Fluid Dynamics :

The science which deals with the geometry of motion of fluids without references to the forces causing the motion is is called kinematics.

The science which deals with the action of the forces in producing or changing motion of fluids is known as kinetics.

Both of this are called as fluid Dynamics.

Types of fluid flow :

1. Steady flow and unsteady flow

In steady flow velocity, pressure, density, temperature at a point do not change with time.

Ex: Flow of water in pipeline due to centrifugal pump run at uniform speed.

In unsteady flow velocity, pressure, density at a point change with respect to time

Ex: Liquid falling under gravity.

2. Uniform and Non-uniform flow:

Uniform flow in which the velocity of any given instant does not change both in magnitude and direction.

Ex: Flow between parallel plates.

Non-uniform flow in which the velocity of flow of fluid changes from one point to another point.

Ex: Flow in converging and diverging pipes.

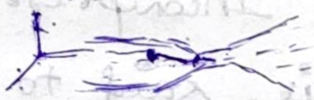
One, two and Three-dimensional flow:

In 1D flow, flow characteristics such as velocity, pressure, density, temperature are function of time and mentioned in one direction.

In 2D flow, flow characteristics are function of time and mentioned in two rectangular space co-ordinates.

Ex: Flow in main stream of a wide river.

In 3D flow, flow characteristics are the function of time and three mutually perpendicular directions.



Ex: Flow in a converging or diverging channel.

Rotational and irrotational flow:

A rotational flow exists when the fluid particles rotate about their mass centers while moving along a streamline.

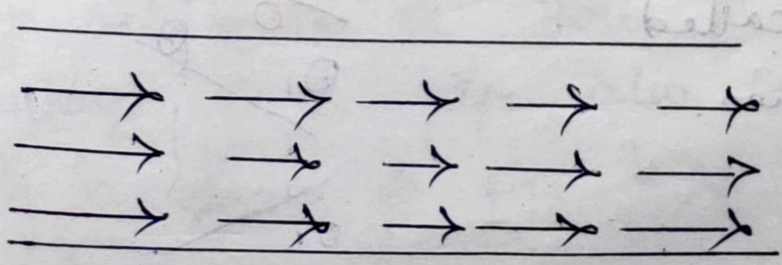
Ex: Liquid in a rotating tank.

In irrotational flow, fluid particles do not rotate about their mass centre while moving along a stream line.

Ex: Flow in a drainhole of wash basin.

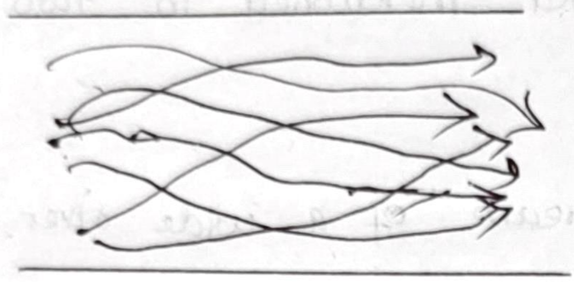
Laminar and Turbulent flow:

A Laminar flow is one in which the fluid particles moves in layers with one layer of fluid sliding smoothly over an adjacent layer. Fluid particles moves in well-defined paths.



Laminar flow

The turbulent flow is one in which the fluid particles moves in an entirely haphazard or erratic manner. Fluid particles moves in an unpredictable path that results in a rapid and continuous mixing of the fluid leading to momentum transfer as flow occurs.



Turbulent flow

Compressible and Incompressible flow :

A fluid is said to be compressible, if the density changes from point to point due to pressure and temperature.

$\rho \neq$ constant

A flow is said to be incompressible, if density is constant in the flow field.

Rate of flow (or) Discharge :

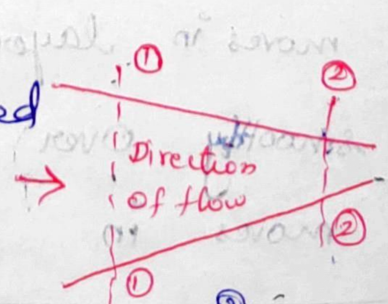
The quantity of fluid flowing per second through a section of a pipe or channel.

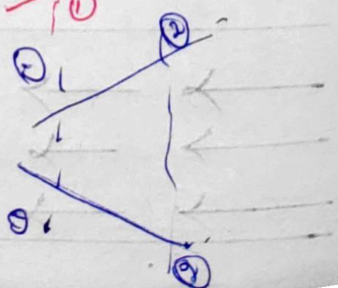
$Q = A \times V$

Continuity Equation :

2

This equation is based on principle of conservation of mass is called continuity equation.





