Principle Stress & Principle Plane

We already discussed the direct tensile and compressive stress as well as shear and also have referred the stress in the plane which is at right angle to the line of action of force. But it have observed that any point in the strained materials there are 3 planes mutually perpendicular to each other which carry direct stress and has no shear stress. It may be noted that out of three stresses one will be maximum, other is minimum. The perpendicular plane which has no shear stress are known as principle plane and direct stress along this plane is known as principle stress.

$$\sigma = \sigma_t + \sigma_b.$$

$$\sigma_t = P/A$$

$$\sigma_b = M/Z$$

$$\tau = 16T/\pi d^3.$$

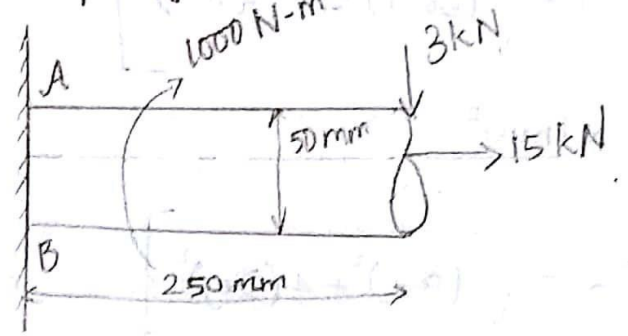
σ_1 – maximum principle stress

$$\sigma_1 = \frac{1}{2}\left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4(\tau_{xy})^2}\right]$$

$$\sigma_2 = \frac{1}{2}\left[(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4(\tau_{xy})^2}\right]$$

$$\tau = \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4(\tau_{xy})^2}$$

1. A shaft is shown in figure is subjected to bending load of 3 kN and pure torque of 1000 Nm. Axial pulley force of 15 kN



$$\sigma_{tA} = P/A$$

$$= \frac{15 \times 10^3}{\pi/4 \times (50)^2}$$

$$= 7.63 \ N/mm^2$$

$$\sigma_{bA} = M/Z$$

$$= \frac{3 \times 10^3 \times 250}{\pi/32 \ (50)^3}$$

$$= 61.1 \ N/mm^2$$

$$T = \frac{\pi}{16} \tau \times d^3$$

$$\tau_{xy} = \frac{16T}{\pi d^3}$$

$$= \frac{16 \times 1000 \times 10^3}{\pi \times (50)^3}$$

$$= 40.7 \ N/mm^2$$

$$\sigma_{xA} = \sigma_t + \sigma_b$$

$$= 68.73 \ N/mm^2$$

$$\sigma_{max\,1A} = \frac{1}{2}\left[\sigma_x + \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2}\right]$$

$$= \frac{1}{2}\left[68.7 + \sqrt{(68.7)^2 + 4(40.7)^2}\right]$$

$$= 87.60 \text{ N/mm}^2$$

$$\sigma_{min\,A} = \frac{1}{2}\left[\sigma_x - \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2}\right]$$

$$= \frac{1}{2}\left[68.7 - \sqrt{(68.7)^2 + 4(40.7)^2}\right]$$

$$= -18.9 \text{ N/mm}^2$$

$$\tau_{max(A)} = \frac{1}{2}\sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2}$$

$$= \frac{1}{2}\sqrt{(68.7)^2 + 4(40.7)^2}$$

$$= 53.29 \text{ N/mm}^2$$

At point B.

$$\sigma_{tB} = 7.63 \text{ N/mm}^2$$

$$\sigma_{bB} = -61.1 \text{ N/mm}^2$$

$$\tau_{xy}(B) = 40.7 \text{ N/mm}^2.$$

$$\sigma_{xB} = 7.63 - 61.1$$

$$= -53.47 \text{ N/mm}^2$$

$$\sigma_{max\,B} = \frac{1}{2}\left[\sigma_x + \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2}\right]$$

$$= \frac{1}{2}\left[-53.47 + \sqrt{(-53.47)^2 + 4(40.7)^2}\right]$$

$$\sigma_{max\,B} = 21.96\ N/mm^2$$

$$\sigma_{min\,B} = \frac{1}{2}\left[-53.47 - \sqrt{(-53.47)^2 + 4(40.7)^2}\right]$$

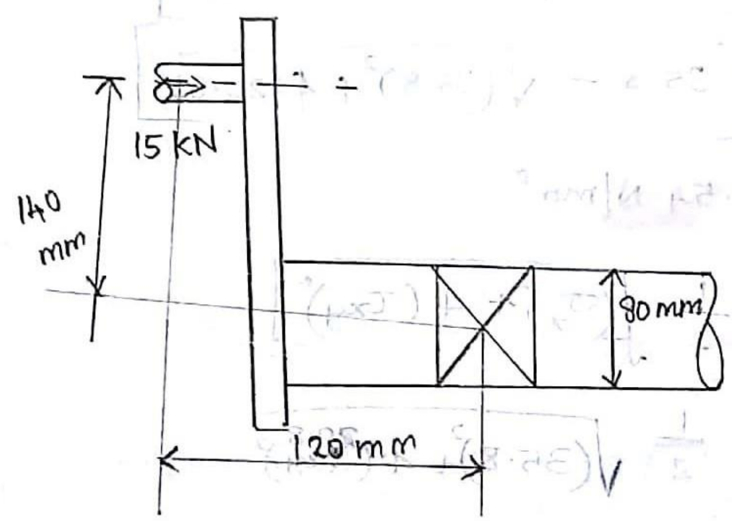
$$= -75.4\ N/mm^2.$$

$$\tau_{max\,(B)} = \frac{1}{2}\left[\sqrt{(-53.47)^2 + 4(40.7)^2}\right]$$

$$= 48.7\ N/mm^2$$

1A/7/16

2. An overhang crank with pin and shaft shown in figure. Tangential load of 15 kN act on a crank pin. Determine the maximum principal stress and minimum principal stress, max. shear stress at the centre of the crank shaft bearing.



