Unit - 3

Flow Through Pipes

A pipe is a closed conduit which is used for carrying fluids under pressure. If the pipe is running partially full, it behaves like an open channel.

Loss of Energy :

1. Major energy losses - Due to friction
2. Minor energy losses - Due to the following
   (i) Sudden enlargement of pipe
   (ii) Sudden Contraction of pipe
   (iii) Bend of pipe.
   (iv) An obstruction in pipe.
   (v) Pipe fittings.

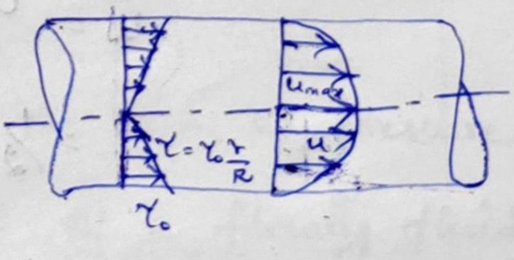
Flow of Viscous Fluid in circular pipes - Hagen Poiseuille Law.

Assumptions :

1. The fluid follows Newton's law of Viscosity
2. There is no slip of fluid particles at the boundary

$$u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2), \quad u = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

$$u_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2$$



Discharge, $Q = \frac{\pi}{2} u_{max} R^2$

Average velocity of flow, $\bar{u} = \frac{u_{max}}{2}$

$$\boxed{(P_1 - P_2) = \frac{32\mu\bar{u}L}{D^2}}$$

Pressure Gradient $\frac{\partial p}{\partial x}$, $\tau_0 = -\frac{\partial p}{\partial x} \cdot R/2$

Reynold's No, $Re = \frac{\rho\bar{u}D}{\mu}$, Friction co.eff $f = 16/Re$

1. A fluid of viscosity 8 poise and specific gravity 1.2 is flowing through a circular pipe of diameter 100mm. The maximum shear stress at the pipe wall is 210 N/m². Find

(i) Pressure gradient,

(ii) Average Velocity

(iii) Reynolds number of flow.

Soln :

$\mu = 8 \text{ poise} = 0.8 \ Ns/m^2$

$S = 1.2$

$\rho = 1.2 \times 1000 = 1200 \ kg/m^3$

$D = 100mm = 0.1m$

$\tau_0 = 210 \ N/m^2$

(i) Pressure gradient $\frac{\partial p}{\partial x}$:

$\tau_0 = -\frac{\partial p}{\partial x} \cdot \frac{R}{2}$

$210 = -\frac{\partial p}{\partial x} \cdot \frac{(0.1/2)}{2}$

$\frac{\partial p}{\partial x} = -\frac{210 \times 4}{0.1}$

$= -8400 \ N/m^2$

(ii) Average velocity :

$\bar{u} = \frac{U_{max}}{2}$

$= \frac{1}{2}\left[-\frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} \cdot R^2\right]$

$= \frac{1}{2}\left[-\frac{1}{4 \times 0.8} \times (-8400) \times (0.1/2)^2\right]$

$= 3.28 \ m/s.$

(iii) Reynold's Number :

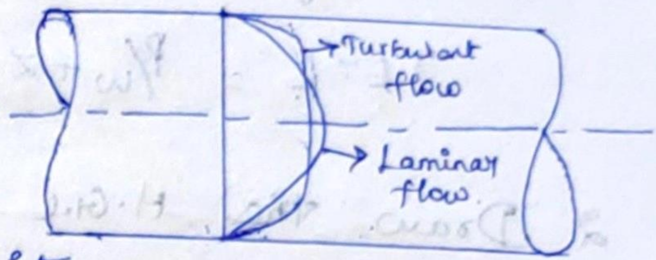
$Re = \frac{\rho v D}{\mu} = \frac{1200 \times 3.28 \times 0.1}{0.8} = 492$

Turbulent Flow in pipes :

Laminar flow Re less than 2000

Turbulent flow Re greater than 4000

(i) The velocity distribution in turbulent flow is more uniform than in laminar flow.



