Unit-1 : Fluid Properties

Density : (Specific mass) (Mass Density)

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \rho = \frac{m}{V}, \quad \text{unit} = \frac{kg}{m^3}$$

Water = 1000 kg/m^3

Specific Weight: (Weight Density)

$$w = \frac{W}{V} = \frac{\text{Weight}}{\text{Volume}} \quad \text{unit} = N/m^3$$

Water, $w = 9.81 \times 10^3 \; N/m^3$ (or) $9.81 \; kN/m^3$

$$w = \frac{\text{Weight}}{\text{volume}}$$

$$= \frac{\text{mass} \times g}{\text{Volume}}$$

$$w = \rho \times g$$

Specific Volume:

$$v = \frac{\text{Volume of fluid}}{\text{Mass of fluid}}$$

$$= \frac{V}{m}$$

$v = 1/\rho$ Unit = m^3/kg Ex: Gases.

Specific Gravity:

$$S = \frac{\text{Specific Weight of fluid}}{\text{Specific Weight of standard fluid (Water)}}$$

$$S = \frac{\rho_f}{\rho_w} \text{ (or) } \frac{w_f}{w_s}$$

For water

$$S = \frac{9.81 \times 10^3}{9.81 \times 10^3} \frac{N/m^3}{N/m^3}$$

$$S = 1$$

1. 2 litre of petrol weighs 14 N. Calculate the specific weight, mass density, specific volume and specific gravity of petrol with respect to water.

Given:

$V = 2$ lit $\qquad 1\,m^3 = 1000$ lit

$= 2 \times 10^{-3}\,m^3 \qquad \frac{1\,m^3}{1000} = 1$ lit

$W = 14\,N \qquad 1$ lit $= 10^{-3}\,m^3$

Soln:

1. Specific Weight:

$w = \frac{\text{Weight}}{\text{Volume}}$

$= \frac{14}{2 \times 10^{-3}}$

$= 7 \times 10^{3}\,N/m^3$

2. Mass Density:

$\rho = w/g \qquad (w = \rho \times g) \qquad \left(\frac{kg}{m^3} \times m/s^2\right)$

$= \frac{7000}{9.81} \qquad \frac{N/m^3}{m/s^2} = \frac{N\,s^2/m}{m^3} = \frac{kg}{m^3}$

$= 713.56\ kg/m^3$

3. Specific Volume:

$v = \frac{1}{\rho} = \frac{1}{713.56}$

$= 1.4 \times 10^{-3}\ m^3/kg$

4. Specific Gravity :

$$S = \frac{\rho_f}{\rho_w} = \frac{713.56}{1000} = 0.7136 \text{ (No unit)}$$

If specific gravity of a liquid is 0.80, make calculations for its mass density, specific volume and specific weight.

Given :

$S_f = 0.80$

Soln :

(i) $S_f = \frac{\rho_f}{\rho_w}$

$0.80 = \frac{\rho_f}{1000}$

$\rho_f = 0.80 \times 1000$

$= 800 \ kg/m^3$

(ii) Specific Volume :

$v = \frac{1}{\rho} = \frac{1}{800}$

$= 0.00125 \ m^3/kg$

(iii) Specific Weight :

$w = \rho \times g = 800 \times 9.81$

$= 7848 \ N/m^3$

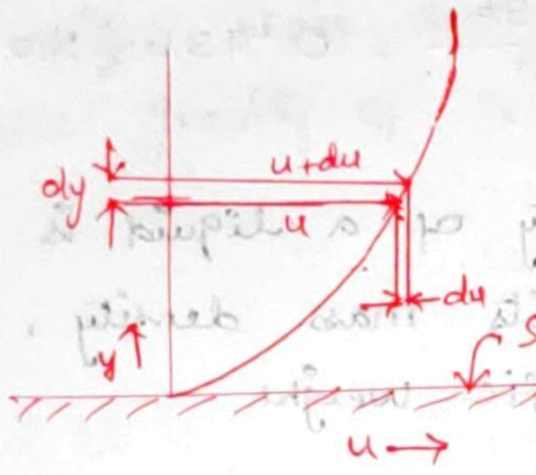
Viscosity :

The property of a fluid which determines its resistance to shearing stress.

