Unit - 2 : Fluid Statics and Dynamics

Pressure of a fluid :

When a fluid is contained in a vessel, it exerts forces at all points on the sides and bottom and top of the container.

The force per unit area is called pressure.

$$p = \frac{F}{A} \text{ (or) } \frac{P}{A}$$

The pressure of a fluid on a surface will always act normal to the surface.

$$p = wh$$

The intensity of pressure in a liquid due to its depth will vary directly with depth.

$$h = \frac{p}{w}$$

The height of the free surface above any point is known as the static head at that point. It will be expressed as,

(i) Force per unit area.
(ii) Equivalent static head (mm, m, cm of liquid)

Hydrostatic law :

Rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point.

$$z = \frac{p}{\rho g} = \frac{p}{w}$$

1. Find the pressure at a depth of 15m below the free surface of water in a reservoir.

Soln :

$h = 15m$, $w = 9.81 \, kN/m^3$

$P = wh = 9.81 \times 10^3 \times 15$

$= 147.15 \, kN/m^2$

$= 147.15 \, kpa$

$1 \, N/m^2 = 1 \, pa$

$1 \, kN/m^2 = 1 \, kpa$

Pascal's Law:

The intensity of pressure at any point in a liquid at rest, is the same in all directions.

Absolute, Gauge and Atmospheric pressures:

The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact and it is known as atmospheric pressure. It is also known as 'Barometric pressure'.

The atmospheric pressure at sea level is called standard atmospheric pressure.

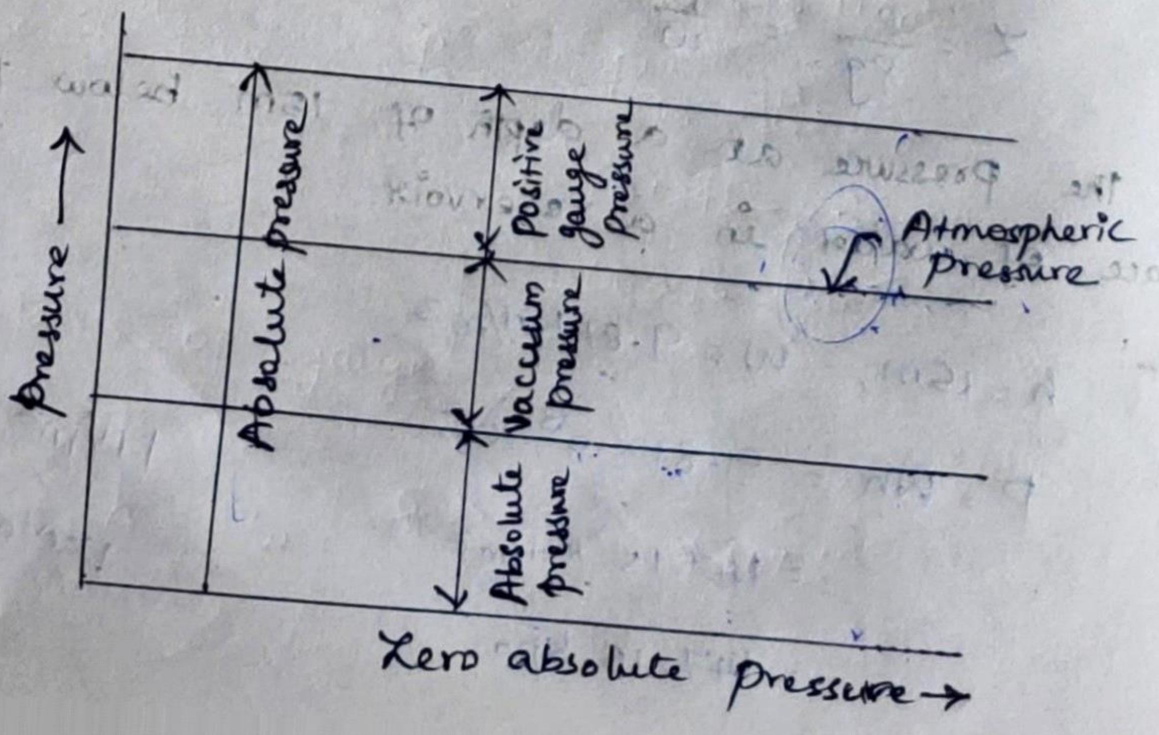
Pressure measured with the help of pressure measuring instrument is called 'Gauge Pressure'. In this atmospheric pressure is taken as datum.

