SNS College of Technology(Autonomous)
Coimbatore-35
Academic Year 2023 – 2024 (Even)

UNIT 2 QUANTITATIVE ABILITY IV

T3: Trignometry

$$\sin\vartheta = \frac{opp}{hyp} \qquad \csc\vartheta = \frac{hyp}{opp}$$

$$\cos\vartheta = \frac{adj}{hyp} \qquad \sec\vartheta = \frac{hyp}{adj}$$

$$\tan\vartheta = \frac{opp}{adj} \qquad \cot\vartheta = \frac{adj}{opp}$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$

Reciprocal Identities

$$\sin\vartheta = \frac{1}{\csc\vartheta} \qquad \csc\vartheta = \frac{1}{\sin\vartheta}$$

$$\cos\vartheta = \frac{1}{\sec\vartheta} \qquad \sec\vartheta = \frac{1}{\cos\vartheta}$$

$$\tan\vartheta = \frac{1}{\cot\vartheta} \qquad \cot\vartheta = \frac{1}{\tan\vartheta}$$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Sum and Difference Formulas

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha\beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Important formula:-

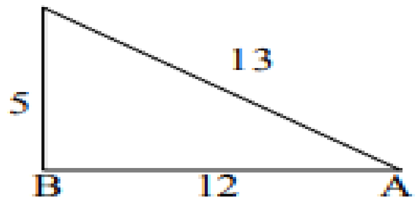
1. $\sin^2\theta + \cos^2\theta = 1$
2. $\mathrm{cosec}^2\theta - \cot^2\theta = 1$
3. $\sec^2\theta = \tan^2\theta = 1$

θ T-ratio	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\mathrm{cosec}\,\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

5. $\sin(90° - \theta) = \cos\theta$
6. $\cos(90° - \theta) = \sin\theta$
7. $\tan(90° - \theta) = \cot\theta \Rightarrow \cot(90° - \theta) = \tan\theta$
8. $\mathrm{cosec}(90° - \theta) = \sec\theta$
9. $\sec(90° - \theta) = \mathrm{cosec}\,\theta$

In a ΔABC right angled at B if AB = 12, and BC = 5 find sin A and tan A, cos C and cot C

Solution: C



$$AC = \sqrt{(AB)^2 + (BC)^2}$$
$$= \sqrt{12^2 + 5^2}$$
$$= \sqrt{144 + 25}$$
$$= \sqrt{169} = 13$$

When we consider t-ratios of∠A we have

Base AB = 12

Perpendicular = BC = 5

Hypotenuse = AC = 13

$$\sin A = \frac{\text{Perpendicular}}{Hypotenuse} = \frac{5}{13}$$
$$\tan A = \frac{\text{Perpendicular}}{Base} = \frac{5}{12}$$

When we consider t-ratios of ∠C, we have

Base = BC = 5

Perpendicular = AB = 12

Hypotenuse = AC = 13

$$\cos C = \frac{Base}{Hypotenuse} = \frac{5}{13}$$
$$\cot C = \frac{Base}{Perpendicular} = \frac{5}{12}$$

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If θ is an acute angle tan θ + cot θ=2 find the value of $\tan^7 + \theta + \cot^7$.

Solution:

$$\tan\theta + \cos\theta = 2$$

$$\tan\theta + \frac{1}{\tan\theta} = 2$$

$$\Rightarrow \tan^2\theta + 1 = 2\tan\theta$$

$$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$$

$$(\tan\theta - 1)^2 = 0$$

$$\tan\theta = 1$$

$$\theta = 45°$$

Now, $\tan^7\theta + \cot^7\theta$.

$$= \tan^7 45° + \cot^7 45°$$

$$= 1 + 1 = 2$$

Find the value of $\frac{\cos 37°}{\sin 53°}$

Solution:

We have

$$\frac{\cos 37°}{\sin 53°} = \frac{\cos(90° - 53°)}{\sin 53°} = \frac{\sin 53°}{\sin 53°} = 1$$

If $\tan 2\theta = \cot(\theta + 6°)$, where 2θ and $\theta + 6°$ are acute angles find the value of θ.

Solution:

We have

$$\tan 2\theta = \cot(\theta + 6°)$$
$$\cot(90° - 2\theta) = \cot(\theta + 6°)$$
$$90 - 2\theta = \theta + 6°$$
$$3\theta = 84°$$
$$\theta = 28°$$

Example 14:

Find the value of $(1 - \sin^2\theta)\sec^2\theta$.

Solution:

We have,

$$(1 - \sin^2\theta)(\sec^2\theta)$$
$$= \cos^2\theta \sec^2\theta$$
$$= \cos^2\theta \times \frac{1}{\cos^2\theta}$$

$= 1$

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Find the value of $[(1 + \cot\theta) - \csc\theta][1 + \tan\theta + \sec\theta]$

Solution:

$= (1 + \cot\theta - \csc\theta)(1 + \tan\theta + \sec\theta)$

$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$
$$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$$
$$= \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta\cos\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$
$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta} = \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$