Unit - 4

Flow in Open Channels.

An 'open channel' may be defined as a passage in which liquid flows with its upper surface exposed to atmosphere. In open channels the flow is due to gravity, thus the flow conditions are greatly influenced by the slope of the channel.

Types of flow in channels:

1. Steady flow and unsteady flow
2. Uniform flow and non-uniform flow
3. Laminar flow and turbulent flow
4. Subcritical flow, critical flow and supercritical flow.

Steady flow and Unsteady flow:

Flow characteristics depth of flow, velocity and flow rate donot change with respect to time, the flow is called steady flow.

$$\frac{\partial y}{\partial t} = 0, \quad \frac{\partial v}{\partial t} = 0, \quad \frac{\partial Q}{\partial t} = 0$$

The flow in which depth, velocity and flow rate change with time.

$$\frac{\partial y}{\partial t} \neq 0, \quad \frac{\partial v}{\partial t} \neq 0, \quad \frac{\partial Q}{\partial t} \neq 0.$$

Uniform flow and Non-uniform flow (varied)

If the depth, slope, C/A remains constant over the length is called uniform flow.

$$\partial y/\partial l = 0, \quad \partial v/\partial l = 0.$$

If the depth, slope is not constant over the length is called non-uniform flow.

$$\partial y/\partial l \neq 0, \quad \partial v/\partial l \neq 0.$$

Rapidly varied flow:

In which depth of flow changes abruptly over a small length.

Ex: Hydraulic jump

Gradually varied flow:

In which the change in depth of flow takes place gradually in a long length of channel.

Laminar flow and Turbulent flow:

$$Re = \frac{\rho V D}{\mu}$$

$Re < 500$ - Laminar flow

$Re > 2000$ - Turbulent flow

$500 < Re < 2000$ - Transctional flow.

Subcritical, critical and super critical flow:

$$F_r = \frac{V}{\sqrt{gD}}$$

$F_r < 1$ - Subcritical flow

$F_r = 1$ - Critical flow

$F_r > 1$ - Supercritical flow.

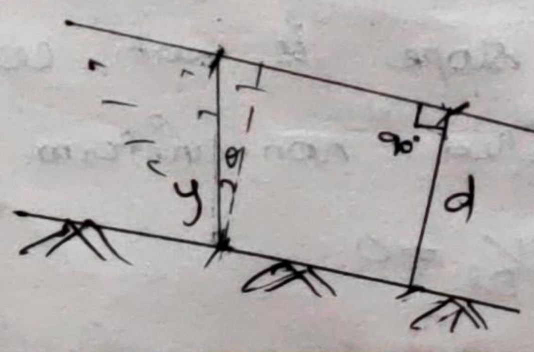
Properties:

1. Depth of flow (y):

It is the vertical distance of the lowest point of a channel from the free surface.

2. Depth of flow section (d):

Depth of flow normal to the bed of the channel.



Top width (T) :
It is the width of the channel section at the free surface [width at top exposed to the atmosphere]

Wetted Area (A) :
It is the C/s Area of the flow section of the channel.

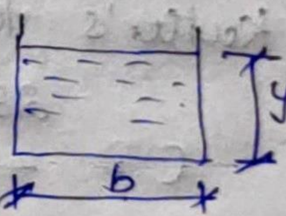
Wetted Perimeter (P) :
It is the length of the channel boundary in contact with the flowing water at any section.

Hydraulic Radius (R) :
Ratio of Area of flow to wetted perimeter.

$$R = \frac{A}{P}$$

(i) Rectangular Open channel :

$$R = \frac{A}{P} = \frac{b \times y}{b + 2y}$$



(ii) Pipe running full :

$$R = \frac{A}{P} = \frac{\pi/4 \times d^2}{\pi d} = d/4$$

(iii) Pipe not running full :

$$R = \frac{A}{P} = \frac{r^2/2 \, (2\theta - \sin 2\theta)}{2 r \theta}$$

Hydraulic Depth (D) :
Ratio of wetted area to top width.

$$D = \frac{A}{T}$$

Open Channel Equations :

1. Chezy's formula
2. Manning's formula.

Chezy's formula :

$$V = C\sqrt{RS}$$

$$Q = AC\sqrt{RS}$$

Chezy's constant C formula :

(i) Bazin's formula :

$$C = \frac{157.6}{181 + \frac{K}{\sqrt{R}}}$$

K = Bazin's constant

(ii) Kutter's formula :

$$C = \frac{23 + \frac{0.00155}{S} + \frac{1}{N}}{1 + \left(23 + \frac{0.00155}{S}\right)\frac{N}{\sqrt{R}}}$$

