Design

of

shaft couplings

- Shafts are usually available up to 7 meters length due to inconvenience in transport.
- In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling

- Shaft couplings are used in machinery for several purposes,
- 1.To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
- 2.To provide for misalignment of the shafts or to introduce mechanical flexibility.
- 3.To reduce the transmission of shock loads from one shaft to another.
- 4.To introduce protection against overloads.
- 5.It should have no projecting parts

- Requirements of a Good Shaft Coupling
- 1.It should be easy to connect or disconnect.
- 2. It should transmit the full power from one shaft to the other shaft without losses.
- 3.It should hold the shafts in perfect alignment.
- 4.It should reduce the transmission of shock loads from one shaft to another shaft.
- 5.If should have no projecting parts.

Types of Shafts Couplings

- 1. Rigid coupling   2. Flexible coupling

1. Rigid coupling : It is used to connect two shafts which are perfectly aligned.

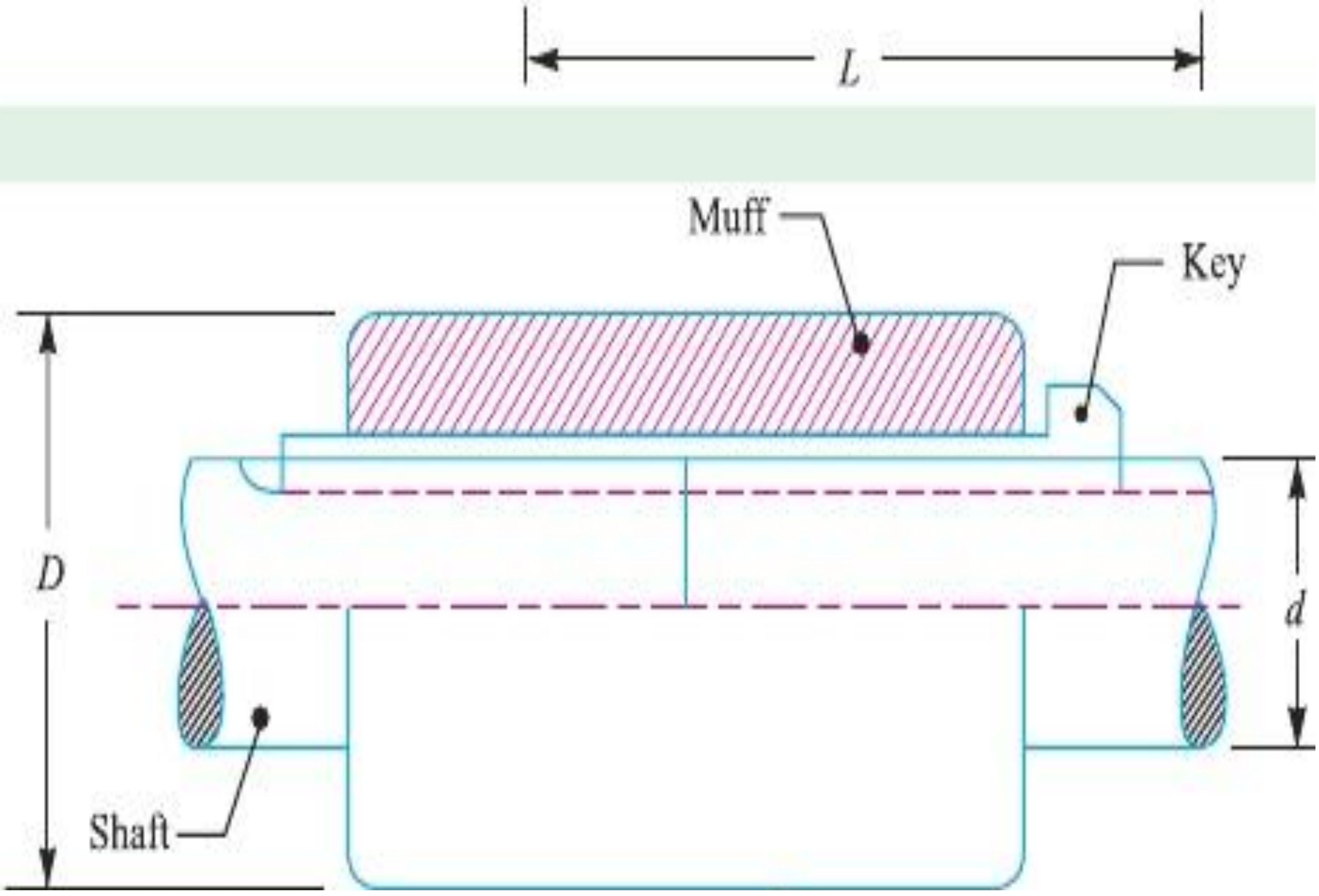
- types of rigid coupling are
- a)Sleeve or muff coupling.
- b)Clamp or split-muff or compression coupling,
- c)Flange coupling

- 2.Flexible coupling : It is used to connect two shafts having both lateral and angular misalignment.