A fluid flowing through the pipe at all the c/s, the quantity of fluid per second is constant.

Rate of flow @ ①-①:

$= \rho_1 A_1 V_1$

Rate of flow @ ②-②:

$= \rho_2 A_2 V_2$

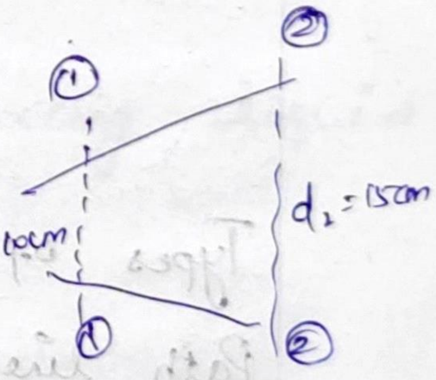
According to the law of conservation of mass.

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \rightarrow$ compressible fluid

$\rho_1 = \rho_2$

$A_1 V_1 = A_2 V_2 \rightarrow$ Incompressible fluid.

1. The diameter of a pipe at the Section ① and ② are 10cm and 15cm. Find the discharge through the pipe is the velocity of water flowing through the pipe at section ① is 5m/s. Determine the velocity of the section ②.



Sol:

$d_1 = 10cm = 0.1m$

$d_2 = 15cm = 0.15m$

$V_1 = 5m/s$

$A_1 V_1 = A_2 V_2$

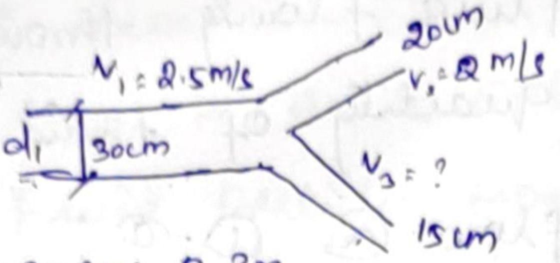
$V_2 = \frac{A_1 V_1}{A_2} = \frac{\pi/4 \times d_1^2 \times V_1}{\pi/4 \times d_2^2 \times V_2}$

$= \frac{0.1^2 \times 5}{0.15^2} \quad \frac{m^2 \times m/s}{m^2}$

$= 2.22$ m/s.

2. A 30cm diameter pipe, conveying water, braches into two pipes of diameter 20cm and 15cm. If the average velocity in the 30cm ϕ pipe is 2.5m/s. Find the discharge in the pipe, also determine the average velocity in 15cm ϕ pipe. If the average velocity is 2m/s.

Soln :



$Q_1 = Q_2 + Q_3$

$Q_1 = A_1 V_1$ $\quad d_1 = 30\text{ cm} = 0.3\text{m}$

$= \pi/4 \times d_1^2 \times 2.5$ $\quad Q_2 = \pi/4 \times d_2^2 \times 2$

$= \pi/4 \times 0.3^2 \times 2.5$ $\quad = \pi/4 \times 0.2 \times 2 \times 0.2$

$= 0.1767 \text{ m}^3/\text{s}$ $\quad = 0.0628 \text{ m}^3/\text{s}$

$Q_1 = Q_2 + Q_3$

$0.1767 = 0.0628 + Q_3$

$Q_3 = 0.1767 - 0.0628$

$= 0.1139 \text{ m}^3/\text{s}$.

$Q_3 = A_3 V_3$

$0.1139 = \pi/4 \times 0.15^2 \times V_3$

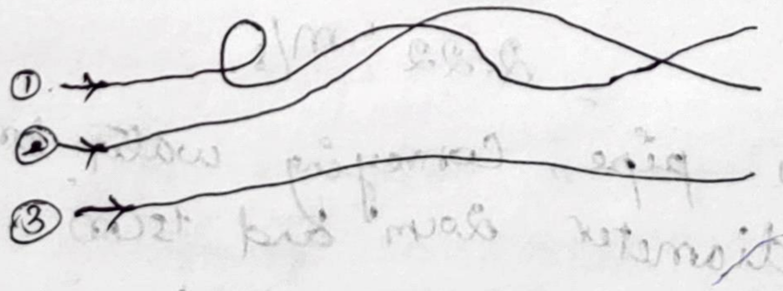
$V_3 = \dfrac{0.1139 \times 4}{\pi \times 0.15^2}$

$= 6.44$ m/s

Types of flow lines :

Path line :

A path line is the path followed by a fluid particle in motion. A path line shows the direction of particular particle as it moves ahead.