N = Kutter's constant

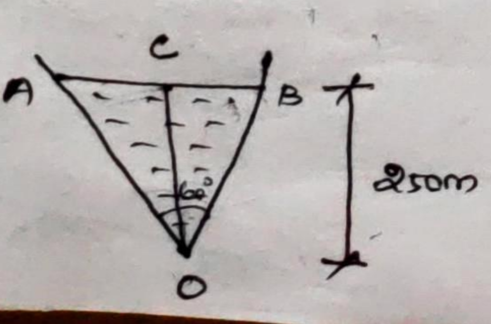
(iii) Manning's formula :

$$C = \frac{1}{N} R^{1/6}$$

N = Manning's constant

1. A triangular gutter, whose sides include an angle of 60° conveys water at a uniform depth of 250mm. If the discharge is 0.04 m^3/s. Determine the gradient of the trough. Take C = 52.

Soln:



$\frac{CO}{AO} = \cos 30^\circ$

$AO = \frac{0.25}{\cos 30^\circ}$

$= 0.288m$

$AO = BO = 0.288m$

$\frac{AC}{CO} = \tan 30^\circ$

$AC = 0.25 \times \tan 30^\circ$

$= 0.144m$

Wetted perimeter $P = AO + BO$

$= 0.288 + 0.288$

$= 0.576m$

Hydraulic Radius, $R = \frac{A}{P} = \frac{0.036}{0.576}$

$= 0.0625\,m$

Using Chezy's formula,

$Q = AC\sqrt{RS}$

$0.04 = 0.036 \times 52 \times \sqrt{0.0625 \times S}$

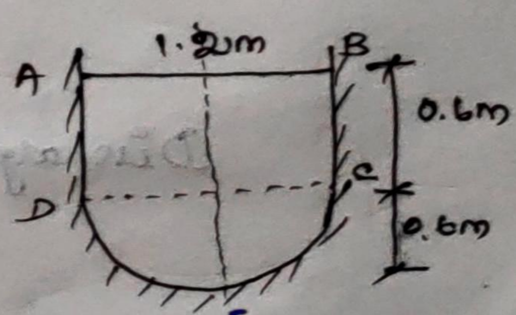
$S = \frac{0.02137^2}{0.0625}$

$= 1/137$

2. Find the discharge of water through the channel as shown. Take the value $C = 60$, $S = 1$ in 950.

Soln :

Area of flow $= (1.2 \times 0.6) + \frac{\pi \times 0.6^2}{2}$



$= 1.285\,m^2$

Wetted Perimeter, $P = AD + DEC + CB$

$= (0.6 \times \pi) + 0.6 + 0.6$

$= 3.085\,m$

Hydraulic mean radius,

$$R = \frac{A}{P} = \frac{1.285}{3.085}$$

$$= 0.416 m$$

Discharge, $Q = AC\sqrt{RS}$

$$= 1.285 \times 60 \times \sqrt{0.416 \times \frac{1}{950}}$$

$$= 1.613 \ m^3/s.$$

3. Find the discharge through a rectangular channel of width 2m, having a bed slope of 4 in 8000. The depth of flow is 1.5m and take the value of N in Manning's formula as 0.012.

Soln:

$b = 2m.$

$d = 1.5m$

$A = b \times d = 2 \times 1.5 = 3 \ m^2$

$P = b + 2d = 2 + (1.5 \times 2) = 5 \ m$

Hydraulic mean depth (R)

$$(m) = \frac{A}{P} = \frac{3}{5} = 0.6$$

Bed slope, $(s), i = 4$ in 8000

$$= 4/8000 = 1/2000$$

Using Manning's formula,

$$C = 1/N \ m^{1/6}$$

$$= 1/0.012 \times 0.6^{1/6}$$

$$= 76.54$$

Discharge $Q = AC\sqrt{mi} = AC\sqrt{RS}$

$$= 3 \times 76.54 \times \sqrt{0.6 \times 1/2000}$$

$$= 3.977 \ m^3/s.$$

Most Economical Section:

A Section of a Channel is said to be most economical where the cost of construction of the channel depends upon the excavation and the lining. To keep the cost minimum, the wetted perimeter for a given discharge should be minimum. It is also called as the best section.

$$Q = AC\sqrt{RS} = AC\sqrt{\frac{A}{P} \times S}$$

$$= K\frac{1}{\sqrt{P}} \quad K \text{ is a constant.}$$

Discharge Q is maximum and the wetted perimeter is minimum.