Gauges:

It records pressure above or below the local atmospheric pressure.

If the pressure of a liquid below the atmospheric pressure, then the gauges are vaccum gauges. The pressure is called 'vaccum pressure'.

Any pressure measured above the absolute zero of pressure is termed as 'absolute pressure'.



Absolute pressure = Atmospheric pressure + gauge pressure

$$P_{abs} = P_{atm} + P_{gauge}$$

Vacuum pressure = Atmospheric pressure − absolute pressure

Standard atmospheric pressure values,

101.3 kN/m^2 (or) 101.3 kpa, 10.3m of water,

760mm of mercury

2. Given that,

Barometer reading = 740mm of mercury

Specific gravity of mercury = 13.6

Intensity of pressure = 40 kpa

Express the intensity of pressure in S.I units. both gauge and absolute.

Soln:

(i) Gauge Pressure:

i) $P = 40\text{ kpa} = 40\text{ kN/m}^2 = 0.4 \times 10^5\text{ N/m}^2 = 0.4\text{ bar}$

(ii) $h = \frac{P}{w} = \frac{0.4 \times 10^5}{9.81 \times 10^3} = 4.077\text{m of water}$

(iii) $h = \frac{P}{w} = \frac{0.4 \times 10^5}{9.81 \times 10^3 \times 13.6} = 0.299\text{m of mercury}$.

(ii) Absolute Pressure:

Barometer reading (atmospheric Pressure)

$= 740\text{mm of mercury}$

$= 740 \times 13.6\text{ mm of water}$

$= \frac{740 \times 13.6}{1000}\text{ m of water}$

$= 10.06\text{m of water}$

Absolute pressure = Atmospheric pressure + Gauge Pressure

$= 10.06 + 4.077$

$= 14.137\text{m of water}$

$P = wh$

$= 9.81 \times 10^3 \times 14.137$

$= 1.38 \times 10^5 \ N/m^2 = 1.38$ bar

$P_{abs} = \frac{14.137}{13.6} = 1.039$ m of mercury.

3. On the Suction side of a pump a gauge Shows a negative pressure of 0.35 bar.

Express this pressure in terms of :

(i) Intensity of pressure, kpa.

(ii) N/m^2 absolute

(iii) Metres of water gauge

(iv) Metres of oil (Sp. gravity 0.82) absolute

(v) cm of mercury gauge.

Take atmospheric pressure as 76cm of Hg and relative density of mercury as 13.6.

Vacuum gauge = 0.35 bar.

Soln :

(i) Intensity of pressure $= 0.35$ bar

$= 0.35 \times 10^5 \ N/m^2$

$= 35 \times 10^3 \ N/m^2$

$= 35$ kpa.

(ii) Absolute pressure ; $P_{atm} = 76$ cm of Hg.

$w \times h$

$P_{atm} = \frac{76}{100} \times (13.6 \times 9810)$ m of water

$= 101396 \ N/m^2$

Absolute Pressure $P_{abs} = P_{atm} - P_{vacuum}$

$= 101396 - 35000$

$= 66396 \ N/m^2$

(iii) Metres of gauge.

$P = wh$

$h = P/w = \frac{0.35 \times 10^5}{9810} = 3.567$ m

(iv) Metres of oil

$$h_{oil} = \frac{66396}{0.82 \times 9810} = 8.254 \text{ m of oil}$$

(v) Centimetres of mercury gauge:

$$h_m = \frac{0.35 \times 10^5}{13.6 \times 9810} = 0.2623 \text{ m of mercury}$$

$= 26.23$ cm of mercury.

4. A diver is working at a depth of 20m below the surface of sea water. (Sp. wt = 10 KN/m³). Calculate the pressure intensity at this depth. What would be the absolute pressure if barometer reads 760mm of mercury column at the sea level?

Soln:

$h = 20m$

$\gamma_w = 10$ KN/m³

$P_{atm} = 760$mm of mercury.

(i) Pressure at given depth ⇒ $P = wh$

$= 10000 \times 20$

$= 200$ kpa

Atmospheric pressure $P_{atm} = 760$mm of mercury

$= 13.6 \times 9810 \times 0.76$

$= 101.4$ kpa.

(ii) Absolute pressure $P_{abs} = P_{atm} + P_g$

$= 101.4 + 200$

$= 301.4$ kpa.