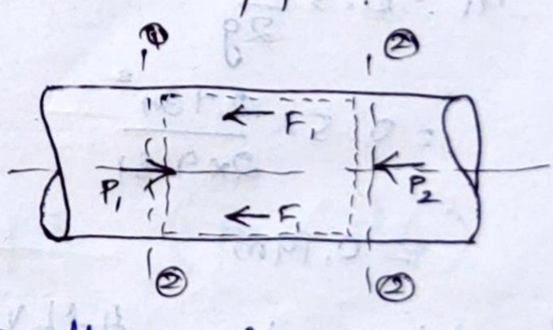
(ii) In turbulent flow, the velocity gradients near the boundary shall be quite large resulting in more shear.

(iii) The velocity distribution is paraboloid in laminar flow, and logarithmic in turbulent flow.

(iv) formation of eddies, mixing and curving of path lines in a turbulent flow results in much greater frictional losses for the same rate of discharge, viscosity and pipe size.

Loss of head due to friction in pipe :

$$h_f = \frac{fLV^2}{2gd}$$



f = friction factor (or) co-eff of friction

Hydraulic and Energy Gradient line :

Hydraulic Gradient Line :

The line which gives the sum of pressure head (P/w) and datum head (z) of a flowing fluid in a pipe with respect to some reference line.

$$H.G.L = P/w + z$$

Total Energy line :

The line which gives the sum of pressure head (P/w), datum head (z) and kinetic head ($V^2/2g$)

with respect to some reference line.

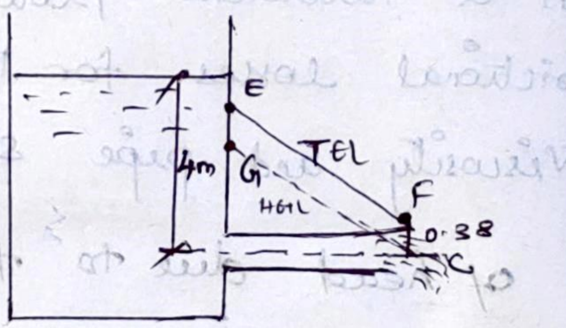
It is obtained by adding kinetic head with hydraulic gradient line.

$$T.E.L = P/w + Z + V^2/2g$$

2. Draw the H.G.L and T.E.L of a pipe of ∅ 20cm and length 50m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and height of water in the tank is 4m above the centre of the pipe. f = 0.009

Sol:

$V = 2.734\ m/s$



$L = 50m,\ H = 4m$

$f = 0.09$

$$h_i = 0.5\frac{V^2}{2g}$$

$$= 0.5 \times \frac{2.734^2}{2 \times 9.81}$$

$$= 0.19m$$

head loss, $h_f = \frac{4fLV^2}{2gd}$

$$= \frac{4 \times 0.009 \times 50 \times 2.734^2}{0.2 \times 2 \times 9.81}$$

$$= 3.428m$$

Total Energy line:

At A, $\frac{P}{\rho g} + \frac{V^2}{2g} + Z = 0 + 0 + 4.0 = 4m$

@ B, $4.0 - 0.19 = 3.81m$

@ C, $\frac{2.734^2}{2 \times 9.81} = 0.38m$

Hydraulic Gradient Line:

@ A, $4 - V^2/2g = 4 - 0.38 = 3.62m$

@ B, $3.81 - 0.38 = 3.16m$

@ C, $0.38 - 0.38 = 0m$

Loss of head due to friction:

Darcy-Weisbach Equation:

The head loss due to friction is calculated from Darcy-Weisbach equation.

$$h_f = \frac{4fLV^2}{2gd}$$

f = co-eff of friction

$$f = \frac{16}{R_e} \text{ for } R_e < 2000 \text{ (Viscous flow)}$$

$$f = \frac{0.079}{R_e^{1/4}} \text{ for } R_e \text{ varies } 4000 \text{ to } 10^6$$

Chezy's formula:

$$V = C\sqrt{mi}$$

m: Hydraulic mean depth, $(m = d/4)$

i: Loss of head per unit length.

3. Find the head lost due to friction in a pipe of ϕ 300mm and length 50m through which water is flowing at a velocity of 3 m/s. Using
(i) Darcy formula (ii) Chezy's formula. Take C = 60.
ν = 0.01 stoke.