Most Economical Rectangular Channel:

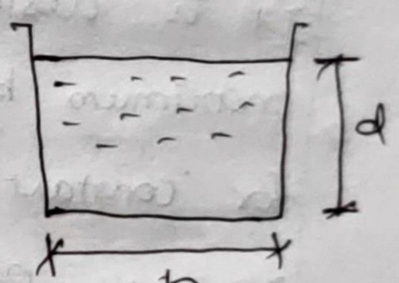
width $= b$

depth $= d$

$A = b \times d \longrightarrow (1)$

$P = d + b + d = b + 2d \rightarrow (2)$

$b = \frac{A}{d}$



$(2) \Rightarrow P = b + 2d = \frac{A}{d} + 2d$

for most economical section, P should be minimum,

$$\frac{dP}{d(d)} = 0$$

$$\frac{d\left(\frac{A}{d} + 2d\right)}{d(d)} = 0$$

$$-\frac{A}{d^2} + 2 = 0$$

$$A = 2d^2$$

$$A = d \times b = 2d^2$$

$$b = 2d$$

$m = \frac{A}{P} = \frac{b \times d}{b + 2d}$ $\quad b = 2d$

$= \frac{2d \times d}{2d + 2d}$

$= \frac{2d^2}{4d}$

$= d/2$

So, For most economical section,

$b = 2d$ (or) $m = d/2$.

4 A rectangular channel 4m wide has depth of water 1.5m. The slope of the bed of the channel is 1 in 1000 and value of Chezy's constant $c = 55$. It is desired to increase the discharge to a maximum by changing the dimensions of the section for constant area of c/s, slope of the bed and roughness of the channel. Find the new dimensions of the channel and increase in discharge.

Soln :

$b = 4m, \quad d = 1.5m$

$A = 4 \times 1.5 = 6 m^2$

$i = 1/1000 = S$

$C = 55$

$P = b + d + d$

$= 4 + 1.5 + 1.5$

$= 7m$

Hydraulic mean depth,

$R = \frac{A}{P} = \frac{6}{7} = 0.857$

Discharge, $Q = AC\sqrt{RS}$

$= 6 \times 55 \sqrt{0.857 \times 1/1000} = 9.66 \, m^3/s$

For maximum discharge,

b' = new breath

d' = new depth

$A = b' \times d'$

$= 6\ m^2$

$b' \times d' = 6\ m^2 \rightarrow (1)$

$b' = 2d'$

$2d' \times d' = 6$

$2d'^2 = 6$

$d'^2 = 3$

$d' = \sqrt{3}$

$d' = 1.732\ m$

$b' = 2 \times 1.732$

$= 3.464\ m$

$P' = b' + 2d'$

$= 1.732 + 1.732 + 3.464$

$= 6.928\ m$

Hydraulic mean depth,

$R = \frac{A}{P'} = \frac{6}{6.928}$

$= 0.866\ m$

$m' = d'/2 = \frac{1.732}{2}$

$= 0.866\ m$

Max. discharge, $Q' = AC\sqrt{m' i}$

$= 6 \times 55 \times \sqrt{0.866 \times 1/1000}$

$= 9.71\ m^3/s$

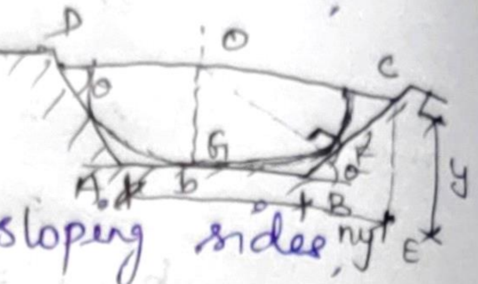
Increase in discharge, $= Q' - Q$

$= 9.71 - 9.66$

$= 0.05\ m^3/s.$

Most Economical Trapezoidal Channel:

Conditions for most economical section.



(i) $\frac{b+2ny}{2} = y\sqrt{n^2+1}$

Half of top width = One of the sloping sides,

(ii) $R = y/2$

Hydraulic radius equals to half of the depth of ~~depth~~ flow.

(iii) A semicircle drawn from O with radius equal to depth of flow will touch the three sides of the trapezoidal channel.

(iv) Best side slope is 60° to the horizontal $\Rightarrow n = \frac{1}{\sqrt{3}}$

5. A power canal of trapezoidal section has to be excavated through hard clay at least cost. Determine the dimensions of the channel, given discharge equal to 14 m³/sec, best slope 1:2500 and Mannings N = 0.02.