5. The inlet to pump is 10m above the bottom of sump from which it draws water through a suction pipe. If the pressure at the pump inlet is not to fall below 30 KN/m² absolute, workout the minimum depth of water in the tank. Take atmospheric pressure as 100 KN/m².

$P_{vac} = P_{atm} - P_{abs}$

$= 100 - 30$

$= 70 \ KN/m^2$

h = Distance between the pump inlet and the free water surface.

$P = wh$

$70 \times 10^3 = 9810 \times h$

$h = 7.136 \ m$

Minimum depth of water in the sump

$= 10 - 7.136$

$= 2.864 \ m.$

Measurement of Pressure:

The pressure of a fluid may be measured by the following devices.

Manometers:

Manometres are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same on another column of liquid.

(a) Simple manometres:

(i) piezometer

(ii) U-tube manometer

(iii) Single column manometer.

(b) Differential manometer

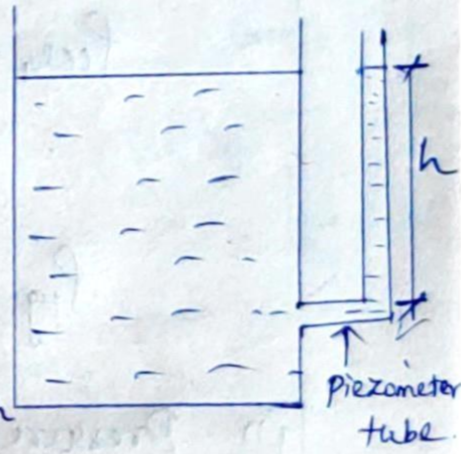
Mechanical Gauges:

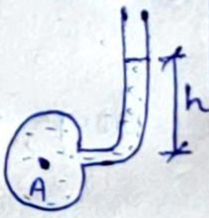
The pressure is measured by balancing the fluid column by spring.

(i) Bourdon tube pressure gauge
(ii) Diapharagm pressure gauge
(iii) Bellow pressure gauge
(iv) Dead-weight pressure gauge.

piezometer :

A piezometer is the simplest form of manometer which can be used for measuring moderate pressures of liquids. It consists of glass tube inserted in the wall of a vessel or of a pipe, containing liquid whose pressure is to be measured.





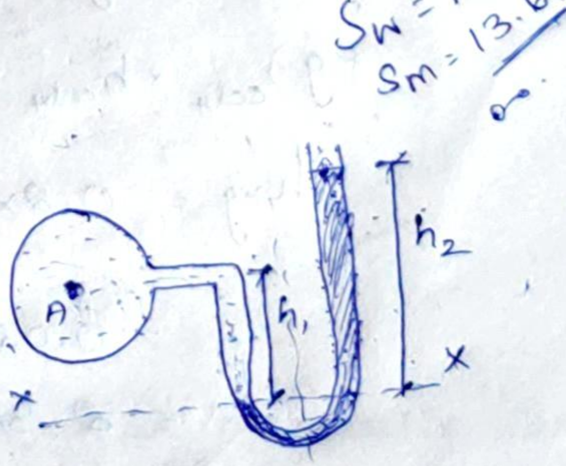
$$P = wh$$

piezometer measure gauge pressure only, since the surface of the liquid in the tube is subjected to atmospheric pressure. Not suitable for measuring negative pressure.

U-tube manometer:

Piezometers cannot be employed when large pressure in the lighter liquids are to be measured.

$S_w = 1$
$S_m = 13.6$



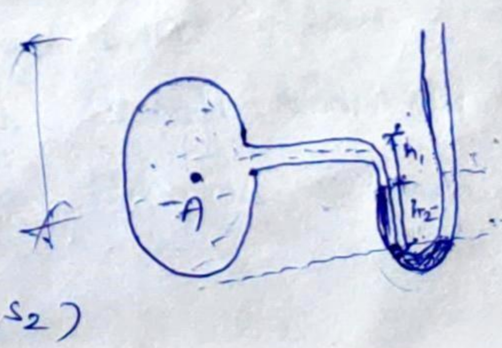
For positive :

pressure in left limb $= h + h_1 S_1$

Pressure in right limb $= h_2 S_2$

$$h + h_1 S_1 = h_2 S_2$$

$$h = h_2 S_2 - h_1 S_1$$



For negative :

$$h + h_1 S_1 + h_2 S_2 = 0$$

$$h = -(h_1 S_1 + h_2 S_2)$$

1. In a pipeline water is flowing. A manometer is used to measure the pressure drop for flow through the pipe. The difference in level was found to be 20cm. If the manometric fluid is CCl_4, find the pressure drop in SI units ($\rho_{CCl_4} = 1.596\ gm/cm^3$) If the manometric fluid is changed to mercury ($13.6\ gm/cm^3$) What will be the difference in level?