Soln:

$d = 0.3m$

$L = 50m$

$V = 3$ m/s

$C = 60$

$\nu = 0.01$ stokes $= 0.01\ cm^2/s = 0.01 \times 10^{-4}\ m^2/s$.

(i) Darcy Formula:

$$h_f = \frac{4fLV^2}{2gd}$$

$$= \frac{4 \times f \times 50 \times 3^2}{2 \times 9.81 \times 0.3}$$

$$Re = \frac{Vd}{\nu} = \frac{3 \times 0.3}{0.01 \times 10^{-4}} = 9 \times 10^5$$

⇒ Turbulent

$$f = \frac{0.079}{Re^{1/4}} = \frac{0.079}{(9 \times 10^5)^{1/4}} = 0.00256$$

$$h_f = \frac{4fLV^2}{2gd} = \frac{4 \times 0.00256 \times 50 \times 3^2}{0.3 \times 2 \times 9.81}$$

$$= 0.7828\,m$$

(ii) Chezy's formula:

$$V = C\sqrt{mi}$$

$$m = d/4 = \frac{0.3}{4} = 0.075\,m$$

$$C = 60$$

$$V = C\sqrt{mi}$$

$$3 = 60\sqrt{0.075 \times i}$$

$$i = 0.0333$$

$$i = h_f/L$$

$$h_f = i \times L = 0.0333 \times 50$$

$$= 1.665\,m$$

Minor Energy losses :

1. Loss due to sudden enlargement

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

2. Loss due to Sudden contraction

$$h_C = 0.5 \frac{V_2^2}{2g}$$

3. Head loss at the entrance of a pipe

$$h_i = 0.5 \frac{V^2}{2g}$$

4. Loss due to exit :

$$h_o = V^2/2g$$

5. Loss of head due to obstruction

$$h_{ob} = \frac{V^2}{2g}\left(\frac{A}{C_c(A-a)} - 1\right)$$

6. Loss of head due to bend in pipe,

$$h_b = \frac{KV^2}{2g}$$

K = co-eff of bend

7. Loss of head in various pipe fittings,

$$h_f = KV^2/2g$$

K = co-eff of pipe fitting

4. At a sudden enlargement of a water main from 240mm to 480mm ϕ. The hydraulic gradient rises ~~from~~ by 10mm. Estimate the rate of flow.

Soln :

Rise of hydraulic gradient,

$$\left(Z_2+\frac{P_2}{\rho g}\right)-\left(Z_1+\frac{P_1}{\rho g}\right) = 10\,mm$$

$$= \frac{10}{1000}\,m$$

$$= \frac{1}{100}\,m$$

By Applying Bernoulli's equation,

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1 = \frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+h_L \rightarrow (1)$$

Loss due to sudden enlargement,

$$h_e = \frac{(V_1-V_2)^2}{2g} \longrightarrow (2)$$

From continuity equation,

$$A_1V_1 = A_2V_2$$

$$V_1 = \frac{\pi/4 \times D_2^2 \times V_2}{\pi/4 \times D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2$$

$$= \left(\frac{0.48}{0.24}\right)^2 \times V_2$$

$$= 4V_2$$

(2) $\Rightarrow$ $$h_e = \frac{(4V_2-V_2)^2}{2g}$$

$$= \frac{(3V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

① → $\frac{P_1}{\rho g} + \frac{(4V_2)^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \frac{9V_2^2}{2g}$

$\left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right) = 1/100 \text{ m}$

$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \frac{1}{100}$

$\frac{6V_2^2}{2g} = \frac{1}{100}$

$V_2 = \sqrt{\frac{2 \times 9.81}{100 \times 6}}$

$= 0.181 \text{ m/s}$

$Q_2 = A_2 \times V_2$

$= \pi/4 \, D_2^2 \times V_2$

$= \pi/4 \times 0.48^2 \times 0.181$

$= 0.03275 \text{ m}^3/\text{s}$

$= 32.75 \text{ lit/s}.$

5. Water is flowing through a horizontal pipe of diameter 200mm at a velocity of 3 m/s. A circular solid plate of ϕ 150mm is placed in the pipe to obstruct the flow. Find the loss of head due to obstruction in the pipe, if $C_c = 0.62$.