It is a measure of the internal fluid friction which cause resistance to flow.

It is due to the cohesion

An ideal fluid which has no viscosity.



$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy} \quad \left[\mu = \frac{\tau}{\left(\frac{du}{dy}\right)} = \left(\frac{N/m^2}{\frac{m/s}{m}}\right) = \frac{Ns}{m^2} \right]$$

μ = dynamic viscosity , unit = Ns/m^2

$\frac{du}{dy}$ = Rate of Change of Shear stress.

Kinematic viscosity :

$$\nu = \frac{\mu}{\rho} \quad \text{unit} = m^2/s \text{ (or) } 1 \text{ stoke} = 10^{-4}\ m^2/s$$

Newton's law of viscosity :

The shear stress on a fluid element layer is directly proportional to the rate of shear strain.

$$\tau = \mu \frac{du}{dy}$$

The fluid which follows Newton's law is called Newtonian fluid.

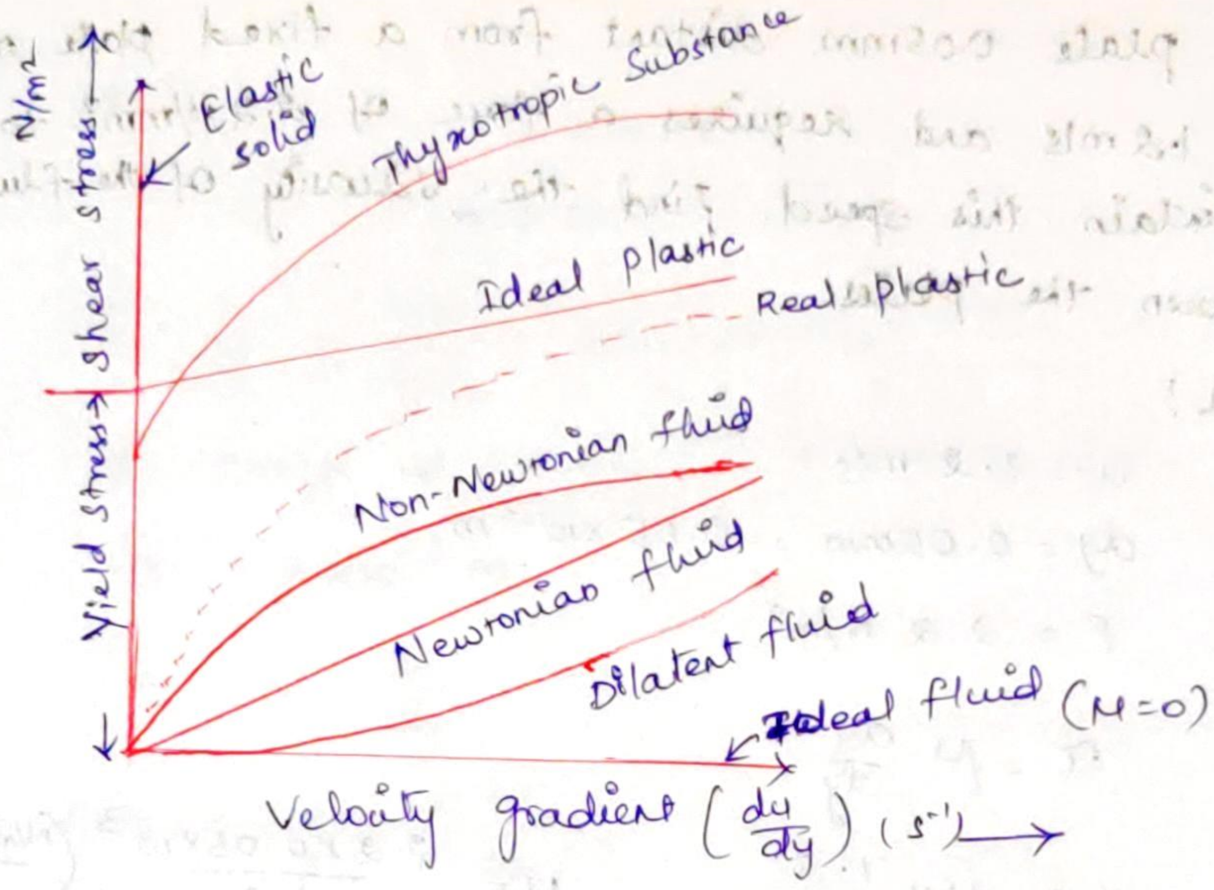
Types of fluids :