- Types of flexible coupling are

- a)Bushed pin type coupling,
- b)Universal coupling, and
- c)Oldham coupling

a. Sleeve or Muff-coupling



- It is the simplest type of rigid coupling, made of cast iron.
- It consists of a hollow cylinder whose inner diameter is the same as that of the shaft **(sleeve).**
- It is fitted over the ends of the two shafts by means of a **gib head key**, as shown in Fig.
- The power is transmitted from one **shaft** to the other shaft by means of a **key** and a **sleeve.**

- **SHAFT - (d, T)**

d = diameter of the shaft , T= torque

- **SLEEVE – <u>(D, L)</u>**

D= Outer diameter of the sleeve

- **KEY- RED**
- l= length, w= width, t=thickness

1. Design for sleeve

- The usual **proportions** of a cast iron sleeve coupling

- Outer diameter of the sleeve, **D =2d + 13 mm**

  length of the sleeve, **L=3.5d**

  Where **d = diameter of the shaft**

- The **sleeve** is designed by considering it as a **hollow shaft.**

- **T**=Torque to be transmitted by the coupling
- **τ_c**=Permissible shear stress for the material of the sleeve which is cast rion.
- Torque transmitted by a hollow section

$$T = (\pi/16)\times\tau_c\times(D^4-d^4)/D$$

$$= (\pi/16)\times\tau_c\times D^3(1-K^4)$$

$$\ldots (\because k = d / D)$$

- From this expression, the induced **shear stress in the sleeve** may be checked

2. Design for key

- The length of the coupling key = length of the sleeve **(i.e. . 3.5d).**
- The coupling key is usually made into two parts
- length of the key in each shaft

  l= L/2=3.5d/2

- After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

- $T = l \times w \times \tau \times (d/2)$

  (Considering shearing of the key)

- $T = l \times t/2 \times \sigma_c \times (d/2)$

  (Considering crushing of the key

Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 rpm. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Solution. Given : $P = 40$ kW $= 40 \times 10^3$ W; $N = 350$ r.p.m.; $\tau_s = 40$ MPa $= 40$ N/mm^2; $\sigma_{cs} = 80$ MPa $=$ 80 N/mm^2; $\tau_c = 15$ MPa $= 15$ N/mm^2

1. ***Design for shaft***

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\pi N} = \frac{40 \times 10^3 \times 60}{2\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 1100 \times 10^3 / 7.86 = 140 \times 10^3 \text{ or } d = 52 \text{ say } 55 \text{ mm } \textbf{Ans.}$$

2. *Design for sleeve*

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm } \textbf{Ans.}$$

and length of the muff,

$$L = 3.5\, d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125} \right]$$

$$= 370 \times 10^3\, \tau_c$$

$$\therefore \quad \tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm^2, therefore the design of muff is safe.

3. *Design for key*

From Table 13.1, we find that for a shaft of 55 mm diameter,

Width of key, $w = 18$ mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

$\therefore$ Thickness of key, $t = w = 18$ mm **Ans.**

We know that length of key in each shaft,

$$l = L / 2 = 195 / 2 = 97.5 \text{ mm}$$ **Ans.**

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \tau_s$$

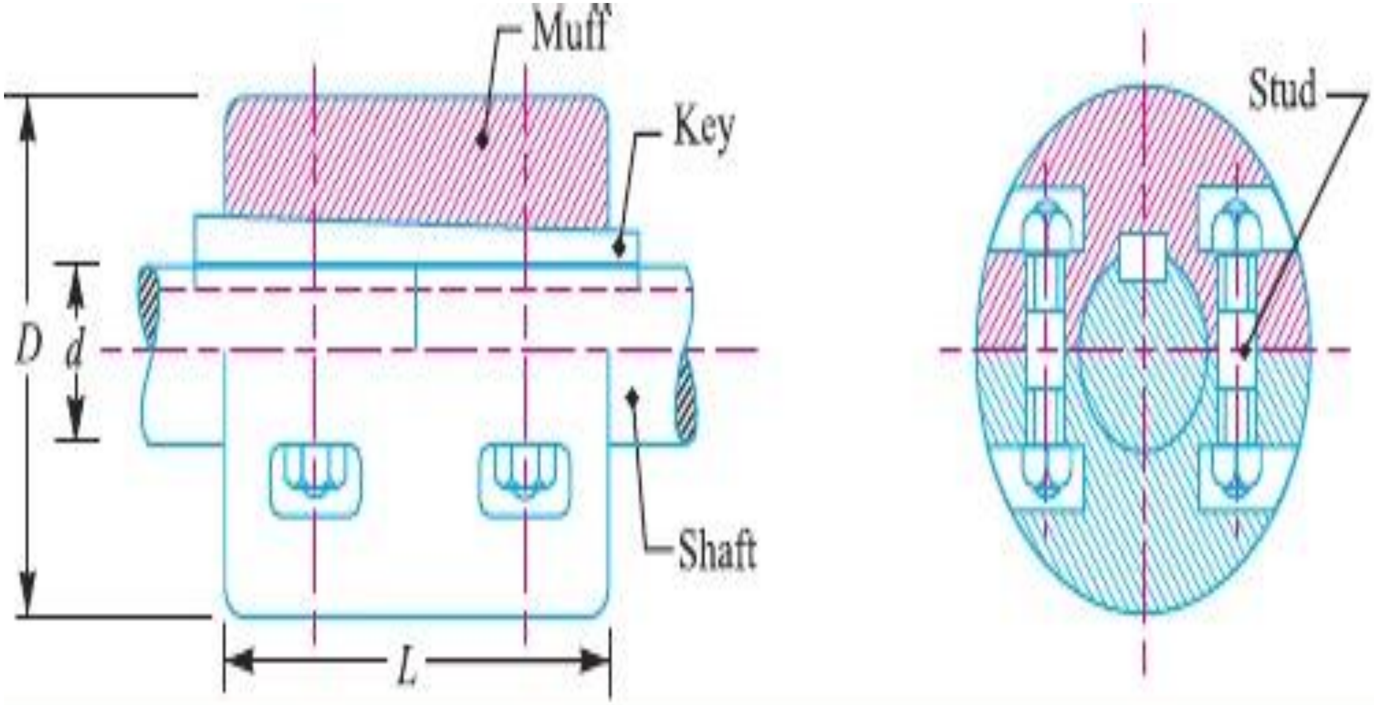
$$\therefore \quad \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \sigma_{cs}$$

$$\therefore \quad \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

b. Clamp or Compression Coupling



- the muff or sleeve is made into two halves and are bolted together.
- The halves of the muff are made of cast iron.
- The shaft ends are made to a butt each other
- a single key is fitted directly in the keyways of both the shafts.
- One-half of the muff is fixed from below and the other half is placed from above.
- Both the halves are held together by means of mild steel studs or bolts and nuts.
- The number of bolts may be two, four or six.
- The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the couplings