Stream line:

A stream line may be defined as an imaginary line within the flow so that the tangent at any point on it indicates the velocity at that point.


Stream line

Streak line:

The streak line is a curve which gives an instantaneous picture of the location of the fluid particles, which have passed through a given point.

Types of Heads (Energies):

1. Potential head:

This is due to position above some suitable datum line. It is denoted by z.

2. velocity head (or) kinetic energy:

This is due to velocity of flowing liquid $V^2/2g$.

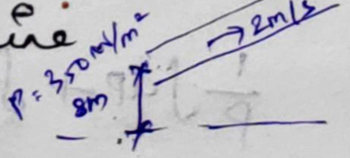
3. Pressure head (or) pressure energy:

This is due to the pressure of liquid. P/w.

Total head:

$$H = z + V^2/2g + P/w$$

3. In a pipe of 90mm ϕ water is flowing with a mean velocity of 2 m/s and at a gauge of 350 kN/m². Determine the total head if the pipe is 8m above the datum line.



Soln:

$\phi = 90mm = 0.09m$

$V = 2 m/s$, $w = 9.81$ kN/m², $P = 350$ kN/m².

$$H = z + \frac{V^2}{2g} + \frac{P}{w}$$

$$= 8 + \frac{2^2}{2 \times 9.81} + \frac{350}{9.81}$$

$$H = 43.88 m$$

Bernoulli's Equation:

In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential energy is constant along a stream line.

$$\frac{P}{w} + \frac{V^2}{2g} + z = \text{Constant.}$$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$

Assumptions:

1. The liquid is ideal and incompressible.
2. The flow is steady and continuous.
3. The flow is along the streamline ($\because$ 1D)
4. The velocity is uniform over the section and is equal to the mean velocity.
5. The only forces acting on the fluid are the gravity forces and the pressure forces.

Euler's Equation for motion:

$$\frac{dP}{\rho} + V.\,dv + g.\,dz = 0$$

By integrating the euler's equation, we get the Bernoulli's equation.

$$\frac{1}{\rho}\int dP + \int V.\,dv + \int g.\,dz = \text{constant.}$$

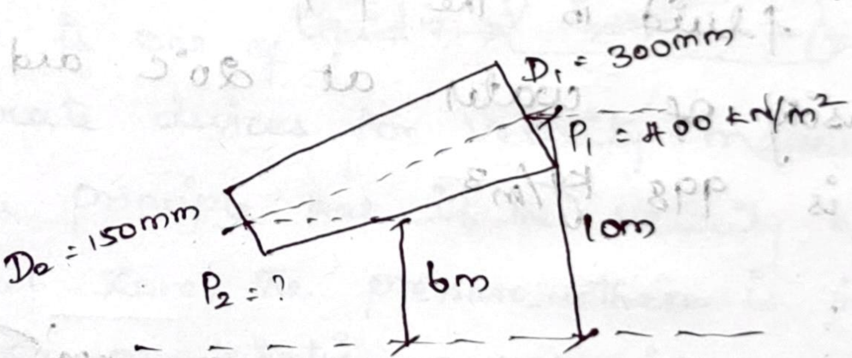
$$\frac{P}{\rho} + \frac{V^2}{2} + gx = \text{Constant}$$

$$\frac{P}{\rho g} + \frac{V^2}{2g} + x = \text{Constant}$$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2 \Rightarrow \text{Bernoulli's Eqn.}$$

4. The water is flowing through a tapering pipe having ∅ 300mm and 150mm at sections ① and ②. The discharge through the pipe is 40 lit/sec. The Section 1 is 10m above datum and Section 2 is 6m above datum. Find the intensity of pressure at section 2 if that at section 1 is 400 kN/m².

Soln :



$Q = 40$ lit/sec

$$= \frac{40 \times 10^3}{10^6} = 0.04\ m^3/sec.$$

$$V_1 = Q/A_1 = \frac{0.04}{\pi/4 \times 0.3^2} = 0.566\ m/s.$$

$$V_2 = Q/A_2 = \frac{0.04}{\pi/4 \times 0.15^2} = 2.264\ m/s.$$

Bernoulli's Theorem,

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + x_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + x_2$$

$$\frac{P_2}{w} = \frac{P_1}{w} + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) + (x_1 - x_2)$$

$$= \frac{400}{9.81} + \left(\frac{0.566}{2 \times 9.81} - \frac{2.264^2}{2 \times 9.81}\right) + (10 - 6)$$