1. Newtonian fluid :

These fluids follows Newton's law of viscosity.

μ does not change with rate of deformation.

Ex : Water, Keroscene



2. Non-newtonian Fluids:

Fluids which do not follow the linear relationship between shear stress and rate of deformation.

Ex: mud flows, blood, slurries.

3. Plastic fluids:

An ideal plastic fluid has a definite yield stress and a constant linear relationship b/w shear stress and rate of angular deformation.

Ex: Sewage sludge.

A thyxotropic substance, which is non-newtonian fluid, has a non-linear relationship between the shear stress and rate of angular deformation.

4. Ideal fluid:

A fluid which is incompressible and has zero viscosity.

1. A plate 0.05mm distant from a fixed plate moves at 1.2 m/s and requires a force of 2.2 N/mm² to maintain this speed. Find the viscosity of the fluid between the plates.

Soln :

$u = 1.2 \text{ m/s}$

$dy = 0.05\text{mm} = 0.05 \times 10^{-3} \text{ m}$

$F = 2.2 \text{ N/m}^2$

$\tau = \mu \frac{du}{dy}$

$2.2 = \mu \times \frac{1.2}{0.05 \times 10^{-3}}$ $\quad \mu = \frac{2.2 \times 0.05 \times 10^{-3}}{1.2}$ $\left(\frac{N/m^2 \times m}{m/s}\right)$

$\mu = 9.16 \times 10^{-5} \text{ Ns/m}^2$ $\qquad$ 1 poise $= \frac{1}{10} \frac{Ns}{m^2}$

$\mu = 9.16 \times 10^{-5} \times 10$ poise $\qquad$ 10 poise $= 1 \frac{Ns}{m^2}$

$\mu = 9.16 \times 10^{-4}$ poise

2. A plate having an area of 0.6m² is sliding down the inclined plane at 30° to the horizontal with a velocity of 0.36 m/s. There is a cushion of fluid 1.8 mm thick between the plane and the plate. Find the viscosity of the fluid if the weight of the plate is 280 N.

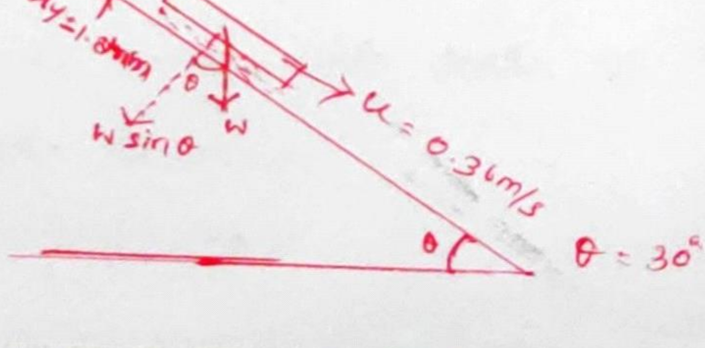
Soln :

$A = 0.6 \text{ m}^2$

$W = 280 \text{ N}$

$u = 0.36 \text{ m/s}$

$t = dy = 1.8 \text{ mm} = 1.8 \times 10^{-3} \text{ m}$



Soln !