1. Design of muff

- The usual **proportions** of a cast iron sleeve coupling
- Outer diameter of the sleeve, **D =2d + 13 mm**
- length of the sleeve, **L=3.5d**

  Where **d = diameter of the shaft**

- The **sleeve** is designed by considering it as a **hollow shaft.**

- **T**=Torque to be transmitted by the coupling
- τ_c=Permissible shear stress for the material of the sleeve which is cast iron.
- τ_c = **14 MPa.**
- Torque transmitted by a hollow section

$$T = (\pi/16)\times\tau_c\times(D^4-d^4)/D$$

$$= (\pi/16)\times\tau_c\times D^3(1-K^4)$$

$$\dots (\because k = d / D)$$

- From this expression, the induced **shear stress in the sleeve** may be checked

2. Design for key

- The length of the coupling key = length of the sleeve **(i.e. . 3.5d).**
- The coupling key is usually made into two parts
- length of the key in each shaft

l= L/2=3.5d/2

- After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted

- $T = l \times w \times \tau \times (d/2)$

  (Considering shearing of the key)

- $T = l \times t/2 \times \sigma_c \times (d/2)$

  (Considering crushing of the key

3. Design of clamping bolts

- T =Torque transmited by the shaft,
- d =Diameter of shaft,
- d_b =Root or effective diameter of bolt
- n=Number of bolts,
- σ_t =Permissible tensile stress for bolt material,
- μ=Coefficient of friction between the muff and shaft, and
- L=Length of muff.

- force exerted by each bolt $(F) = (\pi/4)\,(d_b^{\,2})\,\sigma_t$

- Force exerted by the bolts on each side of the shaft $(F) = (\pi/4)\,(d_b^{\,2})\,(\sigma_t)(n/2)$

- (P)be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface

- P=Force/Projected area

- $P = (\pi/4)\,(d_b^{\,2})\,(\sigma_t)(n/2)/(1/2)Ld$

- ∴Frictional force between each shaft and muff,

F =μ× pressure × area

- $F = (\mu \times (\pi/4)(d_b^2)(\sigma_t)(n/2)/(\cancel{(1/2)Ld}) \times \pi \cancel{(1/2)\,d\,L}$

- $F = \mu \times (\pi^2/8)(d_b^2)(\sigma_t)(n)$

- Torque that can be transmitted by the coupling

$$T = F \times \mathbf{d/2}$$

$$\mathbf{T = \mu \times (\pi^2/8)(d_b{}^2)(\sigma_t)(n) \times d/2}$$

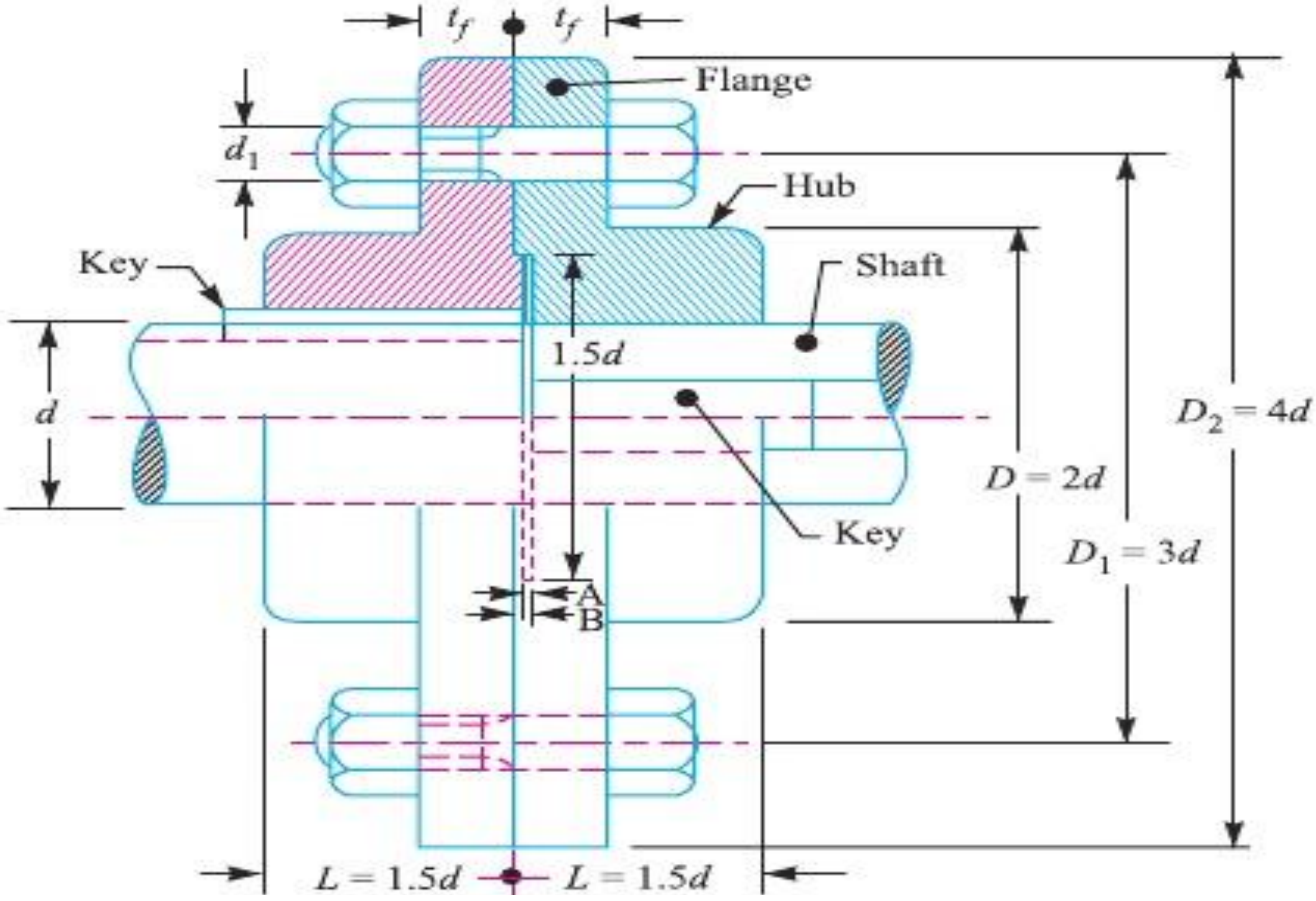
- From this relation, the root diameter of the bolt **(d_b)**