Soln:

$D = 200\text{mm} = 0.2\text{m}$

$v = 3 \text{ m/s}$

$A = \pi/4 \times D^2 = \pi/4 \times 0.2^2 = 0.03141 \text{ m}^2$

ϕ of obstruction = 0.15m

$a = \pi/4 \times d^2 = \pi/4 \times 0.15^2$

$= 0.01767\ m^2$

$C_c = 0.62$

$$h_{ob} = \frac{V^2}{2g}\left(\frac{A}{C_c(A-a)} - 1\right)^2$$

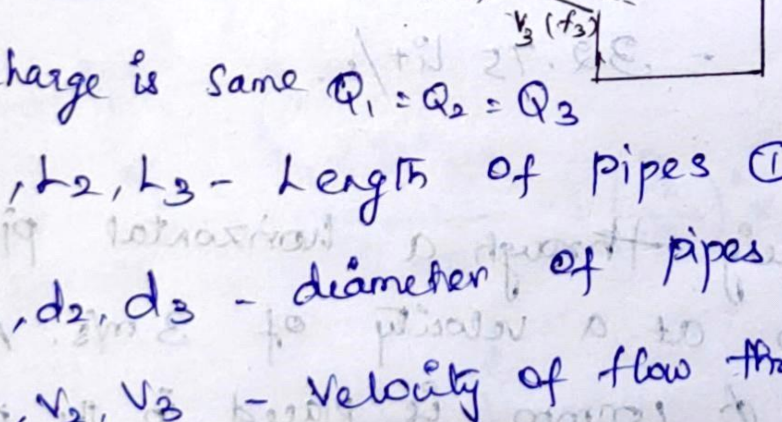
$$= \frac{3 \times 3}{2 \times 9.81}\left(\frac{0.03141}{0.62(0.03141 - 0.01767)} - 1.0\right)^2$$

$$= \frac{9}{2 \times 9.81}(3.687 - 1.0)^2$$

$= 3.311\ m.$

Pipes in Series:

The pipes of different length and diameters connected end to end (series) to form a pipe line



Discharge is same $Q_1 = Q_2 = Q_3$

L_1, L_2, L_3 - Length of pipes ①, ② & ③

d_1, d_2, d_3 - diameter of pipes ①, ② & ③

V_1, V_2, V_3 - Velocity of flow through pipes ①, ② & ③

f_1, f_2, f_3 - Co-eff. of friction for pipes ①, ② & ③

H = Difference of water level.

Neglecting losses ⇒

$$H = \frac{4f_1 L_1 V_1^2}{2g \times d_1} + \frac{4f_2 L_2 V_2^2}{2g \times d_2} + \frac{4f_3 L_3 V_3^2}{2g \times d_3}$$

Same material pipe $f_1 = f_2 = f_3 = f$

$$H = \frac{4fL_1V_1^2}{2gd_1} + \frac{4fL_2V_2^2}{2gd_2} + \frac{4fL_3V_3^2}{2gd_3}$$

$$H = \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]$$

6. The difference in water surface levels in two tanks, which are connected by three pipes in series of length 300m, 170m and 210m and of diameters 300mm, 200mm and 400mm respectively is 12m. Determine the rate of flow of water if Co-eff of frictions are 0.005, 0.0052 and 0.0048.

(i) considering minor losses

(ii) Neglecting minor losses.

Soln:

$H = 12m$,

$L_1 = 300m$, $L_2 = 170m$, $L_3 = 210m$

$d_1 = 0.3m$, $d_2 = 0.2m$, $d_3 = 0.4m$

$f_1 = 0.005$, $f_2 = 0.0052$, $f_3 = 0.0048$

(i) Considering minor losses.