Soln:

$h = 20\ cm = 0.2m$

$\rho_{CCl_4} = 1.596\ gm/cm^3 = 1.596 \times \frac{(100)^3}{1000} = 1.596 \times \frac{10^6}{10^3}$

$= 1.596 \times 10^3\ kg/m^3$

$\rho_{Hg} = 13.6 \times 10^3\ kg/m^3$

(i) Pressure drop, $P_{CCl_4} = wh = (\rho g)h$

$= 1.596 \times 10^3 \times 9.81 \times 0.2$

$= 3131.3\ N/m^2$

$= 3.131\ kpa.$

(ii) The difference in level with mercury:

$P_{CCl_4} = P_{Hg}$

$(\rho g)_{CCl_4} \times h_{CCl_4} = (\rho g)_{Hg} \times h_{Hg}$

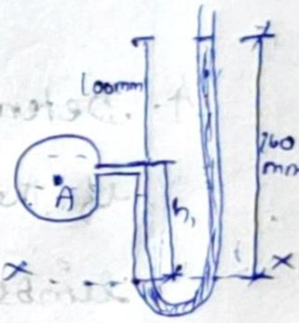
$h_{Hg} = 0.2 \times \frac{1.596 \times 10^3}{13.6 \times 10^3}$

$= 0.02347\ m$

$= 2.347\ cm$

2. A U-tube manometer is used to measure the pressure of oil of Specific gravity 0.85 flowing in a pipe line. Its left end is connected to the pipe and the right limb, is open to the atmosphere. The center of the pipe is 100mm below the level of mercury (sp. gravity = 13.6) in the right limb. If the difference of mercury level in the two limbs is 160mm, Determine, The absolute pressure of The oil in the pipe. Take P_{atm} = 100 kpa.

Soln:



$S_1 = 0.85$

$S_2 = 13.6$

$h_2 = 160mm = 0.16m$

$h_1 = 160 - 100 = 60mm = 0.06m$

Equating the pressure heads above the datum line $x - x$

$$h + h_1 S_1 = h_2 S_2$$

$$h + (0.06 \times 0.85) = 0.16 \times 13.6$$

$$h = 2.125m$$

Pressure, $P = wh$

$= 9.81 \times 2.125$

$= 20.84$ kpa.

$P_{abs} = P_{atm} + P_{gauge}$

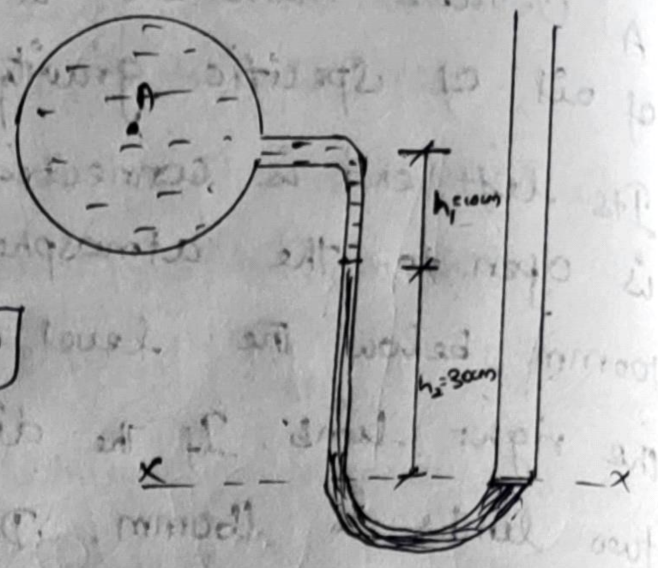
$= 100 + 20.84$

$= 120.84$ kpa.

3. The right limb of a simple U-tube manometer containing mercury is open to atmosphere and the left limb is Connected to a pipe through which flows a fluid of specific gravity 0.8. Make calculations for the vacuum pressure in the pipe, if the difference of mercury level in the two limb is 30cm and the level of fluid in the left limb is 10cm below the Centre of pipe.