may be evaluated . **$\mu = 0.3$**

c. Flange coupling

- A flange coupling usually applies to a coupling having two separate cast iron flanges.
- Each flange is mounted on the shaft end and keyed to it.
- The faces are turned up at right angle to the axis of the shaft
- Flange coupling are
- 1.Unprotected type flange coupling
- 2. Protected type flange coupling
- 3. Marine type flange coupling

1.Unprotected type flange coupling



- In an unprotected type flange coupling each shaft is keyed to the boss of a flange with a counter sunk key and the flanges are coupled together by means of bolts.

- Generally, three, four or six bolts are used

Design of Unprotected type Flange Coupling

- The usual **proportions** for an unprotected type cast iron flange couplings
- d = diameter of the shaft or inner diameter of the hub
- D= Outside diameter of hub **D=2d**
- Length of hub, **L= 1.5d**
- Pitch circle diameter of bolts, **D_1=3d**
- Outside diameter of flange,

  $$D_2 = D_1 + (D_1 - D) = 2D_1 - D = 4d$$

- Thickness of flange **t_f =0.5d**
- **Number of bolts =3, for d upto 40 mm**

  =4, for d upto 100 mm

  =6, for d upto 180 mm

- d =Diameter of shaft or inner diameter of hub,
- τ_s =Allowable shear stress for shaft,

- D=Outer diameter of hub,
- t_f =Thickness of flange
- τ_c=Allowable shear stress for the flange material

- d_1=Nominal or outside diameter of bolt,
- D_1 =Diameter of bolt circle,
- n=Number of bolts,
- $\tau_{b=}$ Allowable shear stress for bolt
- $\sigma_{cb,,}$ =Allowable crushing stress for bolt

- $\tau_{k=}$ Allowable shear stress for key material
- $\sigma_{ck=}$ key material

1. Design for hub

- The hub is designed by considering it as a hollow shaft,
- transmitting the same torque (T) as that of a solid shaft

$$T = T = (\pi/16)\times\tau_c\times(D^4-d^4)/D$$

The outer diameter of hub is usually taken as twice the diameter of shaft.

- The length of hub **(L) = 1.5d**

2. Design for key

The material of key is usually the same as that of shaft. The length of key is taken equal to the length of hub

$$l = L$$

- $T = l \times w \times \tau \times (d/2)$

  (Considering shearing of the key)

- $T = l \times t/2 \times \sigma_c \times (d/2)$

  (Considering crushing of the key

3. Design for flange

The flange at the junction of the hub is under shear while transmitting the torque.

- T =Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub

- $T = \pi D \times t_f \times \tau_c \times D/2$

  $T = \pi \times t_f \times \tau_c \times D^2/2$

  The thickness of flange is usually taken as half the diameter of shaft

4. Design for bolts

- Load on each bolt (F)=

$$(\pi/4)\,(d_1^{\,2})\,(\tau_b)$$

- Total load on all the bolts (F) =

$$(\pi/4)\,(d_1^{\,2})\,(\tau_b)(n)$$

- The bolts are subjected to shear stress due to the torque transmitted

$$(T) = (\pi/4)\,(d_1^{\,2})\,(\tau_b)(n)\,(D_1/2)$$

From this equation, the diameter of bolt **(d_1)** may be obtained.

- We know that area resisting crushing of all the bolts $= n \times d_1 \times t_f$