Discharge is same in series connection,

$$A_1V_1 = A_2V_2 = A_3V_3$$

$$A_1V_1 = A_2V_2$$

$$V_2 = \frac{A_1V_1}{A_2} = \frac{\pi/4 \times d_1^2 \times V_1}{\pi/4 \times d_2^2} = \left(\frac{d_1}{d_2}\right)^2 \times V_1$$

$$V_2 = \left(\frac{0.3}{0.2}\right)^2 \times V_1 = 2.25V_1$$

$$A_1V_1 = A_3V_3$$

$$V_3 = \frac{A_1V_1}{A_3}$$

$$= \frac{\pi/4 \times d_1^2 \times V_1}{\pi/4 \times d_3^2}$$

$$= \left(d_1/d_3\right)^2 \times V_1 = \left(\frac{0.3}{0.4}\right)^2 \times V_1 = 0.5625V_1$$

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2g\times d_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2g\times d_2} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2g\times d_3} + \frac{V_3^2}{2g}$$

$$12 = \frac{0.5V_1^2}{2g} + \frac{4\times0.005\times300\times V_1^2}{0.3\times 2g} + \frac{0.5\times(2.25V_1)^2}{2g} + \frac{4\times0.0052\times170\times(2.25V_1)^2}{0.2\times2g} + \frac{(2.25V_1-0.562V_1)^2}{2g} + \frac{4\times0.0048\times210\times(0.5625V_1)^2}{0.4\times2g} + \frac{(0.5625V_1)^2}{2g}$$

$$12 = \frac{V_1^2}{2g}(0.5+20+2.53+89.505+2.847+3.189+0.316)$$

$$12 = V_1^2/2g\,(118.887)$$

$$V_1 = \sqrt{\frac{12\times2\times9.81}{118.887}}$$

$$= 1.407 \text{ m/s}$$

Discharge $Q = A_1\times V_1$

$= \pi/4 \times d_1^2 \times V_1$

$= \pi/4 \times 0.3^2 \times 1.407$

$= 0.09945 \text{ m}^3/\text{s}$

$= 99.45$ lit/sec.

(ii) Neglecting minor losses:

$$H = \frac{4f_1L_1V_1^2}{d_1\times 2g} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{4f_3L_3V_3^2}{2gd_3}$$

$$12 = \frac{V_1^2}{2g}\left[\frac{4\times 0.005\times 300}{0.3} + \frac{4\times 0.0052\times 170\times 2.25^2}{0.2} + \frac{4\times 0.0048\times 210\times 0.5625^2}{0.4}\right]$$

$$= \frac{V_1^2}{2g}\left[20 + 89.505 + 3.189\right]$$

$$12 = \frac{V_1^2}{2g} \times 112.694$$

$$V_1 = \sqrt{\frac{12\times 2\times 9.81}{112.694}}$$

$$V_1 = 1.445 \text{ m/s}$$

Discharge, $Q = A_1 \times V_1 = 1.445 \times \frac{\pi}{4} \times 0.3^2$

$$= 0.1021 \text{ m}^3/\text{s}$$

$$= 102.1 \text{ lit/sec}$$

Equivalent Pipe:

It is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters.

$$L = L_1 + L_2 + L_3$$

Head loss by neglecting minor losses,

$$H = \frac{4f_1 L_1 V_1^2}{2g d_1} + \frac{4f_2 L_2 V_2^2}{2g d_2} + \frac{4f_3 L_3 V_3^2}{2g d_3}$$

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

7. Three pipes of lengths 800m, 500m, and 400m and diameters 500mm, 400mm and 300mm are connect in series. These pipes are to be replaced by a single pipe of length 1700m. Find the diameter of the single pipe.

Soln:

$L_1 = 800m, \quad d_1 = 0.5m$

$L_2 = 500m, \quad d_2 = 0.4m$

$L_3 = 400m, \quad d_3 = 0.3m$

$L = 1700m$

Equivalent single pipe diameter,

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

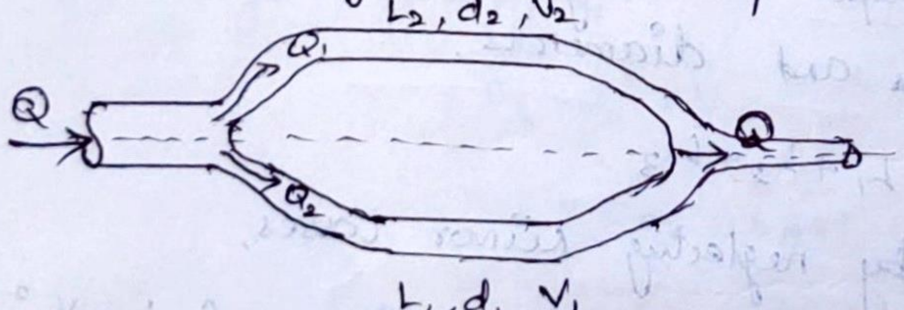
$$\frac{1700}{d^5} = \frac{800}{0.5^5} + \frac{500}{0.4^5} + \frac{400}{0.3^5}$$