Soln:-

$$\sigma_b = M/Z = \frac{P \times e}{Z}$$

$$= \frac{15 \times 10^3 \times 120}{\frac{\pi}{32}(80)^3} = 3.58 \times 10$$

$$= 35.8\ N/mm^2$$

$$T = 15 \times 10^3 \times 140$$
$$= 21 \times 10^5 \text{ N-mm}$$

$$\tau_{xy} = \frac{16T}{\pi d^3}$$
$$= \frac{16 \times 21 \times 10^5}{\pi \times (80)^3}$$
$$= 20.8 \text{ N/mm}^2$$

$$\sigma_x = \sigma_b$$

$$\sigma_{max} = \frac{1}{2}\left[\sigma_x + \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2}\right]$$
$$= \frac{1}{2}\left[(35.8) + \sqrt{(35.8)^2 + 4(20.8)^2}\right]$$
$$= 45.34 \text{ N/mm}^2$$

$$\sigma_{min} = \frac{1}{2}\left[\sigma_x - \sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2}\right]$$
$$= \frac{1}{2}\left[35.8 - \sqrt{(35.8)^2 + 4(20.8)^2}\right]$$
$$= -9.54 \text{ N/mm}^2$$

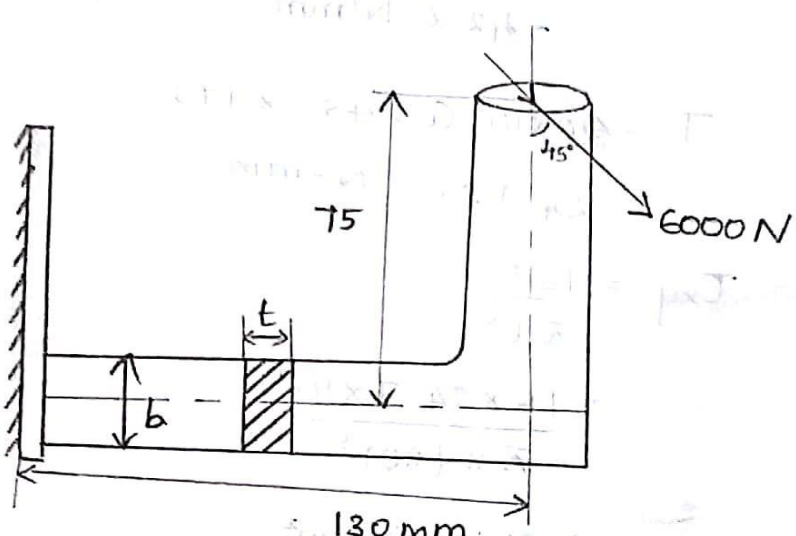
$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x)^2 + 4(\tau_{xy})^2}$$
$$= \frac{1}{2}\sqrt{(35.8)^2 + 4(20.8)^2}$$
$$= 27.4 \text{ N/mm}^2$$

$$\tau_{max} = \frac{1}{2}\sqrt{(41.2)^2 + 4(\tau_{xy})^2}$$

$$= \frac{1}{2}\sqrt{(41.2)^2 + 4(24.5)^2}$$

$$= 32.3\ N/mm^2$$

1. A mild steel bracket is shown in figure. is subjected to axial pull of 6000N acting at 45° to horizontal axis. Bracket has rectangular section whose depth is twice the thickness. Find the cross section dimension of the bracket if the permissible stress in the material of the bracket is 60 MPas.



$$\sigma_T = \sigma_{bv} + \sigma_{bH} + \sigma_d$$

PSG DB 6.2

$$\sigma_{bH} = \frac{M_H}{Z}$$

$$= \frac{6000 \cos 45^\circ \times 75}{\frac{4t^3}{6}}$$

$$= \frac{477297.07}{t^3}\ N/mm^2.$$

b d $\frac{bd^2}{6}$

t b $\frac{tb^2}{6}$

$b = 2t$

$$Z = \frac{t(2t)^2}{6} = \frac{4t^3}{6}$$

$$\sigma_d = \frac{P_v}{A}$$

$$= \frac{6000 \sin 45^\circ}{t \times 2t}$$

$$= \frac{2121.3}{t^2}$$

$$\sigma_{bv} = \frac{6000 \sin 45^\circ \times 130}{\frac{4 \cdot t^3}{6}}$$

$$= \frac{827314.9}{t^3} \ N/mm^2$$

$$60 = \frac{477297}{t^3} + \frac{827314}{t^3} + \frac{2121.3}{t^2}$$

$$\frac{1304611}{t^3} + \frac{2121.3}{t^2} = 60$$

$$\frac{21743.5}{t^3} + \frac{35.355}{t^2} = 1$$

$t = 25$

$1.44 = 1$

$t = 27$

$1.15 = 1$

$t = 28$

$1.03 \approx 1$

So thickness $t = 28$ mm

$b = 2t$

$= 56$ mm