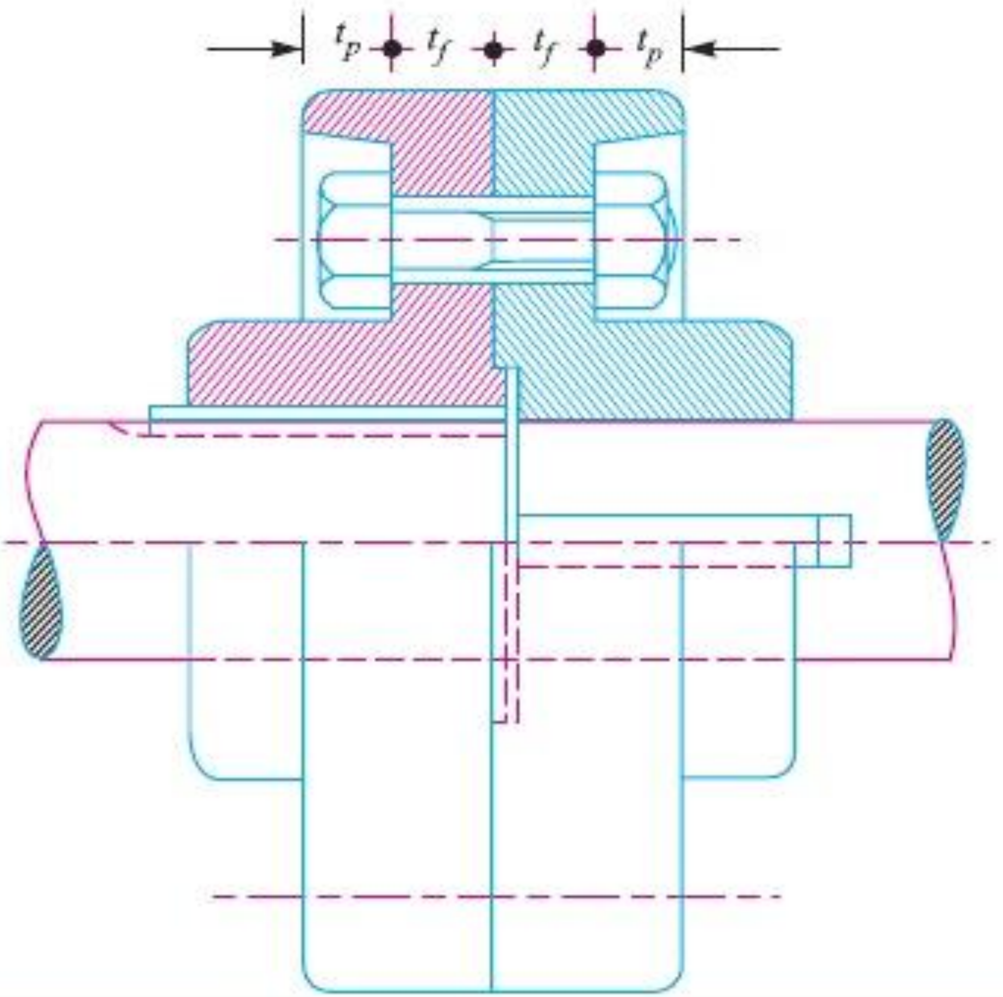
- crushing strength of all the bolts

$$= n \times d_1 \times t_f \times \sigma_{Cb}$$

Torque $= n \times d_1 \times t_f \times \sigma_{Cb} \times (D_1/2)$

- From this equation, the induced crushing stress in the bolts may be checked

Protected type flange coupling

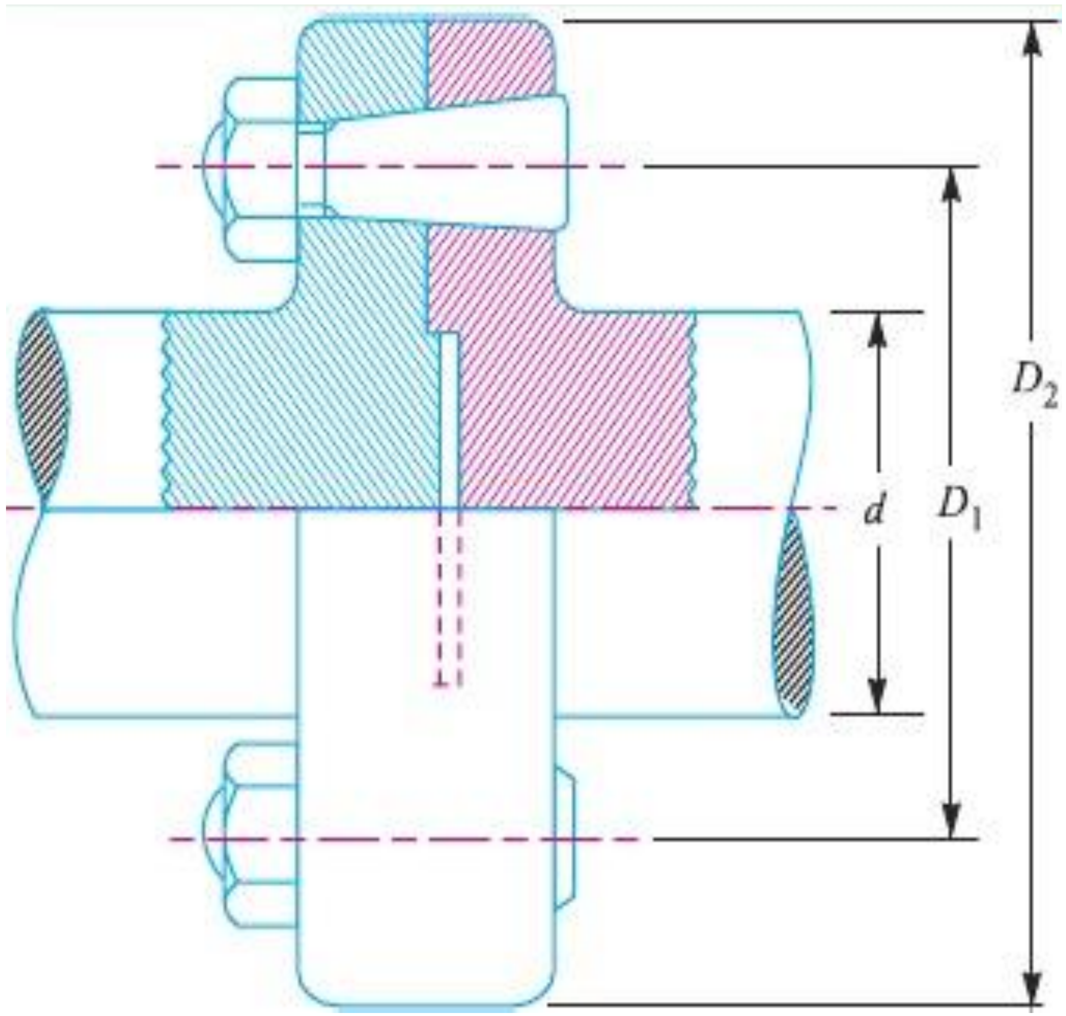


- the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman

$$(t_p) = 0.25d$$

The design of unprotective type is same process of protective type

Marine type flange coupling



- In a marine type flange coupling, the flanges are forged integral with the shafts .
- The flanges are held together by means of tapered head less bolts.
- numbering from four to twelve depending upon the diameter of shaft.

