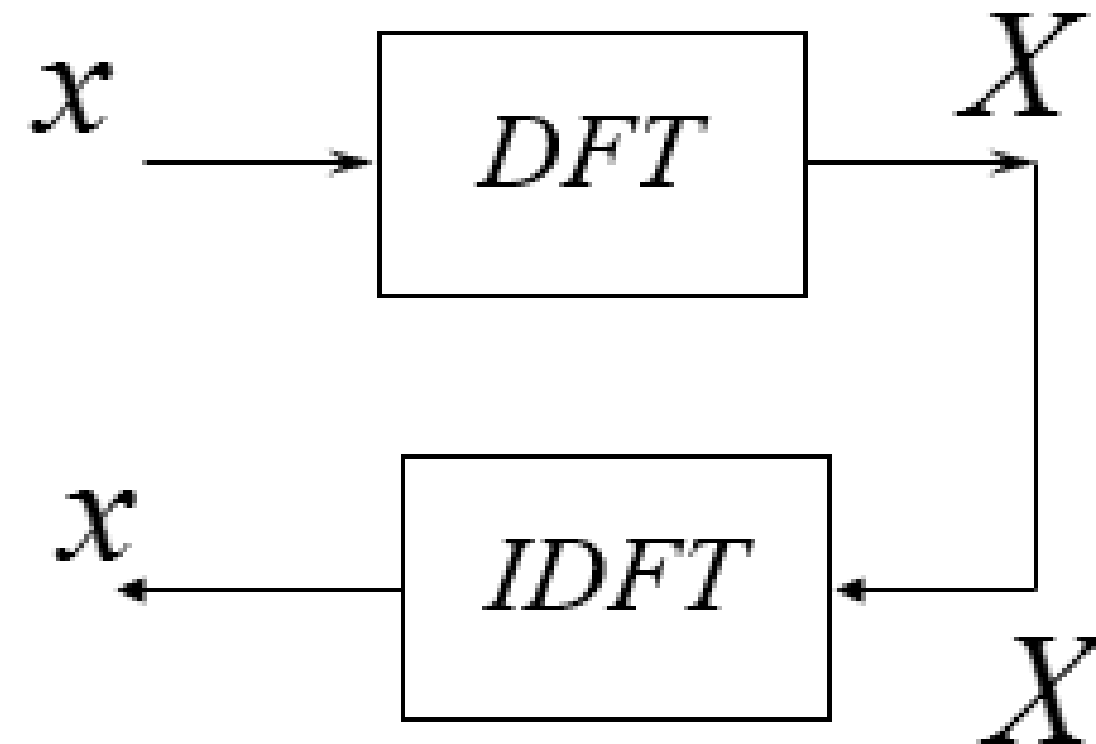




DISCRETE FOURIER TRANSFORM PAIR



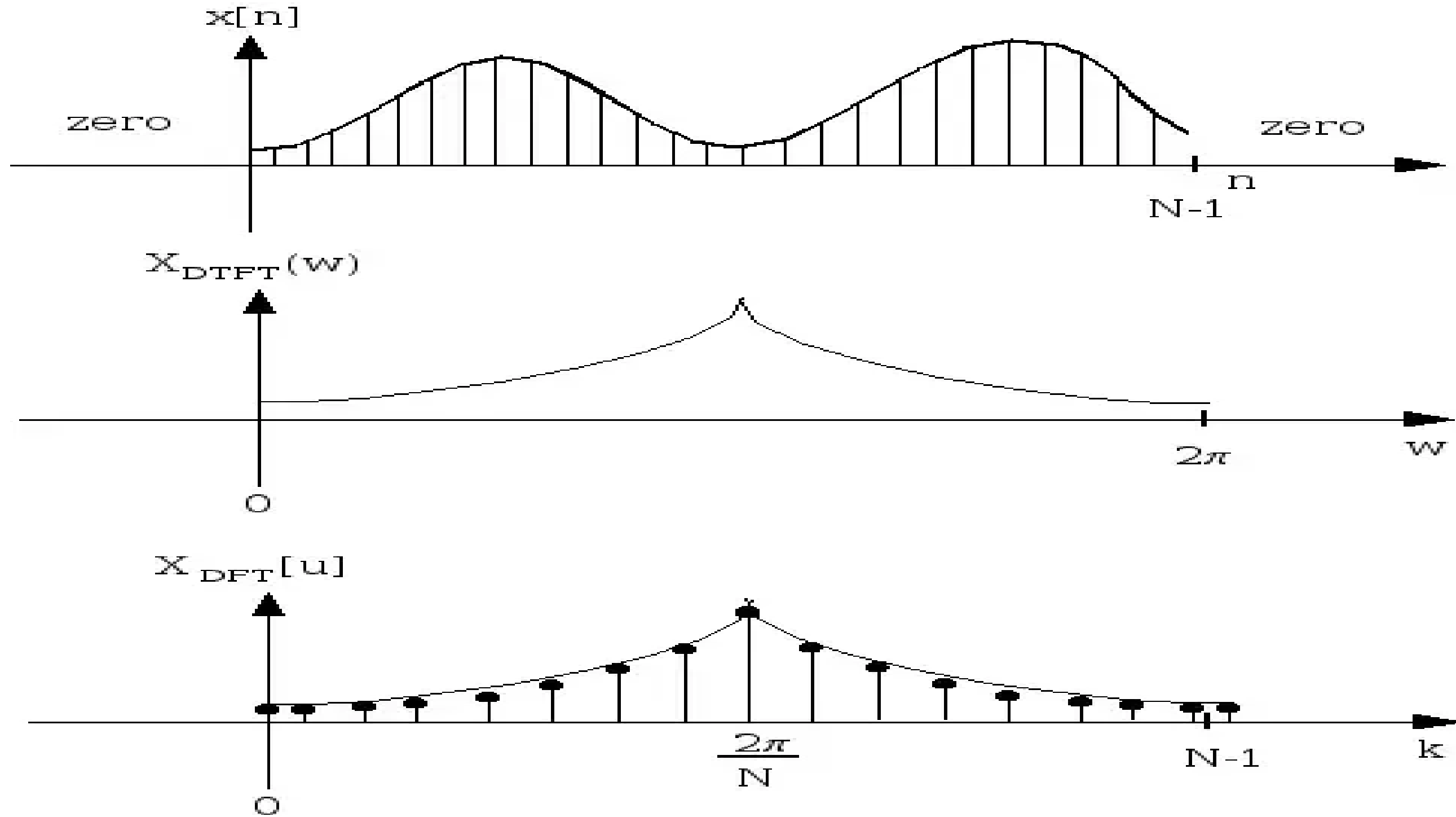
The DFT and the IDFT form a transform pair.



The DFT is a numerical algorithm, and it can be computed by a digital computer.



REPRESENTATION OF DTFT & DFT





PROPERTIES OF DFT



| Property | Time Domain | Frequency Domain |
|------------------------------------|---------------------------|-------------------------------------|
| 1. Linearity | $ax_1[n] + bx_2[n]$ | $aX_1[k] + bX_2[k]$ |
| 2. Time-shifting | $x[n - m]$ | $e^{-j2\pi km} X(k)$ |
| 3. Frequency-shifting (modulation) | $e^{-j2\pi k_0 n/N} x[n]$ | $X(k - k_0)$ |
| 4. Time reversal | $x[-n]$ | $X(-k)$ |
| 5. Conjugation | $x^*[n]$ | $X^*(-k)$ |
| 6. Time-convolution | $x_1[n] \otimes x_2[n]$ | $X_1[k]X_2[k]$ |
| 7. Frequency-convolution | $x_1[n]x_2[n]$ | $\frac{1}{N} X_1[k] \otimes X_2[k]$ |



APPLICATIONS OF DFT



1. Spectral Analysis
2. Image Processing
3. Signal Processing

Other Applications:

1. Sound Filtering
2. Data Compression
3. Partial Differential Equations
4. Multiplication of large integers



DIFFERENCE B/W DFT & IDFT

| DFT (Analysis transform) | IDFT (Synthesis transform) |
|--|---|
| DFT is finite duration discrete frequency sequence that is obtained by sampling one period of FT. | IDFT is inverse DFT which is used to calculate time domain representation (Discrete time sequence) form of x(k). |
| DFT equations are applicable to causal finite duration sequences. | IDFT is used basically to determine sample response of a filter for which we know only transfer function. |
| Mathematical Equation to calculate DFT is given by $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$ | Mathematical Equation to calculate IDFT is given by $x(n) = 1/N \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N}$ |
| Thus DFT is given by $X(k) = [WN][xn]$ | In DFT and IDFT difference is of factor 1/N & sign of exponent of twiddle factor. Thus $x(n) = 1/N [WN]^{-1}[XK]$ |



ASSESSMENT



1. Define DFT
2. What is meant by IDFT.
3. Give some applications of Fourier Transform.
4. Define DFT Pair.
5. *The DTFT (Discrete Time FT) for sequences having ----- duration*
6. *Determine DFT of $x(n) = \{1,0,1,0\}$*



THANK YOU