$$P_2/w = 44.525\ m$$

$$P_2 = 44.525 \times 9.81 = 436.8\ kN/m^2$$

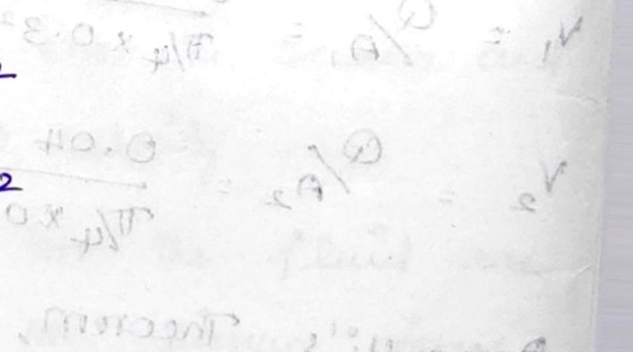
Bernoulli's Equation for Real fluids :

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2 + h_L$$

h_L = Loss of head.

5. In a smooth inclined pipe of uniform ϕ 250mm, a pressure of 50 kpa was observed at section ① which was at elevation 10m. At another section ② at elevation 12m, The pressure was 20 kpa and the velocity was 1.25 m/s. Determine the direction of flow and the head loss between these two sections. The fluid in the pipe is water. The density of water at 20°C and 760mm of Hg is 998 kg/m³.

Soln :



$Z_1 = 10m$,

$Z_2 = 12m$,

$\phi = 250mm = 0.25m$

$P_1 = 50$ kpa $= 50 \times 10^3$ N/m²

$P_2 = 20$ kpa $= 20 \times 10^3$ N/m²

$\rho = 998$ kg/m³

$V_1 = V_2 = 1.25$ m/s.

Total energy @ ①-①

$$E_1 = \frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1$$

$$= \frac{50 \times 10^3}{9.81 \times 998} + \frac{1.25^2}{2 \times 9.81} + 10$$

$$= 15.87m$$

Total energy @ ②-②

$$E_2 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$= \frac{20\times10^3}{998\times9.81} + \frac{1.25^2}{2\times9.81} + 12$$

$$= 14.122\,m$$

$$h_L = 15.187 - 14.122$$

$$= 1.065\,m$$

Direction of flow,

$E_1 > E_2 \Rightarrow$ Flow is from section ① to section ②

Pitot Tube:

Pitot tube is one of the most accurate devices for velocity measurement. It works on the principle that if the velocity of flow at point becomes zero, the pressure there is increased due to conversion of kinetic energy into pressure.

Both the static pressure as well as stagnation pressure can be measured in a device known as pitot-static tube.

$$\frac{P_o}{w} + \frac{V^2}{2g} = \frac{P_s}{w}$$

$$h_o + \frac{V^2}{2g} = h_s$$

$$V = \sqrt{2g(h_s - h_o)}$$

$$V = \sqrt{2g\Delta h}$$

$$\Delta h = y\left(\frac{S_m}{S} - 1\right)$$

$$V = C\sqrt{2g\Delta h}$$

6. A submarine fitted with a pitot tube moves horizontally in sea. Its axis is 12m below the surface of water. The pitot tube fixed infront of the submarine and along its axis is connected to the two limbs of a U-tube containing mercury, the reading of which is found to be 200mm. Find the speed of the submarine.

Take specific gravity of sea water = 1.025 of fresh water

Soln :

Reading of the manometer, $y = 200mm = 0.2m$ of mercury

$S_{hl} = 13.6$

$S_l = 1.025$

$$h = y\left(\frac{S_{hl}}{S_l} - 1\right)$$

$$= 0.2\left(\frac{13.6}{1.025} - 1\right)$$

$$= 2.45m$$

Velocity of the submarine, $V = \sqrt{2gh}$

$$= \sqrt{2 \times 9.81 \times 2.45}$$

$$= 6.93 \text{ m/s}.$$

Practical Applications of Bernoulli's Equation :

1. Venturimeter
2. Orifice meter
3. Rotameter and elbow meter
4. Pitot tube

Venturimeter :

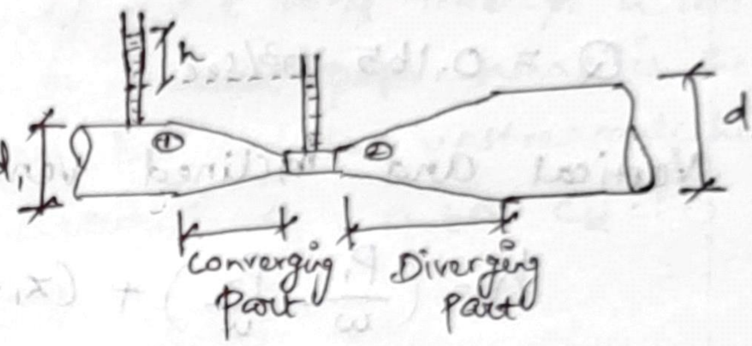
It is an instrument used to measure the rate of discharge in a pipeline and is often fixed permanently at different sections of the pipeline to know the discharge.