Shaft diameter	No. of bolts
35 to 55	4
56 to 150	6
151 to 230	8
231 to 390	10
Above 390	12

- The other **proportions** for the marine type flange coupling
- Thickness of flange = $d / 3$
- Taper of bolt= 1 in 20 to 1 in 40
- Pitch circle diameter of bolts, $D_1 = 1.6d$
- Outside diameter of flange, $D_2 = 2.2d$

Design a cast iron protective type flange coupling to transmit 15 kW at 900 rpm. from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used : Shear stress for shaft, bolt and key material = 40 MPa Crushing stress for bolt and key = 80 MPa Shear stress for cast iron = 8 MPa Draw a neat sketch of the coupling.

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 900$ r.p.m. ; Service factor = 1.35 ; $\tau_s = \tau_b = \tau_k = 40$ MPa $= 40$ N/mm^2; $\sigma_{cb} = \sigma_{ck} = 80$ MPa $= 80$ N/mm^2 ; $\tau_c = 8$ MPa $= 8$ N/mm^2

1. *Design for hub*

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft,

$$T_{max} = 1.35 \times 159.13 = 215 \text{ N-m} = 215 \times 10^3 \text{ N-mm}$$

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3$ or $d = 30.1$ say 35 mm **Ans.**

We know that outer diameter of the hub,

$$D = 2d = 2 \times 35 = 70 \text{ mm}$$ **Ans.**

and length of hub, $L = 1.5\, d = 1.5 \times 35 = 52.5$ mm **Ans.**

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\,147\, \tau_c$$

$\therefore$ $\tau_c = 215 \times 10^3 / 63\,147 = 3.4 \text{ N/mm}^2 = 3.4 \text{ MPa}$

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than the permissible value of 8 MPa, therefore the design of hub is safe.

2. *Design for key*

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 35 mm diameter,

Width of key, $w = 12$ mm **Ans.**

and thickness of key, $t = w = 12$ mm **Ans.**

The length of key (l) is taken equal to the length of hub.

$\therefore$ $l = L = 52.5$ mm **Ans.**

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\ 025\ \tau_k$$

$$\therefore \quad \tau_k = 215 \times 10^3 / 11\ 025 = 19.5 \text{ N/mm}^2 = 19.5 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\ \sigma_{ck}$$

$$\therefore \quad \sigma_{ck} = 215 \times 10^3 / 5512.5 = 39 \text{ N/mm}^2 = 39 \text{ MPa}$$

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. *Design for flange*

The thickness of flange (t_f) is taken as 0.5 d.

$$\therefore \quad t_f = 0.5\, d = 0.5 \times 35 = 17.5 \text{ mm} \quad \textbf{Ans.}$$

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\,713\, \tau_c$$

$$\therefore \quad \tau_c = 215 \times 10^3 / 134\,713 = 1.6 \text{ N/mm}^2 = 1.6 \text{ MPa}$$

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. *Design for bolts*

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

$$n = 3$$

and pitch circle diameter of bolts,

$$D_1 = 3d = 3 \times 35 = 105 \text{ mm}$$

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi}{4}(d_1)^2\ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2\ 40 \times 3 \times \frac{105}{2} = 4950\ (d_1)^2$$

$$\therefore \quad (d_1)^2 = 215 \times 10^3/4950 = 43.43 \text{ or } d_1 = 6.6 \text{ mm}$$

Assuming coarse threads, the nearest standard size of bolt is M 8. **Ans.**

Other proportions of the flange are taken as follows :

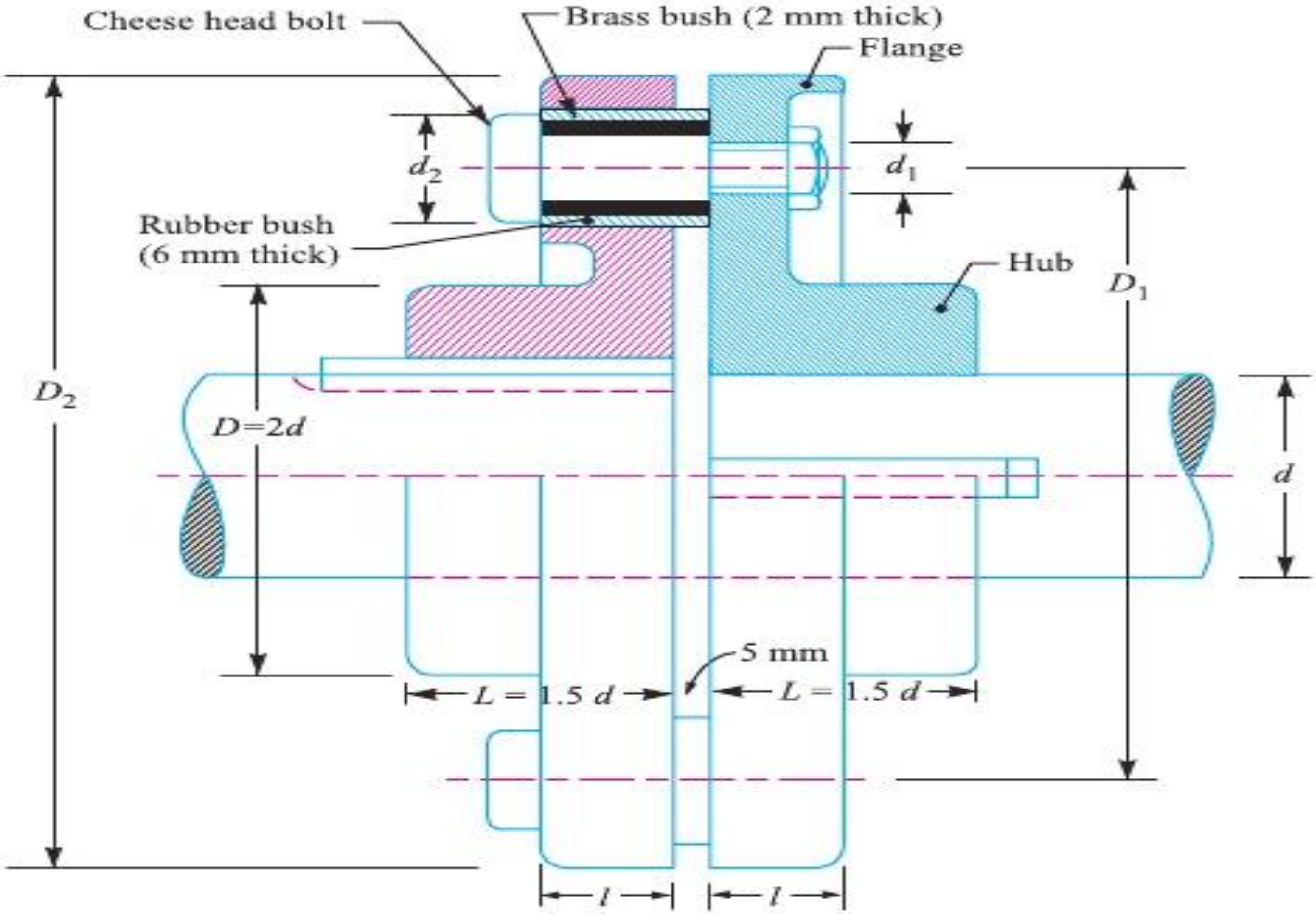
Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 35 = 140 \text{ mm} \quad \textbf{Ans.}$$