Soln:

$$Q = 14\ m^3/s$$

$$S = \frac{1}{2500}$$

$$N = 0.02$$

For least cost $\Rightarrow$ most economical section.

$$n = 1/\sqrt{3}$$

$$\frac{b+2ny}{2} = y\sqrt{n^2+1}$$

$$\frac{b+2\times 1/\sqrt{3}\times y}{2} = y\sqrt{(1/\sqrt{3})^2+1}$$

$$= \frac{2}{\sqrt{3}}y$$

$b = \frac{2}{\sqrt{3}} y \times 2 - \frac{2}{\sqrt{3}} \times y = \frac{2}{\sqrt{3}} y$

Area of flow, $A = (b+ny)y$

$= \left(\frac{2}{\sqrt{3}} y + \frac{1}{\sqrt{3}} y\right) y$

$= \frac{3}{\sqrt{3}} y^2 = \sqrt{3} y^2$

$Q = AC\sqrt{RS}$

$C = \frac{1}{N} R^{1/6}$

$Q = \sqrt{3} y^2 \times \frac{1}{N} R^{1/6} \sqrt{RS}$

$= \sqrt{3} y^2 \times \frac{1}{N} R^{2/3} S^{1/2}$

$14 = 1.732 y^2 \times \frac{1}{0.02} \times \left(\frac{y}{2}\right)^{2/3} \times \sqrt{\frac{1}{2500}}$

$14 = 1.732 y^2 \times y^{2/3} \times \frac{1}{2^{2/3}}$

$14 = 1.09\, y^{8/3}$

$y = \left(\frac{14}{1.09}\right)^{3/8}$

$= 2.6\,m$

$b = \frac{2}{\sqrt{3}} y$

$= \frac{2}{\sqrt{3}} \times 2.6$

$= 3\,m.$

Non-Uniform flow Through open Channel:

Friction force and gravity force are not steady non-uniform in Non-uniform flow.

Causes:

(i) The change in width, depth, bed slope of a channel.

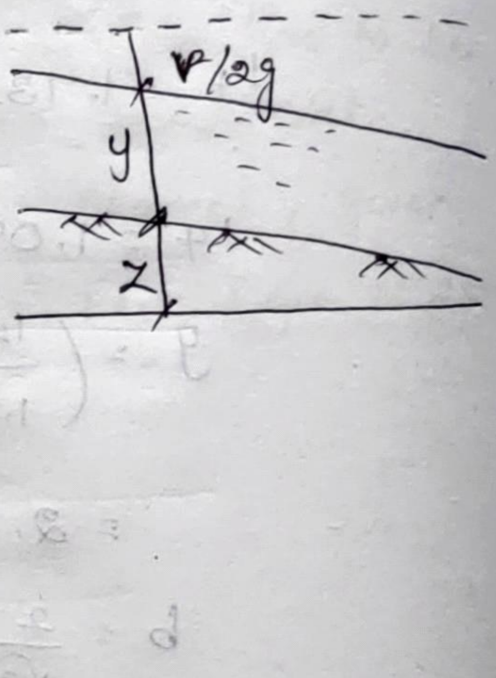
(ii) An obstruction, constructed across a channel of uniform width.

Ex: Waves and Surges.

Specific Energy and Specific Energy curve:

The total Energy per unit weight of liquid is called specific Energy.

$$E = z + y + V^2/2g$$



$z = 0$,

$$E = y + V^2/2g$$

Velocity, $V = \frac{Q}{A} = \frac{Q}{b \times y} = \frac{q}{y}$

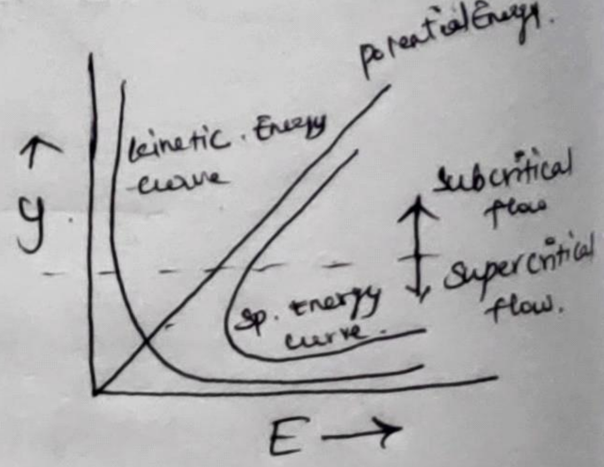
Then, $E = y + \frac{(q/y)^2}{2g}$

$$= y + \frac{q^2}{2gy^2}$$

(i) The curve of potential energy is straight line.