Types :

1. Horizontal Venturimeter
2. Vertical venturimeter
3. Inclined Venturimeter

Horizontal Venturimeter :

1. Short Converging Part
2. Throat
3. Diverging Part



Throat ratio = $\frac{d_2}{d_1}$ varies $\frac{1}{4}$ to $\frac{3}{4}$

$$Q = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

For heavy liquid, $h = y\left(\frac{S_H}{S_P} - 1\right)$

For lighter liquid, $h = y\left(1 - \frac{S_L}{S_P}\right)$

7. A horizontal venturimeter with inlet ϕ 200mm and throat ϕ 100mm is used to measure the flow of water. The pressure at inlet is 0.18 N/mm² and the vacuum pressure at the throat is 280mm of mercury. Find the rate of flow. The value of Cd may be taken as, 0.98.

Soln :

$A_1 = \pi/4 \times d_1^2 = \pi/4 \times 0.2^2 = 0.0314\ m^2$

$A_2 = \pi/4 \times d_2^2 = \pi/4 \times 0.1^2 = 0.00785\ m^2$

$P_1 = 0.18\ N/mm^2 = 180\ KN/m^2$

$\frac{P_1}{w} = \frac{180}{9.81} = 18.3\,m$

Vacuum pressure at the throat,

$\frac{P_2}{w} = -280$ mm of mercury

$= -0.28$ m of mercury $= -0.28 \times 13.6 = -3.8$ m of water

Differential head $= \frac{P_1}{w} - \frac{P_2}{w} = 18.3 - (-3.8) = 22.1\,m$

$$Q = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{0.314 \times 0.00785}{\sqrt{0.314^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times 22.1}$$

$$Q = 0.165 \text{ m}^3/\text{sec}$$

Vertical and inclined Venturimeters :

$$h = \left(\frac{P_1}{w} - \frac{P_2}{w}\right) + (x_1 - x_2)$$

8. A 200mm x 100mm venturimeter is provided in a vertical pipe carrying water, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 220mm. Find the rate of flow. $C_d = 0.98$.

Soln :

$D_1 = 0.2m$, $D_2 = 0.1m$

$$A_1 = \frac{\pi}{4} \times D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

$$A_2 = \frac{\pi}{4} \times D_2^2 = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$$

$S_{hl} = 13.6$, $S_p = 1$, $C_d = 0.98$

$y = 220mm = 0.22 m$

$$h = \left(\frac{P_1}{w} + x_1\right) - \left(\frac{P_2}{w} + x_2\right)$$

$$h = y\left(\frac{S_{hl}}{S_p} - 1\right)$$

$$= 0.22\left(\frac{13.6}{1} - 1\right)$$

$$= 2.77m$$

$$Q = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}$$

$$= 0.98 \times \frac{0.0314 \times 0.00785}{\sqrt{0.0314^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times 2.77}$$

$$Q = 0.0584 \text{ m}^3/\text{s}$$

Orificemeter :

Orificemeter is a device employed for measuring the discharge of fluid through a pipe. It also works on the same principle of a venturimeter.

9. Water flows at the rate of 0.015 m^3/s through a 100mm diameter orifice used in a 200mm pipe. What is the difference of pressure head between the upstream section, and the vena contracta section? $C_c = 0.60$, $C_v = 1.0$

Soln :

$$A_1 = \frac{\pi}{4} \times D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \, m^2$$

$$A_0 = \frac{\pi}{4} \times D_0^2 = \frac{\pi}{4} \times 0.1^2 = 0.00785 \, m^2$$

$$C_d = C_c \times C_v$$

$$= 0.60 \times 1.0 = 0.6$$

$$Q = C_d \times \frac{A_0 A_1}{\sqrt{A_1^2 - A_0^2}} \sqrt{2gh}$$

$$0.015 = 0.6 \times \frac{0.00785 \times 0.0314}{\sqrt{0.0314^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times h}$$

$$0.015 = \frac{0.6 \times 0.001093 \sqrt{h}}{0.030420}$$

$h = 0.484$ m of water.