Thickness of the protective circumferential flange,

$$t_p = 0.25\,d = 0.25 \times 35 = 8.75 \text{ say } 10 \text{ mm} \quad \textbf{Ans.}$$

Bushed-pin Flexible Coupling



It is a modification of the rigid type of flange coupling.

- The coupling bolts are known as pins. The rubber or leather bushes are used over the pins.
- The two halves of the coupling are dissimilar in construction.
- A clearance of 5 mm is left between the face of the two halves of the coupling.

- the bearing pressure on the rubber or leather bushes and it should not exceed 0.5 N/mm^2

Pin and bush design

- l=Length of bush in the flange,
- d_2=Diameter of bush,
- p_b=Bearing pressure on the bush or pin,
- n=Number of pins,
- D_1=Diameter of pitch circle of the pins

Pin and bush design

- bearing load acting on each pin,

  $W = p_b \times d_2 \times l$

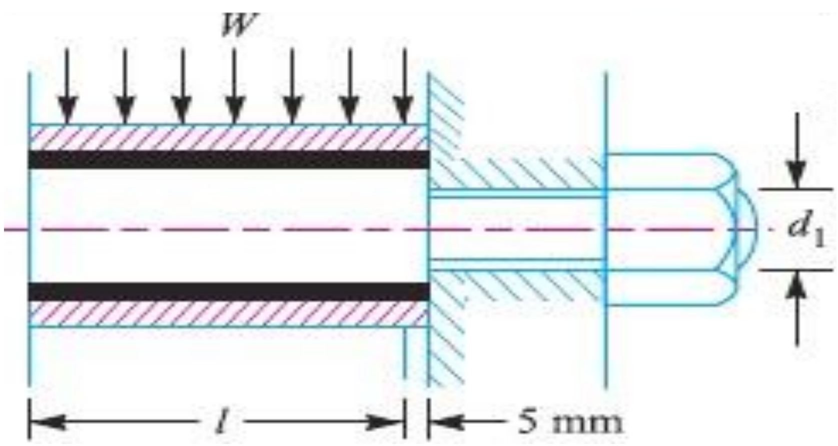
- ∴Total bearing load on the bush or pins

  $W \times n = p_b \times d_2 \times l \times n$

- torque transmitted by the coupling=

  $T = W \times n \times (D_1/2)$

  $T = p_b \times d_2 \times l \times n \times (D_1/2)$



- Direct shear stress due to pure torsion in the coupling halve

$$\tau = W / [(\pi/4)(d_1^2)]$$

- maximum bending moment on the pin

$$M = W (l/2 + 5mm)$$

- bending stress

$$\sigma = M / Z$$

$$= W (l/2 + 5mm) / (\pi/32)(d_1^3)$$

- Maximum principal stress

$$= 1/2[\sigma + (\sigma + 4\tau^2)^{1/2}]$$

- maximum shear stress on the pin

$$= 1/2(\sigma + 4\tau^2)^{1/2}$$

- The value of maximum principal stress varies from 28 to <u>42 MPa</u>

Design a bushed-pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32 kW at 960 rpm. The overall torque is 20 percent more than mean torque. The material properties are as follows :

(a) The allowable shear and crushing stress for shaft and key material is 40 MPa and 80 MPa respectively.

(b) The allowable shear stress for cast iron is 15 MPa.

(c) The allowable bearing pressure for rubber bush is 0.8 N/mm^2.