(ii) The curve of kinetic energy is parabola.

(iii) At critical depth specific energy is minimum



(iv) For every value of specific energy other than minimum there are two possible depths of flow, one greater and other lesser than critical depth 'y_c'. These two depths are referred to as alternate or conjugate depth.

(i) Critical Depth y_c:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

(ii) Critical velocity V_c:

$$V_c = \sqrt{g y_c}$$

(iii) Minimum Specific Energy E_{min}:

$$E_{min} = \frac{3 y_c}{2}$$

$$y_c = \frac{2}{3} E_{min}$$

(iv) Critical flow:

In critical flow specific energy is minimum.

$$\frac{V_c}{\sqrt{g y_c}} = 1 \Rightarrow F_r = 1$$

(v) Subcritical flow:

In subcritical flow, the depth of flow is greater than critical depth. $F_r < 1$.

(vi) Super critical flow:

In which the depth of flow is less than the critical depth. $F_r > 1$.

For maximum discharge specific energy should be minimum and depth of flow should be critical.

6. A 8m wide channel conveys 15 m^3/s of water at a depth of 1.2m. Calculate,

(i) Specific Energy of flowing water.

(ii) Critical depth, Critical velocity and minimum Specific energy.

(iii) Froude No and state whether flow is subcritical or supercritical.

Soln :

$Q = 15\ m^3$

$b = 8m$

$y = 1.2\ m$

$$V = \frac{Q}{b \times y} = \frac{15}{8 \times 1.2} = 1.5625\ m/s.$$

Discharge per unit width,

$$q = \frac{Q}{b} = \frac{15}{8} = 1.875\ m^3/s \text{ per m.}$$

(i) Specific Energy, $E = y + \frac{V^2}{2g}$

$$= 1.2 + \frac{1.5625^2}{2 \times 9.81}$$

$$= 1.324\ m$$

(ii) Critical depth (y_c) :

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1.875^2}{9.81}\right)^{1/3}$$

$$= 0.71\ m/s.$$

Critical velocity (V_c) :

$$V_c = \sqrt{g y_c} = \sqrt{9.81 \times 0.71}$$

$$= 2.64\ m/s$$

Minimum Specific Energy (E_{min}) :

$$E_{min} = \frac{3}{2} y_c = \frac{3}{2} \times 0.71 = 1.065\ m.$$

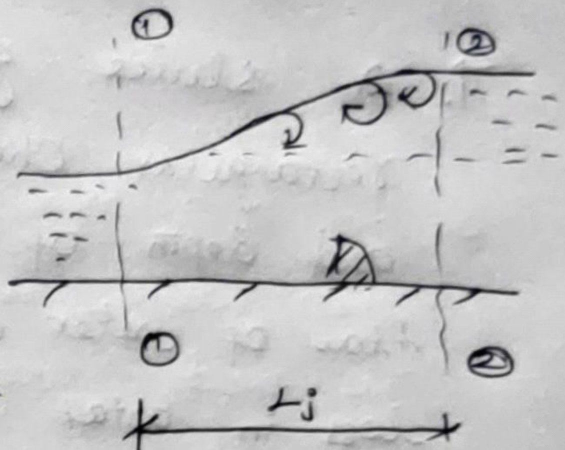
(iii) Froude Number:

$$Fr = \frac{V}{\sqrt{gy}} = \frac{1.565}{\sqrt{9.81 \times 1.2}}$$

$$= 0.455$$

$Fr < 1$, $\Rightarrow$ Subcritical flow (or) tranquil flow.

Hydraulic Jump:



In an open Channel, when rapidly flowing stream abruptly changes to slowly flowing stream, a distinct rise or jump in the elevation of fluid surface takes place is called hydraulic jump.

In this, kinetic energy converts into potential energy.

Analysis of hydraulic jump:

(i) Loss of head due to friction at the walls and Channel bed is negligible.

(ii) The flow is uniform and the pressure distribution is hydrostatic before and after the jump.

(iii) The Channel is horizontal or it has a very small slope. The weight component in the direction of flow is negligible.

(iv) The momentum correction factor (β) is unity.