Soln:

For negative



$h = -(h_1 S_1 + h_2 S_2)$

$= -[(0.1 \times 0.8) + (0.3 \times 13.6)]$

$= -(4.16)\,m$.

Pressure, $P = wh$

$= -(\times 9810 \times 4.16)$

$= ~~32640~~ N/m^2 = -40809\ N/m^2$

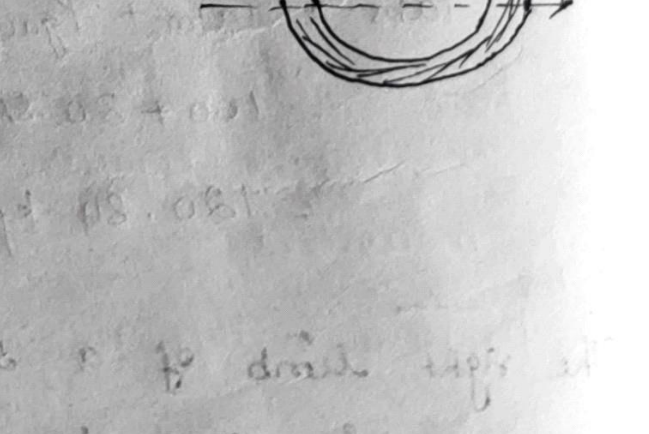
$= -40.809\ KN/m^2$

4. Determine the pressure of water in the mainline if the difference in the level of mercury in the limbs of a U-tube is 10cm and the free surface of mercury is in level with the centre of pipe. If the pressure of water in the pipeline reduce to $10 KN/m^2$, calculate the new difference in the level of mercury. Take specific weight of water as $10\ KN/m^3$.

Soln:

$h_1 = h_2 = 10\,cm$

$= 0.1m$



(i) $h + h_1 S_1 = h_2 S_2$

$h = h_2 S_2 - h_1 S_1$

$= (S_2 - S_1) \times h_1$

$= (13.6 - 1) \times 0.1$

$= 1.26m$

$P = wh$

$= 10 \times 10^3 \times 1.25$

$= 12.5\ KN/m^2$

(ii) If pressure $P = 10\ KN/m^2$

$10 = 10 \times h$

$P = wh$

$= w(-h_1 S_1 + h_2 S_2)$

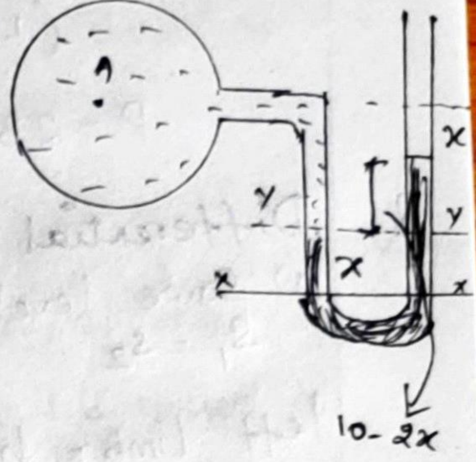
$10 = 10\left((0.1 - x)\times 1) + (0.1 - 2x)\times 13.6\right]$

$10 = 10\left[0.1 - x + 1.36 - 27.2x\right]$

$1 = 1.46 - 28.2x$

$0.46 = 28.2x$ Check

$x = 0.$



$P = wh$

$P = w(h_2 S_2 - h_1 S_1)$

$10 = 10\left[(0.1 - 2x)13.6 - (0.1 - x)1\right]$

$1 = \left[1.36 - 27.2x - 0.1 + x\right]$

$1 = \left[1.26 - 26.2x\right]$

$26.2x = 0.26$

$x = 0.26 / 26.2$

$x = 0.0176\ m$

$h = 0.1 - (2 \times 0.0176) =$

$h = 6.48\ cm.$

$x = 0.0099m$

$h = 0.1 - 2\times 0.0099$

$= 0.081m$

$= 8.1\ cm$

5. A simple U-tube manometer is installed across an orificemeter. The manometer is filled with mercury (13.6) and the liquid above the mercury is Carbon tetrachloride (1.6). The manometer reads 200mm. What is the pressure difference over the manometer in N/m^2.