$$d^5 = \frac{1700}{239037} = 0.007118$$

$d = 0.3718 m$

$d = 371.8 mm$

Flow through Parallel Pipes:



Consider a main pipe which divides into two or more branches as shown in fig.

Head loss is same,

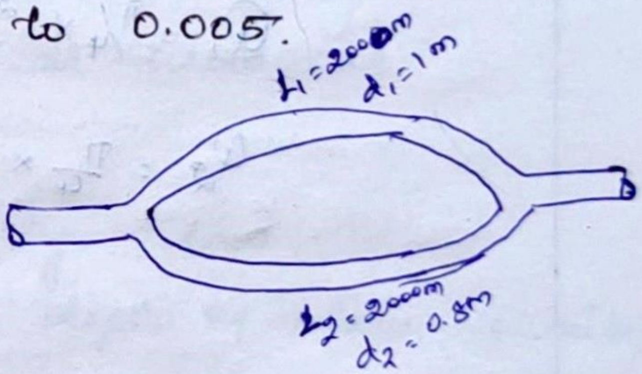
$$Q = Q_1 + Q_2$$

$$\frac{4 f_1 L_1 V_1^2}{2 g d_1} = \frac{4 f_2 L_2 V_2^2}{2 g d_2}$$

$$f_1 = f_2$$

$$\frac{L_1 V_1^2}{d_1 \times 2g} = \frac{L_2 V_2^2}{d_2 \times 2g}$$

8. A main pipe divides into two parallel pipes which again forms one pipe as shown in figure. The length and dia of 1st parallel pipe are 2000m and 1m respectively, while the length and ϕ of 2nd pipe are 2000m and 0.8m. Find the rate of flow in each parallel pipe, if total flow in the main is 3 m^3/s. The co-eff of friction for each parallel pipe is same and equal to 0.005.



Soln :

$L_1 = 2000m$

$d_1 = 1m$

$L_2 = 2000m$

$d_2 = 0.8m$

$Q = 3\ m^3/s$

$f_1 = f_2 = f = 0.005$

$Q = Q_1 + Q_2 = 3\ m^3/s$

$$\frac{4 f_1 L_1 V_1^2}{2g \times d_1} = \frac{4 f_2 L_2 V_2^2}{2g \times d_2}$$

$$\frac{4 \times 0.005 \times 2000 \times V_1^2}{1 \times 2 \times 9.81} = \frac{4 \times 0.005 \times 2000 \times V_2^2}{0.8 \times 2 \times 9.81}$$

$$\frac{V_1^2}{1} = \frac{V_2^2}{0.8}$$

$$V_1^2 = \frac{V_2^2}{0.8}$$

$$V_1 = \frac{V_2}{0.894}$$

$$Q_1 = \frac{\pi}{4} \times d_1^2 \times v_1 = \frac{\pi}{4} \times 1^2 \times \frac{V_2}{0.894}$$

$$Q_2 = \pi/4 \times d_2^2 \times V_2 = \frac{\pi}{4} \times 0.8^2 \times V_2$$

$$Q = Q_1 + Q_2$$

$$3 = \left[\pi/4 \times 1^2 \times \frac{V_2}{0.894}\right] + \left[\pi/4 \times 0.8^2 \times V_2\right]$$

$$V_2 = 2.17 \text{ m/s}$$

$$V_1 = \frac{V_2}{0.894} = \frac{2.17}{0.894} = 2.427 \text{ m/s.}$$

$$Q_1 = \pi/4 \times 1^2 \times 2.427 = 1.906 \text{ m}^3/\text{s}$$

$$Q_2 = \frac{\pi}{4} \times 0.8^2 \times 2.17 = 1.094 \text{ m}^3/\text{s}$$