Height of hydraulic Jump:

$$H_j = y_2 - y_1$$

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2V_1^2 y_1}{g}}$$

$$y_2 = -\frac{y_1}{2}\left(\sqrt{1 + 8Fr^2} - 1\right)$$

Length of hydraulic Jump:

$$L_j = 5 \text{ to } 7\, H_j.$$

Loss of energy due to hydraulic Jump:

$$E_L = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$$

7. A sluice gate discharges water into horizontal rectangular channel with a velocity of 10 m/s and depth of flow of 1m. Determine the depth of flow of water after the jump and consequent loss in total head.

Soln:

Discharge per unit width, $q = V_1 \times y_1$

$$= 10 \times 1$$

$$= 10\ m^3/s \text{ per } m$$

The depth of flow after the jump,

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{g y_1}}$$

$$= -\frac{1}{2} + \sqrt{\frac{1^2}{4} + \frac{2 \times 10^2}{9.81 \times 1}}$$

$$= 4.043\ m$$

Loss in total head E_L:

$$E_L = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$$

$$= \frac{(4.043 - 1)^3}{4 \times 1 \times 4.043}$$

$$= 1.742\ m.$$

Gradually Varied Flow:

A flow in which the depth changes gradually over a long distance.

Assumptions:

(i) The channel is prismatic

(ii) The bed slope is small.

(iii) The flow is steady and discharge is constant

(iv) The pressure distribution over the channel section is hydrostatic

(v) The energy correction factor is (α) unity

(vi) The roughness co-eff is constant for the length of the channel.

(vii) The chezy and Manning correlations are equally applicable to gradually varied flow.

$$\frac{dy}{dx} = \frac{S_0 - S_e}{(1 - Fr^2)}$$

S_b = slope of bed

S_e = Slope of energy line

(i) $\frac{dy}{dx} = 0$ ⇒ Free water surface is parallel to the channel bed.

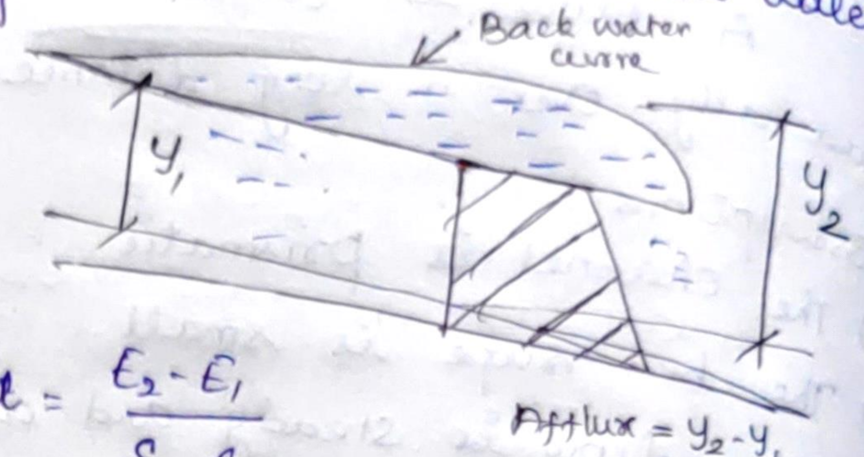
(ii) $\frac{dy}{dx} > 0$ ⇒ depth increase in flow direction. The profile of water is "back water curve".

(iii) $\frac{dy}{dx} < 0$ ⇒ Depth decreases in the direction of flow. The profile of water is "Drop down curve".

Back water curve and Afflux:

- y_1 is the depth of water at the point, where the water rising up and y_2 is the maximum height of rising water from the bed, then this increase in depth $(y_2 - y_1)$ is known as afflux.

The curved surface of the liquid with its concavity upwards, is known as 'back water curve'.



Length of back water curve } $l = \dfrac{E_2 - E_1}{S_b - S_e}$

8. In a rectangular channel 12m wide and 3.6m deep water is flowing with a velocity of 1.2m/s. The bed slope of the channel is 1 in 4000, the flow of water through the channel is regulated in ~~which~~ such a way that energy line is having a slope of 0.00004. Find the rate of change of depth of water in the channel.

Soln:

$b = 12m$

$y = 3.6m$

$V = 1.2\ m/s$

$S_b = 1/4000 = 0.00025$

$S_e = 0.00004$

$$\frac{dy}{dx} = \frac{S_b - S_e}{\left(1 - \frac{V^2}{gy}\right)}$$

$$= \frac{0.00025 - 0.00004}{\left(1 - \frac{1.2^2}{9.81 \times 3.6}\right)}$$

$$= 0.0002189.$$