$F = W \sin\theta = 280 \sin 30°$

$= 140\ N$

$\tau = \frac{F}{A} = \frac{140}{0.6} = 233.33\ N/m^2$

(F.V − I.V)

du = change of velocity $= u - 0 = 0.36\ m/s$

$dy = 1.8 \times 10^{-3}\ m$

$\tau = \mu \cdot \frac{du}{dy}$

$233.33 = \mu \times \frac{0.36}{1.8 \times 10^{-3}}$

$\mu = 1.166\ Ns/m^2$

$\mu = 11.66$ poise

3. The space between two square flat parallel plates is filled with oil. Each side of the plate is 720mm. The thickness of the oil film is 15mm. The upper plate, which moves at 3m/s requires a force of 120 N to maintain the speed. Determine,

(i) The dynamic viscosity of the oil.

(ii) The kinematic viscosity of oil if the specific gravity of oil is 0.95.

Soln :

Each side, $a = 0.72\ m$

$dy = 15mm = 0.015\ m$

Velocity of the upper plate $= u = 3\ m/s$

Change of velocity, $du = 3 - 0 = 3\ m/s$

Force, $F = 120\ N.$

Shear stress $\tau = \frac{F}{A} = \frac{120}{0.72 \times 0.72} = 231.5\ N/m^2$

(i) Dynamic viscosity:

$\tau = \mu \frac{du}{dy}$

$231.5 = \mu \times \frac{3}{0.015}$ $\quad \mu = 1.16\ Ns/m^2$

(ii) Kinematic Viscosity ν :

$$S = \frac{\text{Wt. density of oil}}{\text{Wt. density of water}}$$

$$0.95 = \frac{w_o}{9.81 \times 10^3}$$

$$w_o = 0.95 \times 9.81 \times 10^3$$

$$= 9320 \ N/m^3$$

Mass density of oil, $\rho_o = \frac{w_o}{g} = \frac{9320}{9.81}$

$$= 950 \ kg/m^3$$

$$\nu = \frac{\mu}{\rho} = \frac{1.16}{950} \quad \left[\frac{\left(\frac{Ns}{m^2}\right)}{kg/m^3} \quad \frac{\frac{kg\,m\,s}{s^2 \times m^2}}{kg/m^3} = \frac{m^3}{ms} = m^2/s \right]$$

$$= 0.00122 \ m^2/s$$

4. The velocity distribution for flow over a plate is given by $u = 2y - y^2$ where u is the velocity in m/s at a distance y metres above the plate. Determine the velocity gradient and shear stress at the boundary and 1.5m from it. Take $\mu = 0.9 \ Ns/m^2$

Soln :

$$u = 2y - y^2$$

$$\frac{du}{dy} = 2 - 2y \quad \left(\frac{m/s}{m}\right) = 1/s$$

$$\left(\frac{du}{dy}\right)_{y=0} = 2 - 2(0) = 2/s$$

$$\left(\frac{du}{dy}\right)_{y=0.15m} = 2 - (2 \times 0.15) = 1.7/s$$

Shear Stress, $\tau = \mu \left(\frac{du}{dy}\right)$

$$(\tau)_{y=0} = 0.9 \times 2 = 1.8 \ N/m^2 \qquad (Ns/m^2) \times 1/s = N/m^2$$

$$(\tau)_{y=0.15m} = 0.9 \times \overset{1.7}{\cancel{0.15}} = 0.9 \times 1.7 = 1.53 \ N/m^2$$

5. The velocity distribution of flow over a plate is parabolic with vertex 30cm from the plate, where the velocity is 180 cm/s. If the viscosity of the fluid is 0.9 Ns/m^2. find the velocity gradients and shear stresses at distances of 0, 15cm, and 30cm from the plate.