(d) The material of the pin is same as that of shaft and key. Draw neat sketch of the coupling

Solution. Given : $P = 32$ kW $= 32 \times 10^3$ W ; $N = 960$ r.p.m. ; $T_{max} = 1.2\ T_{mean}$; $\tau_s = \tau_k = 40$ MPa $= 40$ N/mm^2 ; $\sigma_{cs} = \sigma_{ck} = 80$ MPa $= 80$ N/mm^2 ; $\tau_c = 15$ MPa $= 15$ N/mm^2 ; $p_b = 0.8$ N/mm^2

1. *Design for pins and rubber bush*

First of all, let us find the diameter of the shaft (*d*). We know that the mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{32 \times 10^3 \times 60}{2\pi \times 960} = 318.3 \text{ N-m}$$

and the maximum or overall torque transmitted,

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 318.3 = 382 \text{ N-m} = 382 \times 10^3 \text{ N-mm}$$

We also know that the maximum torque transmitted by the shaft (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\ d^3$$

$$\therefore \qquad d^3 = 382 \times 10^3 / 7.86 = 48.6 \times 10^3 \text{ or } d = 36.5 \text{ say } 40 \text{ mm}$$

We have discussed in rigid type of flange coupling that the number of bolts for 40 mm diameter shaft are 3. In the flexible coupling, we shall use the number of pins (*n*) as 6.

$$\therefore \qquad \text{Diameter of pins, } d_1 = \frac{0.5\,d}{\sqrt{n}} = \frac{0.5 \times 40}{\sqrt{6}} = 8.2 \text{ mm}$$

In order to allow for the bending stress induced due to the compressibility of the rubber bush, the diameter of the pin (d_1) may be taken as 20 mm. **Ans.**

The length of the pin of least diameter *i.e.* $d_1 = 20$ mm is threaded and secured in the right hand coupling half by a standard nut and washer. The enlarged portion of the pin which is in the left hand coupling half is made of 24 mm diameter. On the enlarged portion, a brass bush of thickness 2 mm is pressed. A brass bush carries a rubber bush. Assume the thickness of rubber bush as 6 mm.

∴ Overall diameter of rubber bush,

$$d_2 = 24 + 2 \times 2 + 2 \times 6 = 40 \text{ mm} \quad \textbf{Ans.}$$

and diameter of the pitch circle of the pins,

$$D_1 = 2\,d + d_2 + 2 \times 6 = 2 \times 40 + 40 + 12 = 132 \text{ mm} \quad \textbf{Ans.}$$

Let l = Length of the bush in the flange.

We know that the bearing load acting on each pin,

$$W = p_b \times d_2 \times l = 0.8 \times 40 \times l = 32\,l \text{ N}$$

and the maximum torque transmitted by the coupling (T_{max}),

$$382 \times 10^3 = W \times n \times \frac{D_1}{2} = 32\,l \times 6 \times \frac{132}{2} = 12\,672\,l$$

$$\therefore \quad l = 382 \times 10^3 / 12\,672 = 30.1 \text{ say } 32 \text{ mm}$$

and $$W = 32\,l = 32 \times 32 = 1024 \text{ N}$$

∴ Direct stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2} = \frac{1024}{\frac{\pi}{4}(20)^2} = 3.26 \text{ N/mm}^2$$

Since the pin and the rubber bush are not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin. Assuming a uniform distribution of load (W) along the bush, the maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5\right) = 1024\left(\frac{32}{2} + 5\right) = 21\ 504 \text{ N-mm}$$

and section modulus, $$Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(20)^3 = 785.5 \text{ mm}^3$$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{21\ 504}{785.5} = 27.4 \text{ N/mm}^2$$

∴ Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[27.4 + \sqrt{(27.4)^2 + 4\,(3.26)^2}\right]$$

$$= 13.7 + 14.1 = 27.8 \text{ N/mm}^2$$

and maximum shear stress

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(27.4)^2 + 4\,(3.26)^2}\right] = 14.1 \text{ N/mm}^2$$

Since the maximum principal stress and maximum shear stress are within limits, therefore the design is safe.

2. *Design for hub*

We know that the outer diameter of the hub,

$$D = 2\,d = 2 \times 40 = 80 \text{ mm}$$

and length of hub, $L = 1.5\,d = 1.5 \times 40 = 60 \text{ mm}$

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(80)^4 - (40)^4}{80}\right] = 94.26 \times 10^3\ \tau_c$$

$$\therefore \quad \tau_c = 382 \times 10^3 / 94.26 \times 10^3 = 4.05 \text{ N/mm}^2 = 4.05 \text{ MPa}$$

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than the permissible value of 15 MPa, therefore the design of hub is safe.

3. *Design for key*

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\,\tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 40 mm diameter,

Width of key, $w = 14$ mm **Ans.**

and thickness of key, $t = w = 14$ mm **Ans.**

The length of key (L) is taken equal to the length of hub, *i.e.*

$$L = 1.5\,d = 1.5 \times 40 = 60 \text{ mm}$$

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times w \times \tau_k \times \frac{d}{2} = 60 \times 14 \times \tau_k \times \frac{40}{2} = 16\,800\,\tau_k$$

$$\therefore \quad \tau_k = 382 \times 10^3/16\,800 = 22.74 \text{ N/mm}^2 = 22.74 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 60 \times \frac{14}{2} \times \sigma_{ck} \times \frac{40}{2} = 8400\,\sigma_{ck}$$

$$\therefore \quad \sigma_{ck} = 382 \times 10^3/8400 = 45.48 \text{ N/mm}^2 = 45.48 \text{ MPa}$$

Since the induced shear and crushing stress in the key are less than the permissible stresses of 40 MPa and 80 MPa respectively, therefore the design for key is safe.

4. *Design for flange*

The thickness of flange (t_f) is taken as 0.5 *d*.

$$\therefore \quad t_f = 0.5\, d = 0.5 \times 40 = 20 \text{ mm}$$

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (80)^2}{2} \times \tau_c \times 20 = 201 \times 10^3 \, \tau_c$$

$$\therefore \quad \tau_c = 382 \times 10^3 / 201 \times 10^3 = 1.9 \text{ N/mm}^2 = 1.9 \text{ MPa}$$

Since the induced shear stress in the flange of cast iron is less than 15 MPa, therefore the design of flange is safe.