Soln:

$h = y\left(\frac{S_{h1}}{S_{l2}} - 1\right)$

$= 200\left(\frac{13.6}{1.6} - 1\right)$

$= 1500mm$ of carbon tetrachloride.

$P = wh$

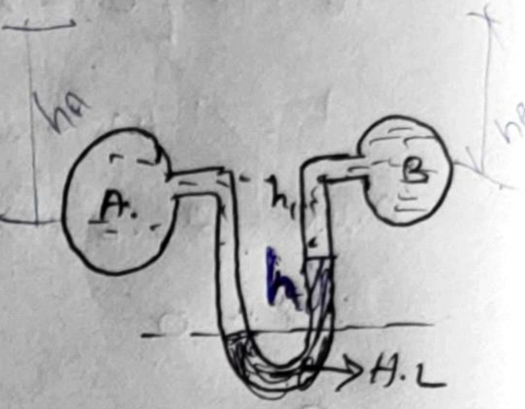
$= (1.6 \times 9810) \times \frac{1500}{1000}$

$P = 23544\ N/m^2$

Differential Manometers :

(i) Same level :

$S_1 = S_2$



left limb $= h_A + (h_1 + h)S_1 \rightarrow (1)$

$= h_A + h_1 S_1 + h S_1$

Right limb $= h_B + h_1 S_1 + h S \rightarrow (2)$

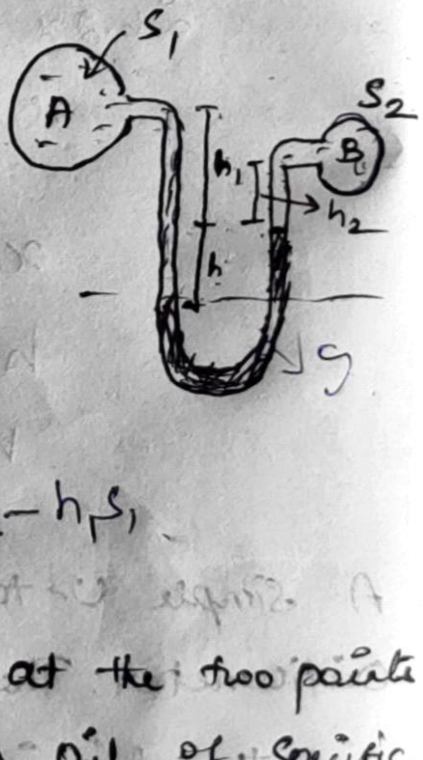
$(1) = (2)$

$h_A + h_1 S_1 + h S_1 = h_B + h_1 S_1 + h S$

$h_A - h_B = hS - hS_1$

$h_A - h_B = h(S - S_1)$

(ii) Different levels :



Left limb $= h_A + h_1 S_1 + h S_1$

Right limb $= h_B + hS + h_2 S_2$

$h_A + h_1 S_1 + h S_1 = h_B + hS + h_2 S_2$

$h_A - h_B = h(S - S_1) + h_2 S_2 - h_1 S_1$

6. A differential manometer connected at the two points A and B in a pipe containing an oil of specific gravity 0.9, shows a difference in mercury level as 150mm. Find the difference in pressure at the two points.

Soln :

$h_A - h_B = h(S - S_1)$

$= 0.150(13.6 - 0.9)$

$= 1.905$ m of water.

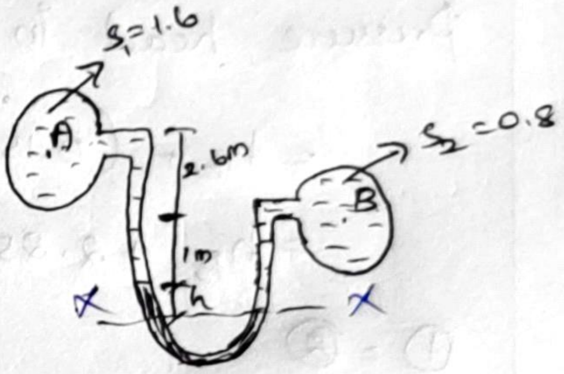
$P_A - P_B = wh$

$= 9.81 \times 1.905$

$= 18.68\ kN/m^2 = 18.68\ kPa.$

7. Figure shows a U-tube differential manometer containing connecting two pressure pipes at A and B. The pipe A contains a liquid of specific gravity 1.6 under a pressure of 110 kN/m². The pipe B contains oil of specific gravity 0.8 under a pressure of 200 kN/m². Find the difference of pressure measured by mercury as fluid filling U-tube.