Soln:

vertex, $y = 30cm$

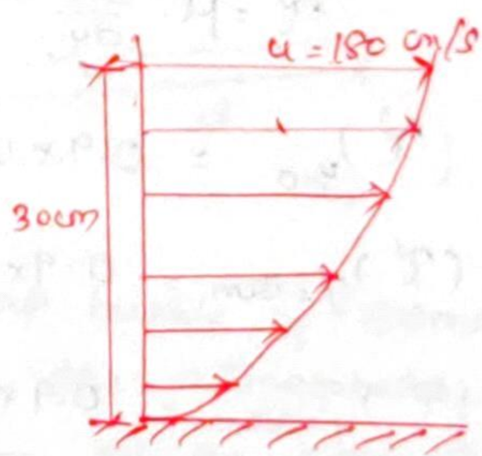
Velocity, $u = 180$ cm/s

Parabolic equation

$$u = ly^2 + my + n \rightarrow ①$$

$[x = ay^2 + by + c]$ (or) $[y = ax^2 + bx + c]$

l, m, n are constants.



(i) At $y = 0$, $u = 0$

(ii) At $y = 30cm$, $u = 180$ cm/s

(iii) At $y = 30cm$, $\frac{du}{dy} = 0$

Sub (i) values in ①,

$$0 = (l \times 0) + (m \times 0) + n$$

$$n = 0$$

Sub (ii) values in ①,

$$180 = l \times 30^2 + m \times 30$$

$$180 = 900l + 30m \rightarrow ③$$

$$u = ly^2 + my + n$$

$$\frac{du}{dy} = 2yl + m \rightarrow ②$$

Sub (iii) values in ②,

$$0 = 2l \times 30 + m$$

$$0 = 60l + m \rightarrow ④$$

Solving ③ & ④

$$l = -0.2, \quad m = 12$$

Sub l, m, n values in (1)

$$u = -0.2y^2 + 12y$$

$$\frac{du}{dy} = -0.4y + 12$$

(i) $\left(\frac{du}{dy}\right)_{y=0} = 12/s$

(ii) $\left(\frac{du}{dy}\right)_{y=15\,cm} = (-0.4 \times 15) + 12 = 6/s$

(iii) $\left(\frac{du}{dy}\right)_{y=30\,cm} = (-0.4 \times 30) + 12 = 0$

Shear Stress :

$$\tau = \mu \cdot \frac{du}{dy}$$

(i) $(\tau)_{y=0} = 0.9 \times 12 = 10.8\ N/m^2$

(ii) $(\tau)_{y=15\,cm} = 0.9 \times 6 = 5.4\ N/m^2$

(iii) $(\tau)_{y=30\,cm} = 0.9 \times 0 = 0$

Surface Tension :

Energy expended per unit area of the surface is called surface tension. (σ)

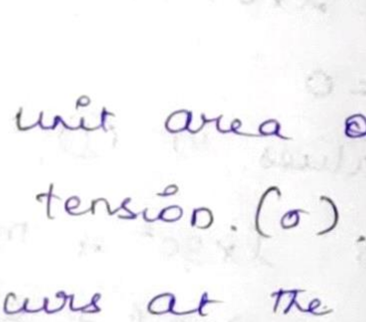
Surface tension occurs at the interface of a liquid and a gas or at the interface of two liquids and is essentially due to inter-molecular forces of cohesion.

· Pressure inside a water droplet :

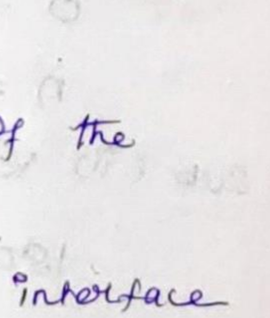
$$\underbrace{(P_i - P_o)\frac{\pi}{4}d^2}_{P.F} = \underbrace{\sigma \times \pi d}_{S.T.F}$$

$$P_i - P_o = \frac{\sigma \times \pi d}{\pi/4 \times d^2}$$

$$\boxed{P_i - P_o = \frac{4\sigma}{d}}$$



Droplet

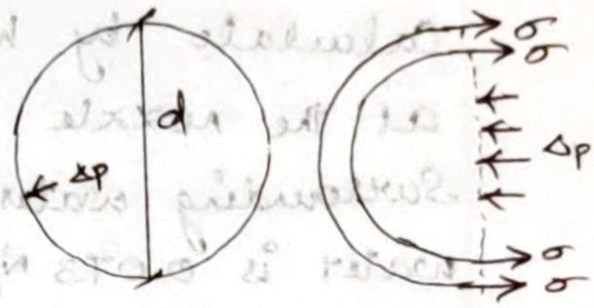


Surface Tension

Pressure inside a Soap bubble :