Soln,



Pressure at A, $P_A = 110\ kN/m^2$

$P_A = wh_A$

$h_A = \frac{110 \times 10^3}{9.81 \times 10^3}$

$= 11.21$ m of water

Pressure at B, $P_B = 200\ kN/m^2$

$h_B = \frac{200 \times 10^3}{9.81 \times 10^3} = 20.38$ m of water.

Taking xx as a datum line

Left limb $= h_A + (2.6 + 1.0) S_1 + h \times 13.6$ m of water → ①

Right limb $= h_B + (1.0 + h) S_2$ m of water. → ②

① = ②

$h_A + 3.6 S_1 + h \times 13.6 = h_B + (1.0 + h) S_2$

$11.21 + (3.6 \times 1.6) + h \times 13.6 = 20.38 + (1 \times 0.8) + h \times 0.8$

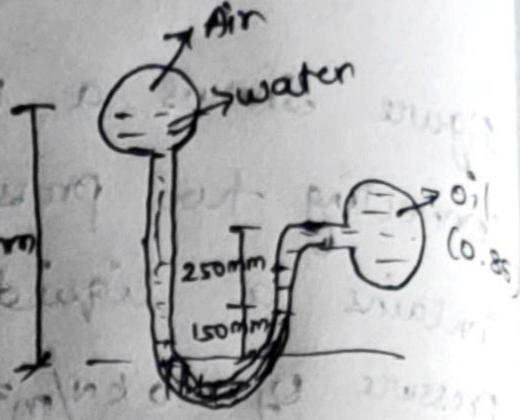
$13.6h - 0.8h = 20.38 + 0.8 - 11.21 - 5.76$

$12.8h = 4.21$

$h = 0.329$ m

$h = 329$ mm

8. Figure shows a differential manometer connected at two points A and B. At A air pressure is 100 kN/m². Find the absolute pressure at B.



Soln :

$P_A = wh_A$

$h_A = \frac{100 \times 10^3}{9.81 \times 10^3}$

$= 10.2m$

Pressure head in left limb,

$= 0.65 + 10.2 = 10.85m \rightarrow ①$

Pressure head in right limb,

$= h_B + (0.15 \times 13.6) + (0.25 \times 0.85)$

$= h_B + 2.25 \rightarrow ②$

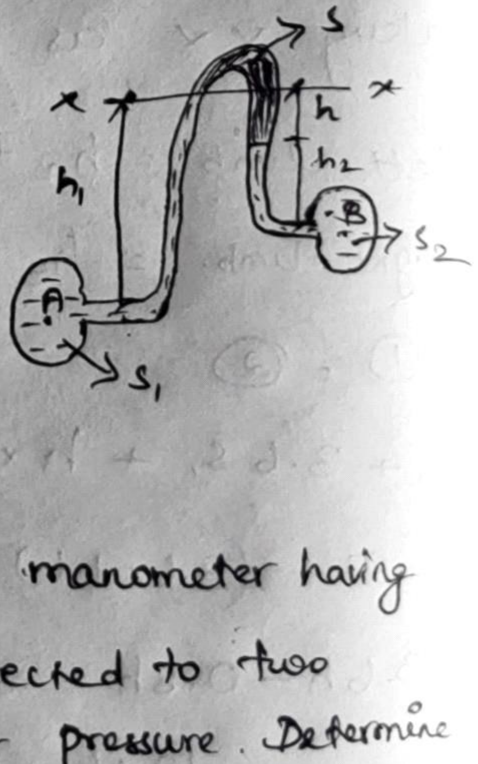
① = ②

$10.85 = h_B + 2.25$

$h_B = 8.6m$

$P_B = wh_B = (9810 \times 8.6) = 84.36$ kpa.

Inverted U-tube manometer :



Left limb = $h_A - h_1 s_1$

Right limb = $h_B - h_2 s_2 - hs$

$h_A - h_1 s_1 = h_B - h_2 s_2 - hs$

$h_A - h_B = h_1 s_1 - h_2 s_2 - hs$

9. Figure shows an inverted differential manometer having an oil of specific gravity 0.8 connected to two different pipes carrying water under pressure. Determine the pressure in the pipe B. The pressure in pipe A is 2m of water.