A soap bubble has two surfaces in contact with air. One inside and other outside.

$$(P_i - P_o)\frac{\pi}{4}d^2 = 2(\sigma \times \pi d)$$

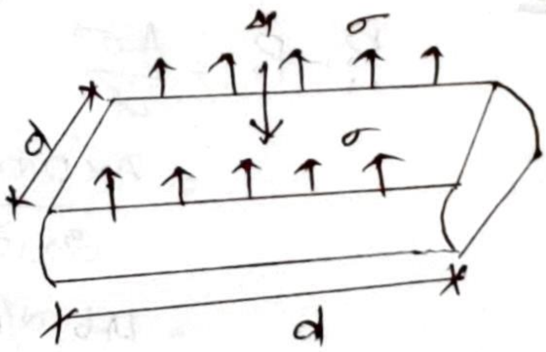


$$P_i - P_o = \frac{2(\sigma \times \pi d)}{\pi/4 \times d^2}$$

$$\boxed{P_i - P_o = \frac{8\sigma}{d}}$$

Pressure inside a liquid jet :

$$(P_i - P_o)ld = \sigma \times 2l$$



$$P_i - P_o = \frac{\sigma \times 2l}{ld}$$

$$\boxed{P_i - P_o = \frac{2\sigma}{d}}$$

1. The pressure inside a soap bubble of 50mm diameter is 2.5N/m² above the atmosphere. Estimate the surface tension of the soap film.

Soln :

$d = 50\text{mm} = 50 \times 10^{-3}$ m

$P = 2.5$ N/m²

$$P_i - P_o = \frac{8\sigma}{d}$$

$$2.5 = \frac{8 \times \sigma}{50 \times 10^{-3}}$$

$$\sigma = \frac{50 \times 10^{-3} \times 2.5}{8}$$

$\sigma = 0.01562$ N/m.

2. Air is introduced through a nozzle into a tank of water to form a stream of bubbles. If the bubbles are intended to have a diameter of 2mm Calculate by how much the pressure of the air at the nozzle must exceed that of the surrounding water. Assume that surface tension of water is 0.073 N/m. What would be the absolute pressure inside the bubble if the surrounding water is at 100 kpa?

Soln:

$$P_i - P_o = \frac{4\sigma}{d}$$

$$= \frac{4 \times 0.073}{2 \times 10^{-3}}$$

$$= 146 \ N/m^2$$

$$P_o = 100 \ kpa = 100 \times 10^3 \ N/m^2$$

$$= 100000 \ N/m^2$$

$$P_i - P_o = 146$$

$$P_i - 100000 = 146$$

$$P_i = 100000 + 146$$

$$= 100146 \ N/m^2$$

$$= 100.146 \ kpa.$$

3. In measuring the unit energy of a mineral oil (Sp. gravity = 0.85) by the bubble method, a tube having an internal diameter of 1.5mm is immersed to a depth of 1.25cm in the oil. Air is forced through the tube forming a bubble at the lower end. What magnitude of the unit surface energy will be indicated by a maximum bubble pressure intensity of 150 N/m^2?

Soln:

$P_i = 150 \, N/m^2$

$P_o = wh = (S \times g W_s) \times h \quad \left[S = \frac{W_f}{W_w} \right]$

$= 0.85 \times 9810 \times 0.0125 \quad (N/m^3 \times m) = N/m^2$

$= 104.3 \, N/m^2$

$P = P_i - P_o = 150 - 104.3$

$= 45.7 \, N/m^2$

$$P_i - P_o = \frac{4\sigma}{d}$$

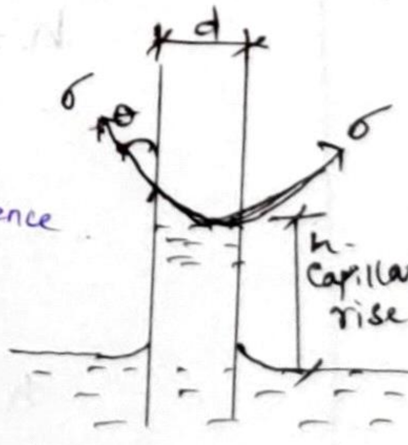
$$45.7 = \frac{4 \times \sigma}{1.5 \times 10^{-3}}$$

$$\sigma = 0.0172 \, N/m$$

Capillary:

Capillarity is a surface tension effect that depends upon the relative inter-molecular attraction between different substances. It is due to both cohesion and adhesion.

$$\left.\begin{array}{l}\text{Weight of water} \\ \text{raised / lowered}\end{array}\right\} = \left\{ \text{Surface tension force} \right.$$



$$\left.\begin{array}{l}\text{Area of tube} \times \text{rise/fall} \times \\ \text{Specific weight}\end{array}\right\} = \sigma \cos\theta \times \text{circumference}$$

$$\frac{\pi}{4} \times d^2 \times h \times w = \pi d \, \sigma \cos\theta$$

$$h = \frac{4\sigma \cos\theta}{wd}$$

(or)

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

If r given, put $(d = 2r)$

1. Calculate the capillary effects in mm in a glass tube of 4mm diameter, when immersed in (i) water (ii) in mercury. The temperature of the liquid is 20°c and the values of surface tension of water and mercury at 20°c in contact with air are 0.0735 N/m and 0.48 N/m. The contact angle for water $\theta = 0°$ and mercury $\theta = 130°$.

Soln:

$\sigma_w = 0.0735$ N/m, $\sigma_m = 0.48$ N/m

$\theta_w = 0°$ $\theta_m = 130°$.

(i) Water:

$$h = \frac{4\sigma\cos\theta}{wd}$$

$$= \frac{4 \times 0.0735 \times \cos 0}{9810 \times 0.004}$$

$$= 7.51 \times 10^{-3} m = 7.51 mm$$

(ii) Mercury:

$$h = \frac{4 \times 0.48 \times \cos 130°}{(9810 \times 13.6) \times 0.04}$$ $w_f = S_f \times w_w$

$$= -2.46 \times 10^{-3} m$$

$$= -2.46 mm$$

$h = 2.46$mm (depression)

Compressibility & Bulk Modulus

The property by virtue of which fluids undergo a change in volume under the action of external pressure is known as compressibility.

$$\text{Compressibility} = \frac{1}{K}$$

Bulk Modulus is defined as the ratio of compressive stress to volumetric strain.

$$K = -\frac{dp}{(dv/v)}$$

1. When the pressure of a liquid is increased from 3.5 MN/m² to 6.5 MN/m². Its volume is found to decreased by 0.08 %. What is the bulk modulus of elasticity of the liquid.

<u>Soln</u>:

$\sigma_i = 3.5\ MN/m^2$

$\sigma_f = 6.5\ MN/m^2$

Increase in pressure, $dp = 6.5 - 3.5 = 3\ MN/m^2$

$= 3 \times 10^6\ N/m^2$

$dv = 0.08$

$$K = -\frac{dp}{\left(\frac{dv}{v}\right)} = -\frac{3 \times 10^6}{\left(\frac{0.08}{100}\right)}$$

$$= 3.75 \times 10^9\ N/m^2$$

$$K = 3.75\ GN/m^2$$

2. A Gas A is compressed isothermally at 125 kPa (abs) and gas B is compressed isentropically ($\gamma = 1.4$) at 100 kPa (abs) which gas is more compressible.

(i) Isothermal, $\chi = 1/p = 1/125 = 0.008\ m^2/kN$

(ii) Isentropical, $\chi = 1/\gamma p = \frac{1}{1.4 \times 100} = 0.0071\ m^2/kN$

Gas A is more compressible.